



# **The use of fuzzy logic for the study of water pollution in the Thermaikos Gulf**

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## **Abstract**

The fuzzy set theory is applied to define the imprecise parameters which are used in a water pollution model. The physico-chemical coefficients and the loads of pollution sources involved, are expressed in the form of triangular fuzzy numbers. A two-dimensional finite element algorithm based on the characteristic-Galerkin technique combined with fuzzy calculus is used for the solution of the advection-dispersion equation, for ten different water quality parameters. The model is applied for the study of distribution of pollutants in the Gulf of Thermaikos, located in Northern Greece.

## **1 Introduction**

The knowledge of distribution of the pollutant loads in the coastal area under different conditions, is necessary to face the pollution problem. Numerical modeling constitutes a powerful tool for the study of advection-dispersion of pollutants for different scenarios. A difficulty arises from the imprecise character of the data related to the values of the loads of pollution sources and of physico-chemical coefficients, because they depend on factors which are changing randomly in space and time like turbulence, temperature and composition of the organic mass (Ducstein & Plate[1], Ganoulis[2]).

Fuzzy set theory and its derivative fuzzy calculus (Zadeh[3], Zimmermann[4]), may be used to introduce imprecise data into a mathematical model in a direct way with minimal input data requirements (Silver[5]). While probabilistic methods like Monte Carlo simulation or Bayesian approach require information about the entire probabilistic distribution like variances, dispersion and correlation for all input variables

(Ferson et al.[6]), in fuzzy modeling only the range and the mean of the input variables are required, so it can be used successfully when the available data are too sparse for a probabilistic method.

In this paper the advection-dispersion equation is used for the study of coastal pollution in the Thermaikos Gulf. In order to minimize the numerical diffusion a characteristic-Galerkin finite element scheme was employed, combined with fuzzy arithmetic for the representation of imprecise parameters in the data and in the functional relationships.

## 2 Fuzzy sets and fuzzy numbers

A fuzzy set is a collection of members which belong to the set up to a certain degree. The degree by which an element belongs to a set is determined by the membership function, which is denoted as  $\mu$  and is expressed by a number between 0 and 1. Number 0 represents the zero confidence, while number 1 the maximum confidence, for an element to belong to the set X.

A fuzzy number  $\tilde{A}$  is a fuzzy set of real numbers whose membership function reaches always the maximum value of 1 and is convex, i.e. it consists of an increasing and a decreasing part. Arithmetic rules for fuzzy numbers are described in detail by Kaufmann & Gupta[7]. The simplest type of fuzzy number is the triangular fuzzy number (T.F.N). A T.F.N can be defined by a triplet  $(x_1, x_2, x_3)$ .

## 3 Mathematical formulation

The ecological model contains the advection-dispersion equations, which describe the variation of concentration of ten different quality parameters with respect to time, under biochemical interactions. These parameters are: chlorophyll-a, coliforms, organic, ammonia, nitrite and nitrate nitrogen, organic and inorganic phosphorus, BOD, and dissolved oxygen. The model was described in detail by Anagnostopoulos et al.[8]. Since the values of the various coefficients and of loads of pollution sources do not remain fixed but vary between certain limits, two sets of computational tests were conducted, using the upper and lower values of all these parameters. However, although the concentration of the various pollutants was higher when the maximum values of the various parameters were used, the simultaneous use of minimum and maximum values of the various parameters in each computation does not yield the extreme values of the output pollution loads.

For the determination of the extreme values of the various quality parameters of the model which is the scope of the present study, fuzzy analysis was employed. All coefficients and loads of pollution sources were expressed in the form of T.F.N. Some of these fuzzy coefficients (dispersion coefficient,  $D_x$ , growth rate of chlorophyll-a,  $\mu$ , coliforms die-off rate,  $k_5$ , the rate constant for hydrolysis of organic nitrogen to ammonia nitrogen,  $\beta_3$ , the reaeration coefficient,  $k_2$ , and the coefficient for organic nitrogen settling rate,  $\sigma_4$ ), are presented in figure 1.

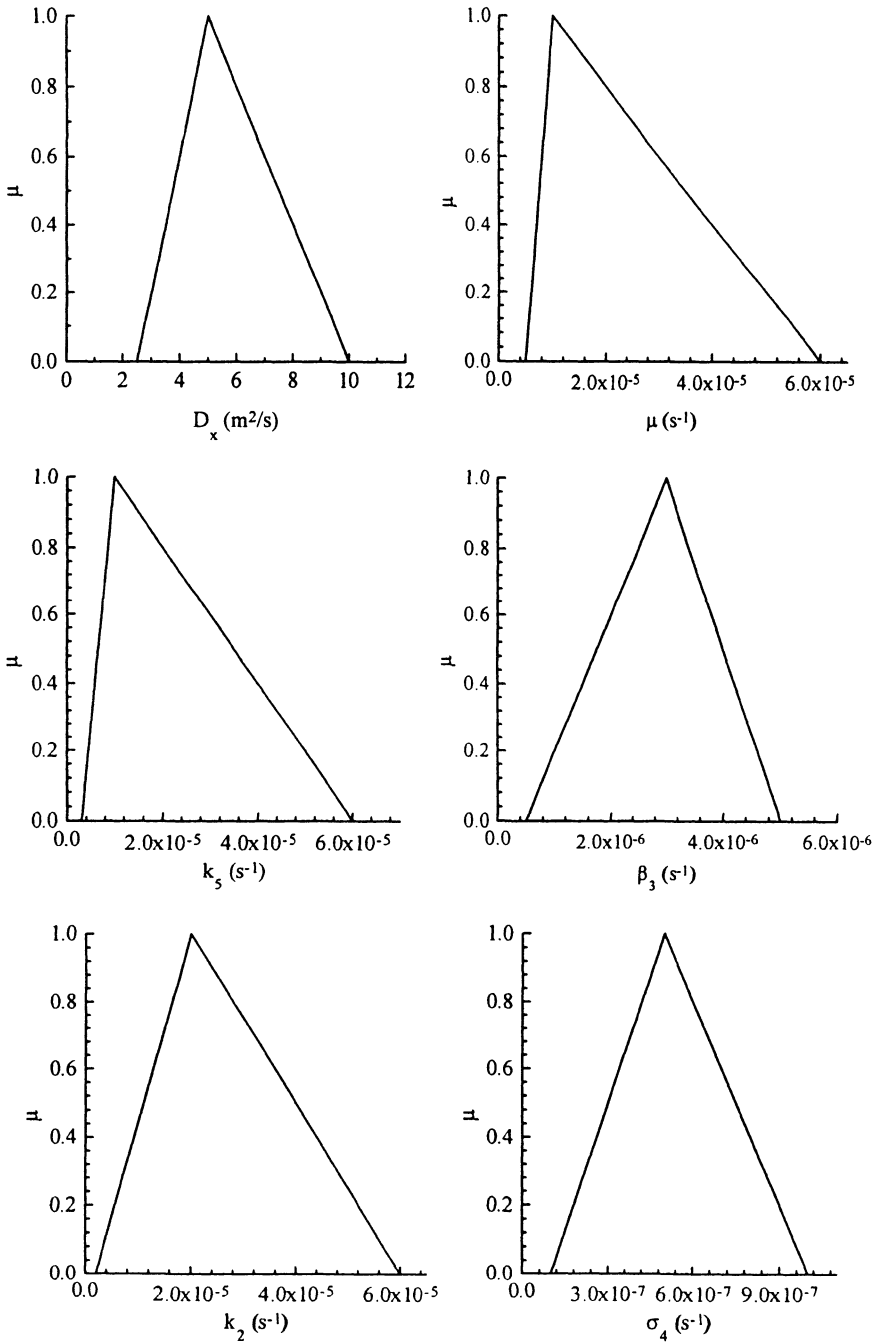


Figure 1: Membership function of fuzzy coefficients.

## 4 Numerical procedure

An explicit finite element scheme was used for the solution of the ten equations of the ecological model, which are of advection-dispersion type in a non-conservative form. To overcome the problem of numerical diffusion which is caused by the advection term in the equations, the characteristic-Galerkin method was used (Zienkiewicz & Taylor[9]).

Since the coefficients and the input loads of pollution are expressed as T.F.Ns the output concentrations  $c_i$  are also fuzzy, so the ten equations set becomes a set of fuzzy equations. At a given  $\alpha$ -level cut in the interval  $[0, 1]$  of the membership function  $\mu(x)$ , each T.F.N  $\tilde{A}$  is represented by an interval number  $\bar{A}_\alpha = [x_1, x_3]$ , which is called interval of confidence (Kaufmann & Gupta[7]). Since multiplication and division operations can be performed only in the intervals of confidence  $\bar{A}_\alpha$ , the system of equations under study becomes a set of interval equations at each  $\alpha$ -level cut.

The operations on the intervals  $\bar{A} = [\alpha_1, \alpha_2]$  and  $\bar{B} = [\beta_1, \beta_2]$  may be calculated (Moore[10], Alefeld[11], Nickel[12]) as:

$$\bar{A} + \bar{B} = [\alpha_1 + \beta_1, \alpha_2 + \beta_2], \bar{A} - \bar{B} = [\alpha_1 - \beta_2, \alpha_2 - \beta_1], -\bar{A} = -[\alpha_1, \alpha_2] = [-\alpha_2, -\alpha_1],$$

$$\bar{A} * \bar{B} = [\min(\alpha_1\beta_1, \alpha_1\beta_2, \alpha_2\beta_1, \alpha_2\beta_2), \max(\alpha_1\beta_1, \alpha_1\beta_2, \alpha_2\beta_1, \alpha_2\beta_2)],$$

$$\bar{A} / \bar{B} = [\min(\alpha_1/\beta_1, \alpha_1/\beta_2, \alpha_2/\beta_1, \alpha_2/\beta_2), \max(\alpha_1/\beta_1, \alpha_1/\beta_2, \alpha_2/\beta_1, \alpha_2/\beta_2)].$$

For the solution of the system of equations, interval calculations are carried out at five different  $\alpha$ -level cuts (0, 0.25, 0.50, 0.75, 1) of T.F.Ns. At each  $\alpha$ -level cut an interval value of concentration is computed. From these results the fuzzy concentrations are constructed (Dou et al.[13], Ganoulis et al.[14], Mpimpas et al.[15]).

## 5 Application and results

The pollution sources were placed in the city of Thessaloniki, in the outlets of the rivers Dendropotamos, Galikos, Axios, Loudias and Aliakmonas, and in the outlets of the drainage networks in Xalastra, Margara and Klidi, as shown in figure 2. All loads of pollution sources are expressed as T.F.N. (Mpimpas et al.[15]). The velocity field was obtained by solving the shallow water equations using triangular finite elements in the domain of Thermaikos Gulf for northern wind of velocity 10 m/s (Anagnostopoulos & Mpimpas[16]). The advection-dispersion equation was solved at five different  $\alpha$ -level cuts (0, 0.25, 0.5, 0.75, 1) of the fuzzy input parameters.

The membership functions of concentration for chlorophyll-a, coliforms, organic nitrogen and shortage of dissolved oxygen constructed from the interval output in the form of fuzzy numbers at nodes 42 and 200 of figure 2, are shown in figures 3 to 6. The left frames correspond to node 42 and the right frames to node 200. The results correspond to a time equal to 55 hours from the inception of the computation. Figures 7 and 8 depict the concentration distribution of coliforms and shortage of dissolved oxygen in the Gulf of Thessaloniki corresponding to the zero  $\alpha$ -level cut. The upper

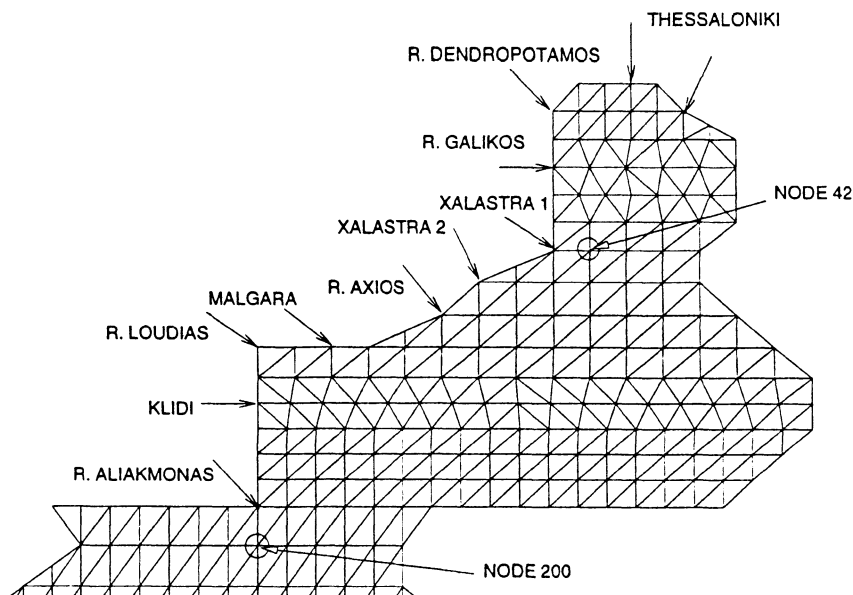


Figure 2: Position of pollution sources on the mesh of Thermaikos Gulf.

frames of figures 7 and 8 illustrate the maximum values of concentration, while the lower frames the minimum values. The loads of the various pollutants are greater in the western part of the Gulf, which can be explained from the distribution of pollution sources portrayed in figure 2.

Without the use of fuzzy analysis, a huge number of computations had to be conducted for the determination of the extreme values of concentration of the various quality parameters, with all possible combinations of the minimum and maximum values of all parameters involved in the model.

## 6 Conclusions

Fuzzy approach combined with the finite element method was employed to express imprecise parameters like biochemical coefficients and loads of pollution sources for the study of pollution in the Thermaikos Gulf. With fuzzy modeling we can represent imprecise data and produce imprecise output in the form of fuzzy numbers, with minimal input data requirements and without the need of a large number of computations. The technique is very powerful for the determination of extreme values of the various quality parameters.

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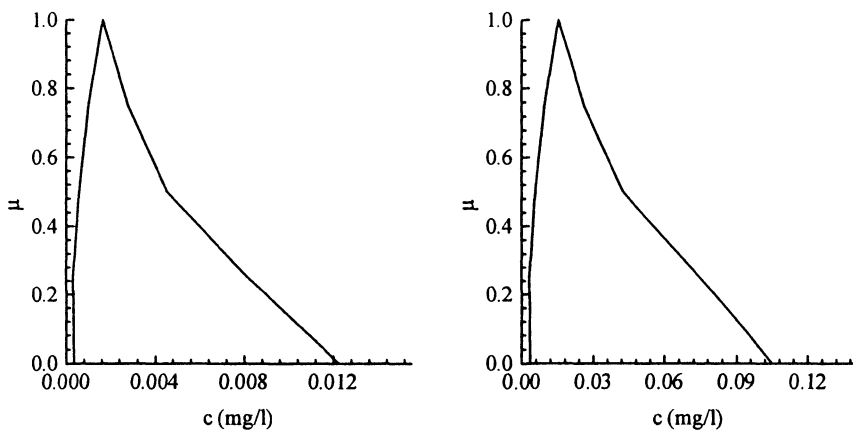


Figure 3: Membership function of chlorophyll-a concentration at nodes 42 and 200.

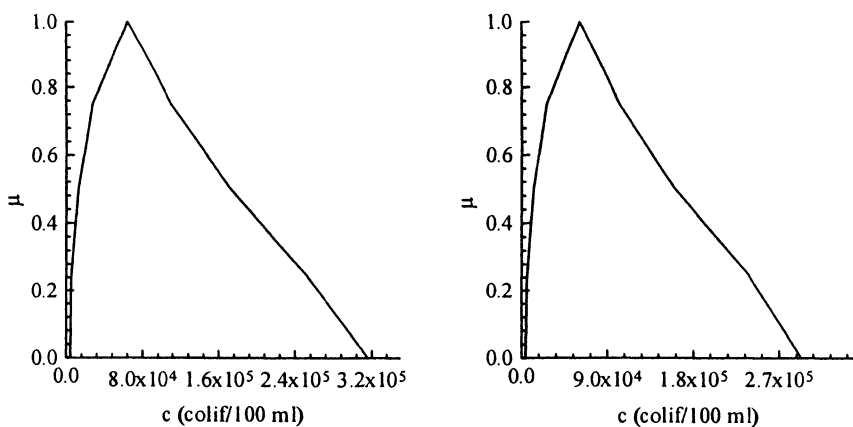


Figure 4: Membership function of coliforms concentration at nodes 42 and 200.

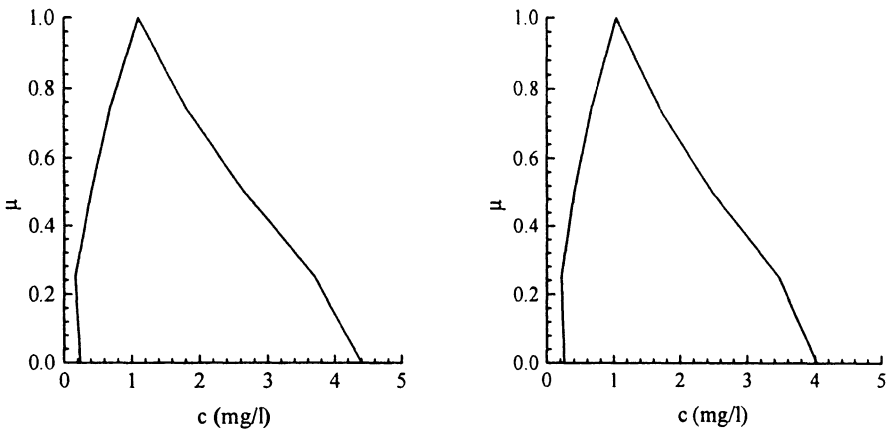


Figure 5: Membership function of organic nitrogen concentration at nodes 42 and 200.

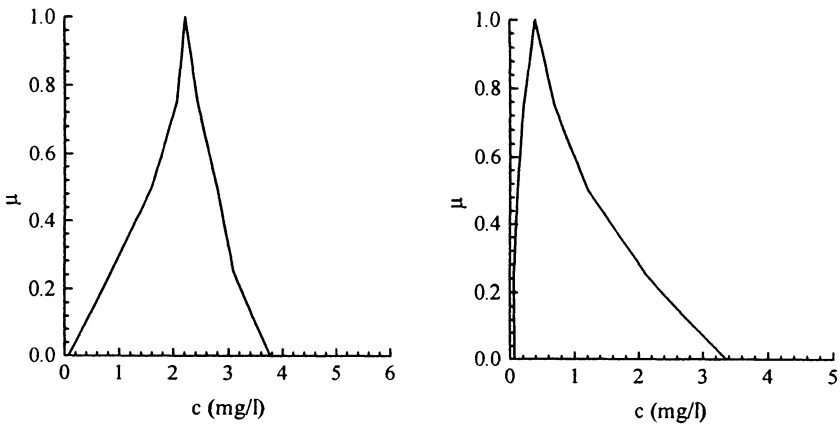


Figure 6: Membership function of Dissolved Oxygen shortage concentration at nodes 42 and 200.

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Figure 7: Distribution of concentration of the extreme values of coliforms in the Gulf of Thessaloniki.

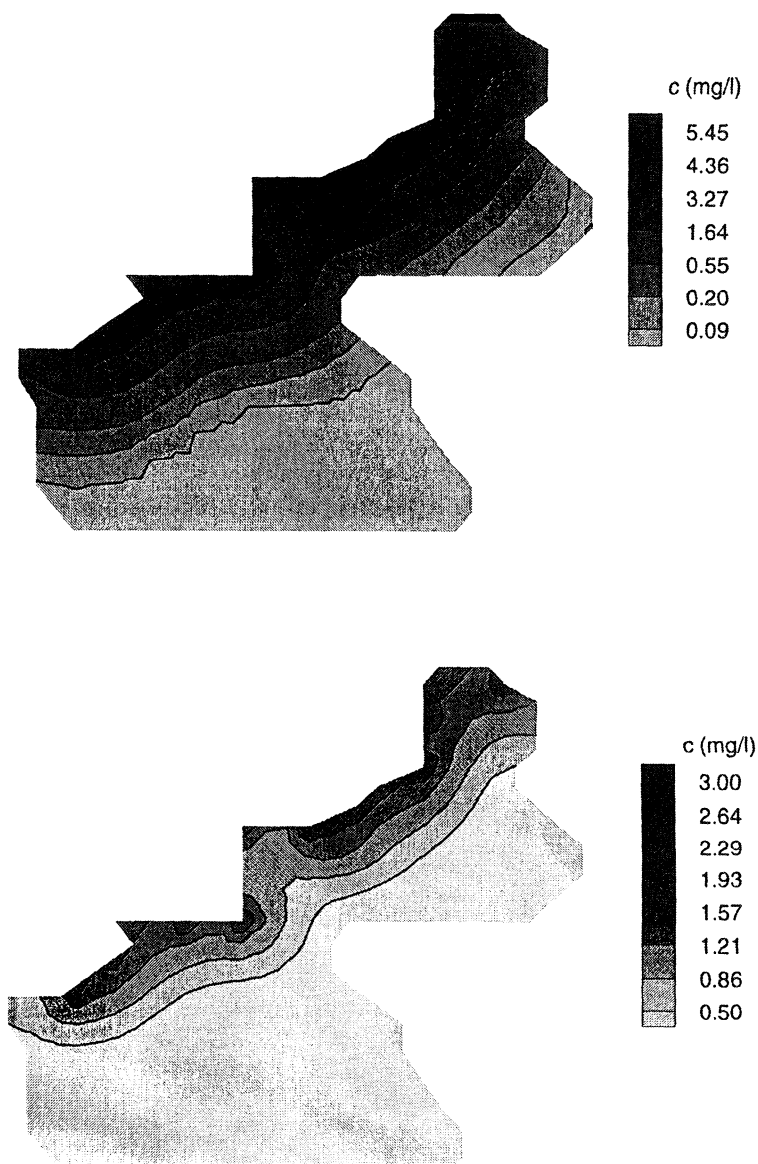


Figure 8: Distribution of concentration of the extreme values of Dissolved Oxygen shortage in the Gulf of Thessaloniki.



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