Introduction to Heat Transfer

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Nomenclature

\begin{itemize}
\item \(A\) \hspace{1em} area \([\text{m}^2]\)
\item \(a\) \hspace{1em} thermal diffusivity \([\text{m}^2/\text{s}]\), (eq. (4.5))
\item \(a_\lambda\) \hspace{1em} monochromatic absorption coefficient \([-]\), (eq. (12.48))
\item \(\text{Bi}\) \hspace{1em} Biot number \([-]\), (eq. (4.5))
\item \(\text{Bo}\) \hspace{1em} boiling number \([-]\), (eq. (14.69))
\item \(b\) \hspace{1em} length, thickness \([\text{m}]\)
\item \(C\) \hspace{1em} heat capacity flow rate \([\text{W/K}]\), (eq. (15.4))
\item \(C_D\) \hspace{1em} drag coefficient \([-]\), (eq. (11.1))
\item \(C_F\) \hspace{1em} shear stress coefficient \([-]\), (eq. (7.30))
\item \(C_p\) \hspace{1em} pressure coefficient \([-]\), Fig. 11.3
\item \(c\) \hspace{1em} specific heat \([\text{J/(kg K)}]\)
\item \(c\) \hspace{1em} propagation velocity for electromagnetic wave motion \([\text{m/s}]\)
\item \(c_p\) \hspace{1em} specific heat at constant pressure \([\text{J/(kg K)}]\)
\item \(D\) \hspace{1em} diameter \([\text{m}]\)
\item \(D_h\) \hspace{1em} hydraulic diameter \([\text{m}]\), (eq. (8.8))
\item \(d_{ij}\) \hspace{1em} devi\c{a}tric stress tensor \([\text{N/m}^2]\), (eq. (6.12))
\item \(E\) \hspace{1em} energy \([\text{J}]\)
\item \(\dot{E}\) \hspace{1em} energy per time unit \([\text{W}]\)
\item \(E\) \hspace{1em} emitted radiation energy, emissive power \([\text{W/m}^2]\)
\item \(E\) \hspace{1em} voltage \([\text{V}]\)
\item \(E_B\) \hspace{1em} emitted black body radiation \([\text{W/m}^2]\), (eq. (12.10))
\item \(E_\lambda\) \hspace{1em} emitted monochromatic radiation emissive power \([\text{W/(m}^2\text{m)}]\)
\item \(e\) \hspace{1em} specific energy \([\text{J/kg}]\)
\item \(e_{ij}\) \hspace{1em} rate of strain tensor \([1/\text{s}]\), (eq. (6.15))
\item \(F\) \hspace{1em} force \([\text{N}]\)
\item \(F\) \hspace{1em} correction factor \([-]\)
\item \(F_{ij}\) \hspace{1em} angle factor, view or shape factor \([-]\), (eq. (12.26))
\item \(F_0\) \hspace{1em} Fourier number \([-]\), (eq. (4.5))
\item \(F_r\) \hspace{1em} Froude number \([-]\), (eq. (14.56))
\item \(f\) \hspace{1em} Darcy friction factor \([-]\), (eq. (8.6))
\item \(f_s\) \hspace{1em} vortex frequency \([\text{s}^{-1}]\), (eq. (11.2))
\item \(G\) \hspace{1em} incident radiation \([\text{W/m}^2]\), (eq. (12.5))
\item \(G\) \hspace{1em} average mass velocity \([\text{kg/(m}^2\text{s)}]\)
\item \(\text{Gr}\) \hspace{1em} Grashof number \([-]\), (eq. (10.4))
\end{itemize}
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Equation</th>
</tr>
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<tr>
<td>$Gr^*$</td>
<td>modified Grashof number [–], (eq. (10.51))</td>
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<tr>
<td>$g$</td>
<td>gravity constant [m/s²]</td>
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<tr>
<td>$H$</td>
<td>enthalpy [J]</td>
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<td>$\dot{H}$</td>
<td>enthalpy per time unit [W]</td>
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<tr>
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<td>specific enthalpy [J/kg]</td>
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<tr>
<td>$h_{fg}$</td>
<td>latent heat [J/kg]</td>
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<tr>
<td>$I$</td>
<td>radiation intensity [W/m² sr], (eq. (12.20))</td>
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<td>$I$</td>
<td>current [A]</td>
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<td>$I$</td>
<td>momentum [kg m/s]</td>
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<tr>
<td>$i, j, k$</td>
<td>unit vectors</td>
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<tr>
<td>$J$</td>
<td>radiosity [W/m²], (eq. (12.5))</td>
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<td>$Ja$</td>
<td>Jakob number, (eq. (13.26))</td>
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<td>$j$</td>
<td>Colburn factor [–], $St \cdot Pr^{2/3}$</td>
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<td>$L$</td>
<td>length, thickness [m]</td>
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<td>$L_i$</td>
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<td>$LMTD$</td>
<td>logarithmic mean temperature difference [K], [°C], (eq. (15.9))</td>
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<td>$l_m$</td>
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<tr>
<td>$M$</td>
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<tr>
<td>$m$</td>
<td>mass [kg]</td>
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<tr>
<td>$\dot{m}$</td>
<td>mass flow rate [kg/s]</td>
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<tr>
<td>$N$</td>
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<td>number of transfer units [–], (eq. (15.17))</td>
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<td>$Nu$</td>
<td>Nusselt number [–], (eq. (6.62))</td>
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<td>$n$</td>
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<td>$P$</td>
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<td>$P_{R}$</td>
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<td>pressure [Pa]</td>
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<td>$p'$</td>
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<td>heat transfer rate [W]</td>
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<td>$q$</td>
<td>heat flux [W/m²]</td>
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<td>$R$</td>
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<td>surface roughness [μm]</td>
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<td>$Re$</td>
<td>Reynolds number [–], (eq. (6.60))</td>
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<td>$r$</td>
<td>radius [m]</td>
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<tr>
<td>$r_s$</td>
<td>latent heat melting [J/kg]</td>
<td></td>
</tr>
<tr>
<td>$S_L$</td>
<td>lateral tube pitch [m]</td>
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</table>
\( S_T \) longitudinal tube pitch [m]

\( S_r \) Strouhal number [–], (eq. (11.2))

\( S_T \) Stanton number [–], (eq. (7.42))

\( T \) absolute temperature [K]

\( T^+ \) dimensionless temperature [–], (eq. (9.82))

\( T_R \) thermal resistance [K/W], (eq. (15.1))

\( t \) temperature [°C]

\( t' \) fluctuating temperature [°C]

\( \bar{t} \) time averaged temperature [°C]

\( U \) internal energy [J]

\( \dot{U} \) internal energy per time unit (effekt) [W]

\( U \) (mean) velocity [m/s]

\( U_\infty \) freestream velocity [m/s]

\( U \) overall heat transfer coefficient [W/(m² K)], (eq. (15.1))

\( u, v, w \) local velocity [m/s]

\( u', v', w' \) fluctuating velocity [m/s]

\( \bar{u}, \bar{v}, \bar{w} \) time averaged velocity [m/s], (eq. (9.4))

\( u^+ \) dimensionless velocity [–]

\( u_{FS} \) fictious liquid velocity [m/s], (eq. (14.43))

\( u_{GS} \) fictious gas velocity [m/s], (eq. (14.44))

\( u_m \) mean velocity [m/s]

\( u_f, u^* \) friction velocity [m/s]

\( V \) volume [m³]

\( X \) coordinate [m]

\( X \) Martinelli parameter [–], (eq. (14.52))

\( X_F \) flowing mass fraction [–], (eq. (14.40))

\( X_S \) static mass fraction [–], (eq. (14.41))

\( x, y, z \) coordinates [m]

\( y^+ \) dimensionless coordinate perpendicular to a solid surface [–], (eq. (9.45))

\( W \) work [J]

\( \dot{W} \) work per time unit [W]

\( W_e \) Weber number [–], (eq. (14.23))

\( Z \) width, length [m]

\( \alpha \) heat transfer coefficient [W/(m² K)], (eq. (1.23))

\( \alpha \) absorptance [–], (eq. (12.3))

\( \beta \) thermal expansion coefficient [1/K]

\( \beta \) angle [rad]

\( \gamma \) ratio of specific heats, \( c_p/c_v \) [–]

\( \delta \) angle [rad]

\( \delta \) boundary layer thickness [m]

\( \delta_{ij} \) Kronecker’s delta [–], (eq. (6.12))

\( \varepsilon \) emissivity [–], (eq. (12.11))

\( \varepsilon \) void [–], (eq. (14.39))

\( \varepsilon \) efficiency [–], (eq. (15.14))
εₘ turbulent kinematic viscosity [m²/s], (eq. (9.31))
ε₉ turbulent diffusivity [m²/s], (eq. (9.32))
η dimensionless coordinate [-]
η fin effectiveness [-], (eq. (3.48))
θ dimensionless temperature [-]
θ angle [rad]
θ thermal length [-], (eq. (15.43))
θ temperature [°C]
κ von Karmans constant [-], (eq. (9.51))
λ parameter [-]
λ thermal conductivity [W/(m K)], (eq. (1.1))
λ wavelength [m]
λ₁ mean free path for molecular motion [m]
μ dynamic viscosity [kg/(m s)]
ν kinematic viscosity [m²/s]
ρ density [kg/m³]
ρ reflectance [-], (eq. (12.3))
ρ resistivity [Ω m]
σ Stefan-Boltzmann constant [W/(m² K⁴)], (eq. (12.10))
σ shear stress [N/m²]
σ surface tension [N/m]
σ electric conductivity [1/(Ω m)]
σ area ratio [-], (eq. (15.35))
τ time [s]
τ transmittance [-], (eq. (12.3))
φ angle [rad]
ϕ fin efficiency [-], (eq. (3.49))
Φ radiation energy [W/m²], (eq. (12.21))
ψ stream function [s⁻¹], (eq. (10.21))
ω solid angle [sr], (eq. (12.16))

Index
B bulk, blackbody
C convective boiling
f at film temperature, fluid
g gas
i inner
KK nucleate boiling
m mean
o outer
TF two-phase
w wall
λ monochromatic
∞ freestream
Preface

This book aims as an introduction of heat transfer at undergraduate and graduate levels. Compared to other similar textbooks it differs significantly as it is much more comprehensive in describing the thermal conductivity of various substances, providing deeper analysis of fin heat transfer, and it includes buried pipes. For convective heat transfer the relation to fluid mechanics is much more highlighted. The turbulent convection, evaporation and heat exchanger chapters are all more comprehensive than other general heat transfer textbooks available. The textbook has been used for exchange students and PhD students for several years. The text has been developed and improved over the years.

At the end of the book a number of problems in heat transfer can be found. These can be solved by the methods presented in the various chapters.

Lund in September 2011

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