

Fundamentals of Wavelets

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Preface

Although wavelets analysis is widespread in science and engineering research, the users fail to understand, in many cases, their fundamental concepts. This involves the reasons why wavelets are appropriate to solve certain problems and how and when they should be applied.

This book is designed to answer these questions.

First - In the first place it discusses the fundamentals of wavelets concepts in a way that can be understood by researchers in a variety of disciplines, who are not expert in numerical analysis.

Second – The book also shows how wavelets can filter wave frequencies and can be used as tools for the analysis of a variety of macro and microscopic phenomena. The emphasis is in real world applications in different scientific disciplines without the researcher needing to have detailed theoretical knowledge of the principles.

Third – The reader is then introduced to the basic computational techniques applied in wavelets, to provide the basic knowledge behind the ready made software programs available in the market place.

Fourth – The book has been written for use in different scientific disciplines and concentrates on the most up to date aspects of the theory, without attempting to present a detailed mathematical treatment.

Fifth – The terminology and concepts are introduced and explained in detail for the benefit of all users, as well as mathematicians who may be unaware of their meaning, i.e. words such as: filtering, low-pass, band-pass, high-pass, cost functions and many others.

Sixth – Detailed proofs of the main results are given to provide the researcher with a

clearer understanding of the issues involved. For instance in the case of a series of functions to be integrated, it is shown how each function can be integrated separately and the results added together, using simple mathematics.

Seventh – The text is written using every day words which try to explain all concepts in the simplest terms. The definition of an integral for instance is presented as a continuous sum, introducing the concept of effective intervals and effective parts of a function in Chapter 1 in similar terms. In Chapter 2 the explanation is given of why a wavelet transform automatically has the property to change focal points and can capture a variety of signals. Although the explanation may not always be strictly accurate from the mathematical point of view, they are useful to explain the fundamental concepts to non mathematicians.

Eighth – The book finally presents many practical applications resulting from the authors research work. These include the constructions of compactly supported real wavelets of Chapter 4; the representations of wavelet packet, equal dilation property and choices of the best wavelet packet bases of Chapter 5, the proof of theorem and algorithms in chapter 6 and the proofs of multivariate wavelets and construction of biorthogonal wavelets in Chapter 7 and 9.

The material presented in this book has been taught at graduate level over a period of several years. The authors were inspired by valuable suggestions made by several graduate students; Yin Juncheng, Liang Bing, Wang Li, Yao Suxia, Jiao Shuyun, Shi Zhihui, Song Congwei, Liang Jing, Feng Chengxiang, Hu Ai, Wang Xuefen, Yin Yanni and He Junyun.

I am particularly indebted to my daughter Dr. Di Jing for the help with the English version of this book.

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