

# Use of the graph model for conflict resolution in water resources problems in Brazil

V. de Fátima Malta, J. M. Damázio & P. C. de Magalhães  
*Water Resources Division of the Postgraduate Department of Civil Engineering, COPPE, Federal University of Rio de Janeiro, Brazil*

## Abstract

This paper presents an application of the Graph Model for Conflict Resolution (GMCR), developed by Fang et al. [1], for water resources management problems in Brazil. The main aspects of the formal development of the GMCR model are described in terms of its basic components (decision makers, options, strategies and preferences), key concepts (stable states, equilibrium states and stability criteria), and mathematical representation of conflicts by sets of oriented graphs and payoff functions. The equations for the stability analysis of conflicts with at least two decision makers for different stability criteria are provided. The model is used to analyze the importance of the water management institutional system in the solution of a conflict over the use of the water in the Lima Campos/Orós system of reservoirs located in the Northeast of Brazil.

*Keywords: conflict analysis, water resources management, game theory, graph theory, Graph Model for Conflict Resolution (GMCR).*

## 1 Introduction

It is well known that the Brazilian territory is covered by several big rivers, and presents an average surface water production of approximately  $250.000\text{m}^3/\text{s}$  (the whole Europe produces around  $100.000\text{m}^3/\text{s}$ ). In the Northeast of Brazil, the value of production of surface water is low, and even in the most humid areas long periods of shortage occur frequently. Despite its large amount of surface water, economic and social development in Brazil faces not only shortage of financial resources, but also problems due to multiple use of water (Malta et al. [2]).



Traditionally it has been recommended that studies and analysis aiming the use of water resources follow the next steps: i) definition of the objectives, ii) formulation of quantitative measures of the objectives, iii) generation of alternatives, iv) quantification of the alternatives, and v) selection of the optimal alternative. This traditional systematic approach (e.g. Braga [3]), although it considers the decision making with multiple objectives, assumes ideal situations of problems of decision, where there is only one decision maker, who is responsible for the choice of the best distribution of water uses.

A closer representation of the reality of a more democratic world, according to Brazilian Water Act (law 9.433/97), would be to consider multiple willing decision makers to negotiate a compromise solution. Accordingly there is room for the development and application of techniques of the called Game Theory. The Game Theory dates back to the works by Fermat in 1654 in living room games, and had its more modern base developed by von Neumann [4]. Since this classical work by von Neumann [4], several developments increased significantly the ranger of topics and subtopics dealt with by the Game Theory. The conflict model, developed first by Howard [5], is one of the several branches of the Game Theory.

## 2 Conflict and its modeling

A conflict is a situation in which two or more groups are in dispute over some issue. An example of conflict involving water resources in Brazil is the dispute for the use of water in the Jaguaribe river basin, state of Ceará. Another example is the dispute for the use of electric energy potentials in the Xingu River, a tributary of the Amazon River, that faces the interests of the Indians who live in the area and of the national and international environmentalists.

The objective of the conflict modeling is to supply approximate definitions of real conflicts, highlighting their main features and representing them through a formal mathematical structure. Thus, a conflict model is a general tool for the systematic study of current, historical or hypothetical disputes. For a current dispute the conflict model can simulate the possible movements and counter-movements of each decision maker and to foresee the possible solutions for the conflict. The results of the analysis can be used, for example, to give support to the decision makers in the conflict.

There are several Conflict Models in the literature that differ mainly as for the manner that they represent the main features of a dispute. Howard [5] created the model called Option Form, where the possible actions (options) of every decision makers are used for the model building. In the model by von Neumann and Morgenstein [6], commonly called Normal Form, the indivisible units are the possible combinations of actions of the decision makers (strategies). Kilgour et al [7] improved to the Conflict Analysis of the Graph Theory, creating the called Graph Model of Conflict Resolution or GMCR.

According to the classification criteria of game models, the model GMCR can be classified as a model that is capable of considering more than two decision makers and with any finite number of actions for the decision makers. The



GMCR uses ordinal preferences and it assumes complete and perfect information, in other words, all information is available for all the decision makers. The GMCR is basically a model for non-cooperative games but it can model some cooperation types (bargain and negotiations).

### 3 Components of a conflict model

The conflict model is a systematic structure for describing the main features of a current, historical or hypothetical conflict. The three main components of a conflict model are the decision makers, their options and preferences. In the construction of a conflict model, we would initially to define the group of decision makers who are in disagreement over some issue. A decision maker can consist of a single person or a group of people potentially beneficiaries or harmed in some way by for the possible solutions of the conflict. We can consider as examples of institutions in Brazil, the state representatives and blocks with corporate interests in the National Congress, the users' of water representatives in the Committees of Basin, the Unions, the users of water representatives in the basin committees, the commissions of having reached by dam, etc. We point that in the principles of sustainable development proclaimed in the Dublin Declaration, the water users participation is included in the processes of water resources management. In Brazil, Law 9.433 was instituted as the foundation of the National Water Resources Policy, with participation of the Government, users and communities.

The options of a decision maker are the actions that he can or cannot take in a conflict. The strategy of a given decision maker is his decision making with respect to which options to take and which not to take. The set of available strategies for a decision maker is in principle given by the set of all combinations of his decisions in respect to every option. The GMCR model accepts that the decision makers may change their strategy along the evolution of the conflict, and every time some decision maker or group change his strategy, it is said that the conflict change its stage. The state of a conflict in a certain stage is defined by a set of strategies selected by every decision maker.

In a conflict, every decision maker associates the set of viable states of the conflict to a structure of preferences. In general, during the evolution of the conflict, each decision maker will act trying to change the conflict towards the state of his largest preference. A unilateral change or unilateral move occurs when a decision maker decides to move the conflict changing the selection of his strategy. When the change is made for a state of larger preference it is called unilateral improvement. A state is said to be stable for a decision maker when he does not consider advantage to move the conflict of this state through a unilateral change. If the state is stable for all decision makers, this state is a possible solution of the conflict and is called a balance state.

### 4 Stability analysis

We can calculate the stability of an action through a clear mathematical definition of the human or social behaviors in a conflict situation. The feature of



the decision maker's behavior correspond to several stability criteria used in the literature was described by Fang et al [1] according to table 1. In this table, columns 3 (foresight) and 4 (disimprovements) supply the characterization of the stability criterion, qualitatively. The characterization foresight refers to the ability of the decision maker to think about possible moves that could take place in the future. If the decision maker has a strategic behavior he can temporarily move to a worse state in order to reach a state of larger preference eventually, which is called disimprovement. Disimprovements by opponents mean that a decision maker can move to a worse state in order to block unilateral improvements taken by other decision makers.

Table 1: Solution concepts and human behavior.

Solution Concepts	References	Characteristics	
		Foresight	Disimprovements
Nash stability	Nash (1950, 1951); von Neumann e Morgenstern (1953)	Low	Never
General Metarationality	Howard (1971)	Medium	By opponents
Symmetric Metarationality	Howard (1971)	Medium	By opponents
Sequential stability	Fraser e Hipel (1979,1984)	Medium	Never
Limited-move stability ( $L_h$ )	Kilgour (1985); Kilgour, Hipel e Fang (1987); Zagare (1984)	Variable	Strategic
Nonmyopic stability	Brams and Wittman (1981); Kilgour (1984, 1985); Kilgour, et al (1987)	High	Strategic

FOUNT: Fang et al [1].

Definition of the stability criteria for two decision makers game:

*Nash Stability (R)*: Let  $i \in N$ . A state  $k \in U$  is Nash stable (or individually rational) (R) for the decision maker  $i$ , iff  $S_i^+(k) = 0$ .

*General Metarationality (GMR)*: For  $i \in N$ , a state  $k \in U$  is general metarational (GMR) for the decision maker iff for every  $k_1 \in S_i^+(k)$  at least one  $k_2 \in S_j(k_1)$  with  $P_i(k_2) \leq P_i(k)$ .

*Symmetric Metarationality (SMR)*: Let  $i \in N$ . A state  $k \in U$  is symmetric metarational (SMR) for the decision maker  $i$ , iff for every  $k_1 \in S_i^+(k)$   $k_2 \in S_j(k_1)$ , such that  $P_i(k_2) \leq P_i(k)$  and  $P_i(k_3) \leq P_i(k)$  for all  $k_3 \in S_i(k_2)$ .



*Sequential Stability (SEQ)*: Let  $i \in N$ . A state  $k \in U$  is Sequential (SEQ) for the decision maker  $i$ , iff for every  $k_1 \in S_i^+(k)$  there is  $k_2 \in S_j^+(k_1)$  with  $P_i(k_2) \leq P_i(k)$ .

*Limited-move stability ( $L_h$ )*: Let  $i \in N$ . A state  $k \in U$  is Limited-move  $i$  iff  $G_h(i,k) = k$ . The analysis of the stability  $L_h$  demands the calculation of the values of  $G_h(i,k)$ , for whole  $I \in N$  and for whole  $k \in U$ . For the calculation of  $G_h(i,k)$  first we should remind that if  $S_i(k) \neq \emptyset$  then the state  $k$  is stable  $L_h$  and therefore we needed to just verify the state  $k$  for the ones which  $S_i(k) \neq \emptyset$ . We Suppose therefore such  $k$  that  $S_i(k) \neq \emptyset$ . Let  $V_h(i,k) \in U$  the largest payoff that the decision maker  $i$  can obtain moving the conflict of the state  $k$  and  $A_i(i,k)$  the state for which he should move the conflict to obtain  $V_h(i,k)$ .

*Nonmyopic stability (NM)*: Let  $i \in N$ . A state  $k \in U$  is nonmyopic for the decision maker  $i$  iff there is positive integer  $t'$  such that  $G_t(i,k) = k$  for all  $t \geq t'$ .

## 5 The GMCR model

Let a conflict where  $N = \{1, 2, \dots, n\}$  is the set of indexes of the decision makers and  $U = \{1, 2, \dots, u\}$  the set of indexes of the states of the conflict. For each  $i$ , we can obtain a vector of preference for the states in  $U$ , also called of payoff function,  $P_i: U \rightarrow R$ , where  $R$  is the set of real numbers:

$$P_i = (P_i(1), P_i(2), \dots, P_i(u)) \quad (1)$$

In the GMCR model, the conflict is represented by a set of finite directed graphs, one for each  $i$ , denoted by  $D_i = (U, A_i)$ , with  $i \in N$ . The set of vertices  $U$  contains the possible states of the conflict. Each set of arcs  $A_i$  defines the possible unilateral moves for the decision maker  $i$ . The arc  $(k, q)$  exists in  $A_i$  if decision maker  $i$  can provoke a unilateral change in one step from state  $k$  to state  $q$ . The payoff functions represent the decision maker state ordinal preferences. If  $P_i(k) > P_i(q)$ , then for  $i$  state  $k$  is preferable to state  $q$ . The set of directed graphs and of payoff functions constitute the Graph Model of Conflict (Fang et al [1]).

We can build a reachable matrix of the unilateral movements of  $i$  as the matrix of order  $u \times u$ ,  $R_i$ , where:  $R_i(k, q) = 1$ , iff  $i$  can unilaterally move the conflict in a step from state  $k$  to state  $q$ ,  $R_i(k, q) = 0$ , otherwise and  $R_i(k, k) = 0$  by convention. An equivalent expression of the possibilities of  $i$ , are the lists of unilateral movements from state  $k$ ,  $S_i(k)$ , for all  $k \in U$ . Therefore:

$$S_i(k) = \{q: R_i(k, q) = 1\} \quad (2)$$

We also can define as unilateral improvement from a particular state, for a specific decision maker as: a better state, which he can unilaterally move. To represent unilateral improvements, each decision maker's reachable matrix  $R_i$  can be replaced by  $R_i^+$ .

$$R_i^+(k, q) = 1 \text{ if } R_i(k, q) = 1 \text{ and } P_i(q) > P_i(k); \quad (3)$$

$$R_i^+(k, q) = 0 \text{ otherwise.}$$



Similarly the decision maker's reachable lists  $S_i(k)$  can be replaced by:

$$S_i^+(k) = \{q: R_i^+(k, q) = 1\} \quad (4)$$

As an example of stability analysis consider conflicts of 2 decision makers, denoted  $i$  and  $j$ , and the problem of the decision maker  $i$  in an initial state  $k$ . If  $i$  takes the initiative and he decides to move the conflict for some state  $k1 \in S_i(k)$ , then his opponent perhaps decides to move the conflict from  $k1$ . Depending on what  $i$  hopes  $j$  can do in each  $k1 \in S_i(k)$ ,  $i$  can prefer not to move the conflict, maintaining it in the state  $k$ ; if this happens  $k$  is stable for  $i$ . If a state  $k$  is stable for the two decision makers,  $k$  is a balance, that is to say  $k$  should be persisted to happen.

## 6 The conflict of the Lima Campos/Orós Reservoir System

The GMCR model was applied to analyze the conflict in the use of the water in the Lima Campos/Orós Reservoir System located in the Northeast of Brazil, described in Furtado and Campos [8].

### 6.1 Introduction

According to Furtado and Campos [8], the Lima Campos reservoir was built by DNOCS (National Department of Works Against the Droughts), in 1932 with capacity to accumulate  $66 \times 10^6 \text{m}^3$ . The area of low water close to Lima Campos reservoir was benefited by the politics of DNOCS, for use of the water potential and use of the land. Besides an area for irrigation downstream water of the reservoir was implanted. The Orós reservoir was built by DNOCS between 1960 and 1962. The reservoir has capacity to accumulate  $1,94 \times 10^9 \text{m}^3$ , and the drainage area of the basin is  $25.000 \text{Km}^2$ . The Lima Campos/Orós tunnel was concluded in 1962. This system was conceived with the objective to become the Jaguaribe river perennial and to promote the colonization and the agricultural development of the alluvial plain of Icó (10.000ha), located downstream of the Lima Campos reservoir between the Salgado and Jaguaribe rivers.

With the beginning of the 70's drought, that lasted until the year of 1977 there was an increase of the discharge liberated by the Orós reservoir, accelerating the emptying of the reservoir, and consequently the fall of the level of water of Lima Campos reservoir. The agriculturists who plant the downstream water of the reservoir and other users of the water suffer with the shortage of the water in the drought period, and they demand a high level of Lima Campos reservoir. Although the agriculturists who plant around the reservoir although also suffer the effects of the droughts, with the maintenance of high levels in Lima Campos reservoir suffer loss of area for irrigation, with damage their agricultural production.

### 6.2 Conflict analysis

In the conflict described previously, the COGERH (Company of Water Resources Management of Ceará State) besides represented other users of the



water, acts as referee among agriculturists who plant downstream water of the reservoir, and agriculturists who plant around the reservoir. The agriculturists who plant around the reservoir, called here the  $DM_1$ , intend to maintain the volume of the reservoir in  $33 \times 10^6 m^3$  and the agriculturists who plant downstream water of the reservoir, called here the  $DM_2$ , demand  $50 \times 10^6 m^3$ . The COGERH is called here  $DM_3$ . In a meeting that lasted five hours on May 26, 1995, there was agreement of maintaining the level to a volume of  $39,5 \times 10^6 m^3$ .

For the application of the GMCR model we consider the situation before the May 1995 meeting. In spite of the existence of the real situation, the analysis presented here was done under an academic point of view in the sense that there was not the possibility to consult the decision makers that participated in this dispute as the validity of the modeling.

### 6.2.1 Decision makers, their respective *status quo* and strategies

Table 2 presents the decision makers of the conflict, their *status quo* and options. Table 3 presents the viable strategies of each decision maker in the conflict of the Lima Campos/Orós system.

Table 2: Decision makers of the conflict, their *status quo* and options.

Decision Makers	Options	<i>Status quo</i>
$DM_1$	Accept the COGERH decision	N
$DM_2$	Accept the COGERH decision	N
$DM_3$	Maintaining the volume of the reservoir "Compromise Position"	N
	To back the $DM_1$ maintaining the volume of the reservoir in $33 \times 10^6 m^3$	N
	To back the $DM_2$ maintaining the volume of the reservoir in $50 \times 10^6 m^3$	N

### 6.2.2 Possible states, reachable list of the decision makers of the conflict

The reachable list ( $S_i(q)$ ) of each decision maker was built starting from the possible movements by the decision maker of a certain state for another. We considered every movement is reversible.

Table 4 shows the possible states, the reachable lists and the preference functions for the 4 analyzed cases. In this table, the status quo is represented by the last state,  $q=16$ . The state  $q=1$  (historical result) corresponding to the real solution of the conflict solution (the compromise solution).



Table 3: Decision makers their viable strategies and *status quo*.

Decision makers	Strategies	<i>Status quo</i>
DM <sub>1</sub>	(Y), (N)	(N)
DM <sub>2</sub>	(Y), (N)	(N)
DM <sub>3</sub>	(YNN), (NYN), (NNY), (NNN)	(NNN)

Table 4: Viable states, reachable list, and payoff functions of every decision maker, for the 4 analyzed cases.

Q	States	S <sub>1</sub> (q)	S <sub>2</sub> (q)	S <sub>3</sub> (q)	P <sub>1</sub> (q)			P <sub>2</sub> (q)		P <sub>3</sub> (q)	
					Cases 1,2,3,4	Cases 1,3	Cases 2,4	Cases 1,2	Cases 3,4		
1	(Y) x (Y) x (YNN)	{9}	{5}	{2,3,4}	12	12	6	14	15		
2	(Y) x (Y) x (NYN)	{10}	{6}	{1,3,4}	16	1	1	13	14		
3	(Y) x (Y) x (NNY)	{11}	{7}	{1,2,4}	1	16	16	16	16		
4	(Y) x (Y) x (NNN)	{12}	{8}	{1,2,3}	6	6	11	15	4		
5	(Y) x (N) x (YNN)	{13}	{1}	{6,7,8}	10	11	8	7	11		
6	(Y) x (N) x (NYN)	{14}	{2}	{5,7,8}	14	3	3	4	8		
7	(Y) x (N) x (NNY)	{15}	{3}	{5,6,8}	2	15	15	9	13		
8	(Y) x (N) x (NNN)	{16}	{4}	{5,6,7}	5	8	12	12	3		
9	(N) x (Y) x (YNN)	{1}	{13}	{10,11,12}	11	10	5	6	10		
10	(N) x (Y) x (NYN)	{2}	{14}	{9,11,12}	15	2	2	5	9		
11	(N) x (Y) x (NNY)	{3}	{15}	{9,10,12}	3	14	14	8	12		
12	(N) x (Y) x (NNN)	{4}	{16}	{9,10,11}	8	5	9	11	2		
13	(N) x (N) x (YNN)	{5}	{9}	{14,15,16}	9	9	7	2	6		
14	(N) x (N) x (NYN)	{6}	{10}	{13,15,16}	13	4	4	1	5		
15	(N) x (N) x (NNY)	{7}	{11}	{13,14,16}	4	13	13	3	7		
16	(N) x (N) x (NNN)	{8}	{12}	{13,14,15}	7	7	10	10	1		

The 4 cases were obtained composing hypotheses in relation to the preferences of COGERH and of DM<sub>2</sub>, obtained 4 cases for analysis.

Case 1: Objective COGERH is to obtain an agreement between the contenders and DM<sub>2</sub> appreciates the compromise solution;

Case 2: Objective COGERH is to obtain an agreement between the contenders and DM<sub>2</sub> does not appreciate the compromise solution;





Case 3: Objective COGERH is to have a decision and  $DM_2$  appreciates the compromise solution;

Case 4: Objective COGERH is to have a decision and  $DM_2$  does not appreciate the compromise solution.

### 6.2.3 Stability analysis

Table 5 presents the stable states of the conflict Lima Campos/Orós System, in agreement with all the stability criteria, using the computational implementation of the GMCR, Fang et al [1], according to 4 considered cases.

Table 5: Stable states.

Cases	Stability Criteria								
	R	GMR	SMR	SEQ	L1	L2	L3	L4	NM
1	16	1,4,8,12,16	1,4,8,12,16	1, 16	16	1	1, 16	1, 16	1,16
2	16	1,4,8,12,16	1,4,8,12,16	16	16	16	16	16	16
3	11	1,5,6,9,11,15	1,5,6,9,11,15	1, 11	1, 11	1, 11	1, 11	1, 11	1,11
4	11	1,5,6,9,11,15	1,5,6,9,11,15	5, 11	11	11	11	11	11

One of the interests of the analysis is to verify the stability of the 4 first states of table 4, that correspond to states the conflict solution, with prominence for the state  $q=1$ , that was the historical result of the 1997 meeting. We can observe that the solution of the conflict through the historical result ( $q=1$ ) is only stable for the two cases where the  $DM_2$  appreciate the compromise solution, cases 1 and 3. However in the case 1, where the COGERH is more worried about obtaining an agreement than in taking a decision, the status quo is stable, indicating that the meeting should finish without a decision. In case 3, when the COGERH worries more in leaving the meeting with a decision, there is another stable state, besides the historical result. This state,  $q=11$ , corresponds to a decision for the highest level. The  $DM_2$  conforms (since his claims were assisted), but the  $DM_1$  doesn't. The meeting finishes with a decision, but the conflict is not solved. As this is a state of smaller preference for the COGERH than the state  $q=1$ , it is likely, that the meeting finished with the historical result. The COGERH in the beginning of the meeting is enough to announce his decision for the compromise level. In the two analyzed cases where the  $DM_2$  does not appreciate the compromise solution, there is only one stable state. When the COGERH worries more in getting an agreement, in case 2, the status quo is stable. When the COGERH worries more in leaving the meeting with a decision than with the obtaining of an agreement, case 4, the stable solution is the state  $q=11$ .



## 7 Conclusions

We stand out here the importance of a water management system, therefore this conflict was solved with the participation of the contenders, but also with the power of decision of COGERH, the agency of the State of Ceará, created with the objective "To implant a management system for superficial and underground water of the State, monitoring reservoirs and artesian wells, maintenance and operation of water workmanships and organization of the users in the 11 river basins of the Ceará. The organization and integration of the rude water users, one of the basic aspects for the success of this new politics of water resources, is carried through the creation of the Committees of Basin ". In the case of the Conflict of the Lima Campos/Orós System the pollution of waters was not considered, and not even the question of the payment for the use of the water, that is differentiated for each type of user. However these conflicts may exist in the basin and fit the COGERH, to the Basin Committees and the users, the solution them. In summary, this first inquiry about the potentialities of the use of conflict models in water resources management shows that with GMCR model is possible to identify and to generalize important feature of problems in this area.

## References

- [1] Fang, L., Hipel, K. W. & Kilgour, M., *Interactive Decision Making*, Copyright: New York, 1993.
- [2] Malta, V. F., Damázio, J. M., Magalhães, P. C., *Uso do Modelo Grafo de Solução de Conflitos em Problemas de Recursos Hídricos no Brasil. Revista Portuguesa de Recursos Hídricos*. APRH, Lisboa, vol.24, n.2, 2003.
- [3] Braga Jr., B. P. F., *Técnicas de Otimização e Simulação Aplicadas em Sistemas de Recursos Hídricos (Capítulo 5). Modelos para Gerenciamento de Recursos Hídricos*, Coleção ABRH de Recursos Hídricos. Editora ABRH, Porto Alegre, RS, Vol. I, pp. 427 – 518, 1997.
- [4] Von Neumann, J., *Zur Theorie der Gessellschaftsspiele. Mathematische Annalen*, Vol. 100, pp. 295-320, 1928.
- [5] Howard, N., *Paradoxes of Rationality*, MIT Press, Cambridge, MA, 1971.
- [6] Von Neumann & J., Morgenstern, O., *Theory of Games and Economic Behavior*, 1<sup>a</sup> ed. 1944, 3<sup>a</sup> ed 1953, Princeton University Press: Princeton, NJ, (1944, 1953).
- [7] Kilgour, D. M., Hipel, K. W. & Fang, L., *The Graph Model for Conflicts, Automática*, Vol. 23, N°1, pp. 41-55, 1987.
- [8] Furtado, L. L. S. & Campos, J. N. B., *Manejo e Cobrança de Água no Sistema de Vazanteiros e Irrigantes do Sistema Orós – Lima Campos*. In: XII Simpósio Brasileiro de Recursos Hídricos. Editora ABRH, Porto Alegre, RS, Vol. 1, pp. 9 – 15, 1997.

