Optimal pumping scheduling model for energy cost minimization: two different resolution methods

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Abstract

A large pipeline system is usually one of the components of a water distribution system. This will carry water from the sources to the distribution reservoirs, often by means of costly pumping stations, since local topography may include areas of high ground.

The pumping timetable should be designed to accommodate the fluctuation of energy costs over a twenty-four – hour period, with preference being given to pumping during off-peak hours. This solution may not always be possible, however, because of operating constraints it could be necessary pumping during peak energy-cost periods, and so the optimal solution will have to recommend the most effective period for pumping, bearing in mind the demand for water and the capacity of the system.

This paper presents a model aimed at defining the minimum cost daily pumping schedule. To solve this model two different resolution approaches were implemented: the first being based on the branch-and-bound method and the second using the simulated annealing method.

The application of both methods is illustrated for a hypothetical example. Comparison of the results demonstrated that the branch-and-bound method performed very quickly and also that simulated annealing was very efficient in terms of both solution quality and execution time.
1 Introduction

The complexity involved with interconnecting the components of a water supply system, consisting of pipelines, supply reservoirs and distribution networks, is considerable, because the action of each part of the system affects the functioning of any other part. An effective integrated operating system is therefore essential for the proper regulation of water distribution. We must also consider that if such a range of variables is involved a formidable number of alternatives arises, with the consumption of energy normally implying enormous running costs. Given this situation, it is obviously necessary to develop tools that will guide the decision-making process so that the most economical solution may be arrived at.

The mathematical models for water supply systems are non-linear, and binary variables are used to model pump functioning. These take the value 1 when the pump is working, and 0 when it is not working, and problems such as these are extremely hard to solve, from the operations research perspective. Among those who have tackled this topic are: Sterling & Coulbeck [19], Ferreira & Vidal [7], Lansey & Awumah [9] (dynamic programming); Brion & Mays [2], Ormsbee et al. [13], Pezeshk & Helweg [14] (non-linear programming); Jowitt & Germanopoulos [8], Likeman [10] (linear programming); Creasey [3], Little & McCrodden [11], Sousa & Cunha [17] (mixed integer linear programming). More recently, Simpson et al. [16] used genetic algorithms, and Sakarya et al. [15] experimented with simulated annealing. Dynamic programming is the method that has been most widely used, although it can really only be used for small-scale problems, and most of the other methods mentioned above are system-specific – they are only applicable to a particular system – which is very restrictive, though they may be adapted for application to other systems, which entails considerable complications.

In this work the authors present a general mixed integer linear programming model, intended to define the least-cost daily pumping schedule, to help the operation of supply systems that include pumping stations. Two different resolution methods were implemented to solve this model: the first is based on the branch-and-bound method, and the second uses the simulated annealing method. The branch-and-bound method is widely known as a classical optimization method that is capable of finding the global optimal solution of the proposed model. It is only applicable, however, to models whose objective function and constraints are linear functions of the decision variables. Simulated annealing is a random search method, widely used to solve combinatorial optimization problems (Aarts et al. [1], Dowsland [6]). It is not an exact method, like branch-and-bound is, but, under certain conditions (where a good set of control parameters is chosen), it can produce high quality solutions relatively quickly. But, the main advantage of this method is that it can be used to solve problems with any kind of functions (linear and non-linear). This capacity could be explored in future work that would develop and solve models that are more complete and realistic. Meanwhile, as simulated annealing is a heuristic method,
it is very important to evaluate its performance in terms of both the accuracy of the solution and the computer time required. The efficiency of simulated annealing is assessed by comparing the results obtained from the two approaches (branch-and-bound and simulated annealing).

2 Model for a simple pipeline system

A full description of the proposed model is given in Sousa & Cunha [17], and so only a brief summary is given here.

A simple pipeline system, Figure 1, usually includes an upstream reservoir, a pumping station, the pipeline and a downstream reservoir. The upstream reservoir may be a source of water, or it may be an intermediate reservoir within the system or a reservoir of treated water located downstream from a water treatment plant. The pumping station is where the pumps are installed. There may be one or more pumps, working separately or combined (in series or in parallel). Finally, the downstream reservoir may be an intermediate reservoir within the system, a distribution reservoir or even perform both functions.

The main goal of the model is to define the optimal scheduling of the pumping station over a 24-hour period. For every hour, the solution must identify the pump, or pump combination, which should be working in order to satisfy the water demand at minimum cost. It is based on linear programming and uses binary variables to define the pumping scheduling.

Consider a distribution reservoir being fed by a pipeline with a pumping station (Figure 1). A 24-hour water demand pattern is assumed to be known ($DP_i$ in m$^3$/h, $i = 1, 2, \ldots, 24$). After studying the functioning of the pumping station we can define the possible $NP$ pump combinations (if there is more than one) and predict, for each, the respective discharge ($D_j$ in m$^3$/h, $j = 1, 2, \ldots, NP$) and power consumption ($PC_j$ in kWh, $j = 1, 2, \ldots, NP$). With respect to the downstream reservoir, both the interior surface ($S$ in m$^2$; where the reservoir has more than one compartment, $S$ should be considered the sum of the interior}
surfaces of the operational compartments) and the minimum and maximum operating levels ($L_{\text{min}}$ and $L_{\text{max}}$, respectively) must be known. The downstream reservoir water level at the beginning of the operating period ($L_0$) must be introduced, and a minimum downstream reservoir water level at the end of that period ($L_{\text{end}}$ in m) must be imposed. To evaluate energy costs ($EC$ in PTE) it is also necessary to know how much the distribution company charges ($C_{kWh}$ in PTE/kWh, $i = 1, 2, \ldots, 24$). This information gives the basis for formulating the optimization model which will provide the optimal pumping station scheduling:

\[
\begin{align*}
\text{Min. } EC &= \sum_{i=1}^{24} \sum_{j=1}^{NP} PC_j \cdot C_{kWh} \cdot Y_{ij} \\
\text{subject to } & \\
S \cdot L_{i-1} + S \cdot L_i + \sum_{j=1}^{NP} Y_{ij} \cdot D_j = D_{P_i} & \quad i = 1, 2, 3, \ldots, 24 \\
L_i &\geq L_{\text{min}} & \quad i = 1, 2, 3, \ldots, 24 \\
L_i &\leq L_{\text{max}} & \quad i = 1, 2, 3, \ldots, 24 \\
\sum_{j=1}^{NP} Y_{ij} &\leq 1 & \quad i = 1, 2, 3, \ldots, 24 \\
L_{24} &\geq L_{\text{end}} 
\end{align*}
\]

where: $L_i$ – downstream reservoir water level at each hour $i$ (in m); $Y_{ij}$ – binary variables defining, for each hour $i$, if the pump combination $j$ is on ($Y_{ij}=1$) or off ($Y_{ij}=0$).

### 3 Model application

The application of the proposed model is demonstrated using a hypothetical example of a simple pipeline system (Figure 1). The downstream reservoir is presumed to satisfy the water demand pattern presented in Figure 2. The electricity tariff depends on several factors (season of the year, day of the week, kind of usage, etc.) and varies during the course of the day. The example given here was calculated using the data presented in Table 1.

<table>
<thead>
<tr>
<th>Time</th>
<th>0-7</th>
<th>7-9.30</th>
<th>9.30-12</th>
<th>12-18.30</th>
<th>18.30-21</th>
<th>21-24</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cost (PTE/kWh)</td>
<td>6.03</td>
<td>9.38</td>
<td>16.09</td>
<td>9.38</td>
<td>16.09</td>
<td>9.38</td>
</tr>
</tbody>
</table>

The pumping station in the example has three identical pumps, combined in parallel, and it is assumed that one of them would only operate in an emergency. The model will thus only consider two pumps, with two different pump combinations. The hydraulic study of the pipeline system showed that for each possible combination the running conditions are as follows:
1 pump: \( D_1 = 470 \text{ m}^3/\text{h} \) \( PC_1 = 169 \text{ kWh} \)
2 pumps: \( D_2 = 800 \text{ m}^3/\text{h} \) \( PC_2 = 315 \text{ kWh} \)

Downstream reservoir conditions were assumed to be:
- Minimum operating level: \( L_{\text{min}} = 275 \text{ m} \)
- Maximum operating level: \( L_{\text{max}} = 280 \text{ m} \)
- Interior surface: \( S = 942.5 \text{ m}^2 \)

3.1 Branch-and-bound method

After compiling all the information, the optimization model was built and solved using a commercially available software package, Visual XPRESS [20].

The result of the optimization model was the optimal pumping scheduling, defining pumping station discharge throughout the day (Figure 3), as well as the downstream reservoir water level (Figure 4).

Figure 2: Assumed water demand pattern.

Figure 3: Pumping station discharge throughout the day.
Figure 4: Downstream reservoir water level throughout the day.

Analysing the results, it can be seen that the solution takes maximum advantage of the electricity tariffs, pumping at maximum capacity during the cheapest hours (off peak hours) and avoiding pumping during the most expensive ones (peak hours). The solution provided incurred a daily energy cost of PTE 35 995.75. The time required to solve this example, for a 24-hour period, was 5 seconds.

3.2 Simulated annealing method

This is a random search method which is inspired on the principles of the physical annealing process to find optimum solutions in optimization problems. The process requires the temperature to be increased so that the molecules become mobile, and thus attain different states. The temperature is then allowed to fall gradually, so that the molecules form into a crystalline structure and reach the minimum energy state, conforming to a systematised crystalline arrangement. In the optimization process, energy is replaced by cost, which has to be minimised, energy states correspond to feasible solutions and temperature is simply regarded as a control parameter.

Specialised literature reports that simulated annealing algorithms have been widely used in combinatorial optimization. The authors have already applied simulated annealing to solve various engineering problems (Cunha [4], Cunha & Sousa [5], Sousa et al. [18]), and the results have demonstrated a high level of efficiency when compared with other methods.

The main difference between simulated annealing and classical optimization methods is related to the fact that transitions from low to high cost configurations are not automatically excluded (according to the Metropolis criterion). These transitions will take place or not depending on the difference between costs and on the level of temperature: initially, even very negative (counter-optimum) transitions will be accepted; with falling temperature, the acceptance of such transitions will become increasingly rare. This way, the algorithm tries to avoid being trapped in local optima. This property confers the classification of “global optimization method” on simulated annealing.
The solution procedure used to solve the pumping scheduling problem is described in the scheme presented in Figure 5.

The simulated annealing algorithm was applied to solve the hypothetical example presented in Figure 1. The algorithm found several different schedules with the same energy cost as the solution found by the branch-and-bound method. Analysis of these different solutions shows that some of them imply fewer modifications to the pumps functioning throughout the day. So, to improve the quality of the solution, a small penalty was introduced to minimise the number of these modifications. The best solution that was found with this new implementation is presented in Figures 6 (discharge) and 7 (water level).
Figure 7: Downstream reservoir water level throughout the day.

The algorithm took less than 2 seconds to run and the new solution incurred the same daily energy cost (PTE 35 995.75). Even so, as we can easily see from Figure 6, the new solution presents a more regular functioning of the pumping station, which is always desirable.

4 Conclusions

The various components of the water supply system (reservoirs, pumping stations, pipelines and distribution networks) interact in such a way that operating the system can be extremely complicated. Because the pumping stations use a considerable amount of energy, it is essential for the operation of water supply systems to be optimised.

The mixed integer linear programming model described in this paper is designed to facilitate the operation of systems where pumping stations are included by determining the optimum scheduling solution for a 24-hour period, to meet the demand for water at the lowest cost. Two different resolution methods were implemented to solve this model: the first is based on the branch-and-bound method and the second uses the simulated annealing method. It is well known that branch-and-bound is only applicable to linear models and, in this work, its solution was used to draw conclusions about the efficiency of the simulated annealing algorithm. From a solution quality point of view, it can be seen that simulated annealing performed as well as the branch-and-bound method. With regard to execution time, simulated annealing was faster. These conclusions, together with the ability of simulated annealing to solve any kind of problems (linear and non-linear), encourage the authors to continue their efforts to develop this work to solve models that are more complete, and realistic.
References


