Dynamics of settling velocity distribution of suspended particulate matter during sedimentation processes in rivers

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Abstract

A depth-averaged model for simulating transport and sedimentation is introduced which allows to take into account the dynamic behaviour of the mean particle settling velocity of suspended particulate matter (SPM) in case of sedimentation processes on river reaches. The model calculates the varying parameters of the particle settling velocity distribution and the mass concentration of settling velocity fractions dependent on time and river location.

1 Introduction

Suspended particulate matter plays an important role in spreading and exposition of nutrients and pollutants in natural rivers. As the sorbed pollutants usually appear only on preferred particle fractions, the distribution of suspended sediments to special fractions, e.g. to settling velocity fractions, is also relevant. In the Spree River in Germany, particle bound polychlorinated biphenyls were detected only on light particles having settling velocities less than 30 cm/h.

The mass of suspended sediments in surface waters can be subdivided into fractions characterised by different settling velocity intervals. The k-th fraction (1 ≤ k ≤ N) is defined by the set of all particles having settling velocities s with

\[ s_{k-1} \leq s < s_k, \quad 0 < s_1 < \ldots < s_{N-1} < \infty \text{ and } s_0 = 0, s_N = \infty. \]

The concentration \( c_k \) of a fraction k can be determined by

\[ c_k = c \cdot (F(s_k) - F(s_{k-1})) \]
where \( c \) is the total suspended sediment concentration and \( F \) is a distribution function. The settling velocity is assumed to be log-normally distributed [1-7]

\[
F'(s) = F'_{\mu,\sigma}(s) = \frac{1}{\sqrt{2\pi\sigma}} \exp\left(-\frac{(\ln(s) - \mu)^2}{2\sigma^2}\right).
\]  

(2)

The log-normal distribution function \( F_{\mu,\sigma} \) has two parameters, the expectation parameter \( \mu \) and the deviation parameter \( \sigma \), and it holds

\[
\mu + \sigma^2/2 = \ln(s_q),
\]  

(3)

where \( s_q \) is the mean settling velocity of the suspension (expectation value of the distribution). Transport models which deal with sedimentation of particulate matter in rivers (e.g. [8,9,10]) usually consider the mean settling velocity \( s_q \) to be a constant parameter in time and location. But in reality \( s_q \) is influenced by the composition of released particulate matter, by particle production processes such as biomass production, by exchange processes with the bottom of the river, and by the stream power of the turbulent flow. Especially in slowly flowing river sections, \( s_q \) decreases because of the settling of particle fractions with higher settling velocities. In the years from 1993 to 1995, we have measured settling velocity spectra in old channels of the Spree River in Germany. A decrease of the mean settling velocity of about one order of magnitude could be observed over the flow distance (figure 1) and moreover, the settling velocities are also quite different, from 0.04 m/h to about 6 m/h at the outlet of the channel. Consequently, the mean settling velocity must be considered as field variable and

\[
s_q = s_q(x,t), \quad \mu = \mu(x,t), \quad \sigma = \sigma(x,t).
\]  

Figure 1: Mean settling velocities of suspended particulate matter measured at the Old Channel Freienbrink in the years from 1993 to 1995.
with respect to eqn (3) the distribution parameters $\mu$ and $\sigma$ also vary in time and river location.

In this paper, based on eqns (1),(2),(3), a depth-averaged numerical model is introduced in order to simulate concentration fields of settling velocity fractions of SPM during sedimentation processes in rivers.

### 2 Turbulent flow and mean settling momentum of SPM

The particulate matter of the most rivers is rather non-uniform. Suspended sediments of eutrophic waters consist of sandy particles, of a high amount of organic material incorporating nutrients and pollutants, of living algae groups, of zooplankton, and of heterogeneously formed detritus flocs. Particle size, density, and settling velocity are considered to be continuously distributed. A close interaction can be observed between sedimentation and development of the settling velocity distribution of SPM. The settling velocity spectra are changed by sedimentation of particles and vice versa, sedimentation rates depend on the mean settling velocity of the particles. In [12] was assumed that the settling momentum of suspended sediments in a confined volume of a river section can only be changed, if

- particles kept in suspension by the turbulent flow velocity are carried through the volume or
- particle transformation and production as aggregation, shearing, sorption, and biomass production are taking place.

Based on this assumption, a balance equation of the mean settling momentum has been derived applying means of the continuum mechanics of flowing media. Concerning to that, the depth-averaged equation

$$\frac{dm}{dt} + \text{div} J_m = f \cdot m + J_{m,\text{bot}} / H$$

results which calculates the vertically averaged settling momentum density

$$m = \rho q c$$

(the complete momentum vector is $(0,0,m)$). $H$ is the water depth, $f$ represents the specific rate of particle production as aggregation, shearing, and biomass production, $J_{m,\text{bot}}$ describes the vertical boundary flux at the bottom of the river, and the following designations are used.
Water Pollution

\[ \text{div } u = \sum_{j=1}^{2} \frac{\partial u_j}{\partial x_j} \quad \text{with } u = (u_1, u_2), \quad \text{grad } w = \left( \frac{\partial w}{\partial x_1}, \frac{\partial w}{\partial x_2} \right) \]

and

\[ \frac{dw}{dt} = \frac{\partial w}{\partial t} + v \cdot \text{grad } w, \quad J_w = -D_w \cdot \text{grad } w, \quad D_w = \frac{1}{\sigma_w} \begin{pmatrix} D_1 & 0 \\ 0 & D_2 \end{pmatrix} \]

for arbitrary functions \( u_j = u_j(x,t), w = w(x,t) \) of the river location \( x = (x_1, x_2) \) and the time \( t \). \( \frac{dw}{dt} \) is the total time derivation of \( w \). \( J_w \) stands for the dispersive flux of the function \( w \) in the turbulent flow, where \( D_w \) is the matrix of the horizontal turbulent exchange coefficients \( D_j/\sigma_w \) of \( w \) and \( \sigma_w \) is the Schmidt number.

Applying the product rule of differentiation to the settling momentum flux \( J_m \), the relation

\[ J_m = s_q \cdot \frac{\sigma_c}{\sigma_m} \cdot J_c + c \cdot \frac{\sigma_{s_q}}{\sigma_m} \cdot J_{s_q} \quad (7) \]

is obtained.

Analogously to eqn (7), the vertical boundary flux \( J_{m,bot} \) at the bottom of the river can be quantified by

\[ J_{m,bot} = s_{q,bot} \cdot \frac{\sigma_c}{\sigma_m} \cdot J_{c,bot} + c_{bot} \cdot \frac{\sigma_{s_q}}{\sigma_m} \cdot J_{s_q,bot} \quad (8) \]

The turbulent flow velocity \( v = (v_1, v_2) \) vertically integrated is governed by Reynold's depth-averaged equations [13-17]

\[ \frac{dv}{dt} + \text{div } J_v + g \cdot \text{grad } \eta = \frac{(J_{v,bot} - J_{v,surf})}{H}. \quad (9) \]

Herein \( g \) is the gravity constant \((\text{m}^2/\text{s})\). The water depth \( H \) and the water level elevation \( \eta \) are controlled by the depth-averaged continuity equation

\[ \frac{\partial H}{\partial t} + \text{div } (H \cdot v) = 0 \quad \frac{\partial \eta}{\partial t} = \frac{\partial H}{\partial t}. \quad (10) \]

The release of wind energy at the free surface is modelled by \( J_{v,surf} \). Based on Prandtl's mixing-length formulation, the kinetic energy at the river bed can be described by \( J_{v,bot} = -\frac{g}{c_z^2} \cdot |v| \cdot v \) where \( c_z \) is the Chezy coefficient.
In the years from 1991 to 1995, we have observed and measured the turbulent flow and the spreading and exposition of particulate matter in the Spree River and in some old branches of the river. For the Old Channel Sawall (figure 2) a hydrological situation from September 1991 was prepared as scenario. At this time the discharge of the old channel was relatively low \((q = 0.3 \text{ m}^3/\text{s})\) and the inflow velocity at the bridge amounted to 0.1 m/s. The turbulent flow field of this situation simulated by Reynolds’ equations (9) and (10) is shown in figure 3. Impact of wind and vegetation were neglected herein. The calculated eddy at the left bank near the bridge could be observed in this form. Drift body experiments and flow profile measurements at the pipe bridge (river meter 555) have also certified the agreement of simulated and measured flow velocities.
3 Total concentration and mean settling velocity of SPM

The suspended sediment concentration $c$ of a river compartment depends on exchange processes with neighbouring compartments, on mixing and transport processes caused by the turbulent flow, on river morphology, on bed roughness, and on particle production processes as aggregation, sorption, and biomass production. The governing equation of these processes can be written in the form (e.g. [11,13,18-21]):

$$\frac{dc}{dt} + \text{div} J_c = f \cdot c + J_{c, \text{bot}} / H$$  \hspace{1cm} (11)

where $f$ is the specific rate of particle production ($s^{-1}$), and

$$J_{c, \text{bot}} = R - D$$  \hspace{1cm} (12)

is the net sedimentation rate. $R$ represents the resuspension rate of particles from the river bed into the water column and $D = s^2 c_{\text{bot}}$ is the deposition rate of suspended sediments. The value $c_{\text{bot}} = \alpha c_{\text{bot}}' c$ of the suspended sediment concentration near the bottom can be calculated from a vertical concentration profile. Figure 4 illustrates isolines of the concentration field simulated by eqns (9-12). Corresponding to the weak current which governed the sedimentation process in the Old Channel Sawall in September 1991, deposition of SPM was dominant in contrast to resuspension. About 50% of the incoming SPM was eliminated downstream from the observed channel reach (from river meter 300 to 650).

Figure 4: Simulated suspended sediment concentrations (mg w.w./l), (62.81% water content), Old Channel Sawall, September 1991
Settling momentum equation (4) can be transformed into a settling velocity equation
\[
\frac{ds_q}{dt} + \text{div} J_s + \frac{2}{c} \text{grad} c \cdot J_s = \alpha_{c,\text{bot}} \cdot J_{s,\text{bot}} / H
\]  
(13)

using eqns (7-9) simplified by the assumptions
\[\sigma_m = \sigma_{s_q} = \sigma_c \quad \text{and} \quad s_{q,\text{bot}} = s_q .\]

Figure 5 shows the calculated field distribution of the mean settling velocity \(s_q\) in the Old Channel Sawall from September 1991.

From eqn (12), an approach concerning the flux \(J_{s_q,\text{bot}}\) of the mean settling velocity at the bottom of the water is suggested in the form:
\[
J_{s_q,\text{bot}} = S_A \cdot (s_R - s_q)
\]  
(14)

where \(s_R\) is the mean settling velocity of the resuspended material and \(S_A\) (in m/s) describes the intensity of the mixing process between suspended and resuspended matter near the bottom. By eqn (14), \(s_R\) may be interpreted as mean settling velocity of the particulate matter in equilibrium of deposition and resuspension.

Based on critical values for the beginning of resuspension, the formula
\[
s_R = s_{R,*} \left( \frac{v_*}{v_{R,*}} \right)^a, \quad a = \begin{cases} 
1, & \text{if } v_* < v_{R,*} \\
4, & \text{else}
\end{cases}
\]  
(15)

could be derived [22] to calculate \(s_R\) in that case.
The attraction velocity $s_A$ depends on the turbulent flow. An increasing bottom shear velocity $v_*$ leads to a growing resuspension of particles from the river bed. In this case the adaptation of $s_q$ to $s_R$ is accelerated and $s_q$ tends to $s_R$. Corresponding to this behaviour, the model approach

$$s_A = s_{A,*} \cdot (\exp\left(\frac{v_*}{v_{A,*}}\right) - 1)$$

(16)

has been used and tested.

The empirical parameters $s_{R,*}$, $v_{R,*}$, and $s_{A,*}$, $v_{A,*}$ are to be determined. For the Spree River in Germany, the parameters

$$s_{R,*} = 4.216 \text{ m/h}, v_{R,*} = 58 \text{ cm/s} \quad \text{and} \quad s_{A,*} = 3.3 \text{ cm/s}, v_{A,*} = 0.228 \text{ cm/s}$$

were estimated and used to calculate the settling velocity fields of the suspended and resuspended sediments (figures 5, 6).

Figure 7 illustrates the simulated nontrivial longitudinal profile of the mean settling velocity of SPM at the mid width of the Old Channel Sawall. At the inlet of the channel, the measured value of the mean settling velocity was used as boundary condition.

![Figure 6: Simulated mean settling velocity field (m/h) in equilibrium of deposition and resuspension, Old Channel Sawall, situation from September 1991](image)

At the outlet of the channel the mean settling velocity simulated on the basis of eqn (13) was found to be near the corresponding measured value. The mean settling velocity decreased from 0.588 m/h to 0.047 m/h on the observed channel reach.
The decrease of the suspended particulate matter concentration at the mid width of the channel can be seen in figure 8 in comparison with measured concentration values. Assuming a constant value of 0.588 m/h of the mean settling velocity of suspended sediments unchanged over time and channel location, an unrealistically strong elimination of SPM would result. The simulation of such a scenario shows (figure 8) that the concentration of SPM would tend nearly to zero at the downstream cross section.
4 Transport of settling velocity distributions

The concentration of a settling velocity fraction can be calculated by eqn (1). Applying eqns (1) and (2), at least three further equations are necessary to obtain the total suspended sediment concentration \( c \), and the distribution parameters \( \mu \) and \( \sigma \).

Based on eqns (9), (10), (11), and (13) the flow velocity \( v \), the water depth \( H \), the total concentration \( c \) and the mean settling velocity \( s_q \) of the suspended particulate matter can be calculated depending on time and location in a river system. Eqn (3) can then be used to obtain \( \mu \) afterwards. It remains to look for a functional relation that allows to calculate the deviation parameter \( \sigma \).

During our studies of settling velocity spectra in the Spree River and in old branches of the river, it could be observed that the deviation parameter \( \sigma \) increases with the mean settling velocity \( s_q \) of the corresponding particle suspension.

Based on measurements of settling velocity spectra (figure 9), an exponential regression function

\[
\frac{\sigma}{\sigma_s} = (\frac{s_q}{s_{q,s}})^{r_s} , \quad s_{q,s} = 5.558 \text{ m/h} , \quad \sigma_s = 3.073 , \quad r_s = 0.1183
\]

(17)

could be found. For \( s_q \) - values which are greater than 2 m/h, it was sufficient to use the more simple relation
Generally, beside equation (3) a functional relation such as given by eqns (17) or (18), about an equation like \( G(\sigma, \mu, s_q, c, v) = 0 \), is necessary to establish a transport model for log-normal settling velocity distributions.

The temporary and local development of settling velocity spectra and mean settling velocities of SPM is closely connected. Using eqns (3) and (17), the parameters \( \mu \) and \( \sigma \) of the log-normal settling velocity distribution function \( F \) can be calculated dependent on the mean settling velocity \( s_q \). Therefore, the real basis for modelling the transport of settling velocity distributions is given by eqn (13).

Light SPM - fraction, settling velocities < 30 cm/h

Heavy SPM - fraction, settling velocities > 30 cm/h

Figure 10: Simulated concentrations of SPM - fractions (mg/l), Old Channel Sawall, September 1991
Concluding, the sedimentation process of SPM in the Old Channel Sawall from September 1991 is studied now with respect to the transport mechanisms of settling velocity distributions introduced above. The SPM is considered herein to be subdivided into two settling velocity fractions, a light one with settling velocities less than 30 cm/h and a heavy one with settling velocities greater than 30 cm/h. At the inlet of the Old Channel Sawall a mean settling velocity of \( s_q = 0.588 \) m/h and a total concentration of \( c = 12.1 \) mg/l were measured in September 1991. From eqns (3) and (17), \( \mu = -3.306 \) and \( \sigma = 2.356 \) follow, and the probability

\[
F_{\mu,\sigma}(0.3) = G\left(\mu - \frac{\ln(0.3) - \mu}{\sigma}\right) = G(0.8923) = 0.814
\]

is obtained, where \( G \) is the Gaussian distribution. By eqn (1), inlet concentrations of the light and the heavy fraction result to be

\[
c_{\text{calc}}^{\text{light}} = c \cdot (F_{\mu,\sigma}(0.3) - F_{\mu,\sigma}(0)) = 12.1 \cdot (0.814 - 0) = 9.85 \text{ mg/l}
\]

\[
c_{\text{calc}}^{\text{heavy}} = c \cdot (F_{\mu,\sigma}(\infty) - F_{\mu,\sigma}(0.3)) = 12.1 \cdot (1 - 0.814) = 2.25 \text{ mg/l}
\]

Corresponding measuring values are \( c_{\text{meas}}^{\text{light}} = 8.74 \) mg/l and \( c_{\text{meas}}^{\text{heavy}} = 3.36 \) mg/l.

In this way concentration fields of the settling velocity fractions were calculated using the mean settling velocities simulated by eqn (13) and applying eqns (3) and (17) afterwards. The simulated concentration fields of the light and the heavy SPM fraction are illustrated in figure 10.

Figure 11 shows the settling velocity spectra of the Old Channel Sawall measured at the inlet (river meter 300) and at the outlet (river meter 650) of the channel in September 1991 compared with corresponding fitted and calculated settling velocity spectra.

![Diagram](image-url)

Figure 11: Concentration of settling velocity fractions of the SPM at the inlet (river meter 300) and at the outlet (river meter 650) of the Old Channel Sawall, September 1991
5 Summary

The dynamics of suspended sediments in flowing waters can be modelled by a system of depth-averaged partial differential equations for the turbulent flow, for the suspended particulate matter concentration, and for the mean settling velocity of the particles. Frequently, the load of nutrients and pollutants is sorbed on preferred particle fractions, especially on preferred settling velocity fractions. Consequently, the distribution of suspended particulate matter to settling velocity fractions is also of interest. Therefore, a model describing the transport of settling velocity spectra in rivers was developed and introduced.

In flowing waters, a close interaction exists between sedimentation of particles and development of the settling velocity spectra of suspended sediments. Accordingly, the parameters of the settling velocity distribution function vary in time and location of a river section. Supposing a log-normal settling velocity distribution function, two parameters, the expectation parameter $\mu$ and the deviation parameter $\sigma$, were to be determined with respect to the transport model.

It was shown that both parameters, $\mu$ and $\sigma$, can be considered as function of the mean settling velocity $s_q$. Therefore, the time- and location-dependent calculation of the mean settling velocity by a partial differential equation became the essential presupposition for the transport modelling of settling velocity distributions.

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References


