Model of bacterial pollution of sea water discharged by submarine outfall

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Abstract

This paper presents mathematical model of expansion of bacterial pollution discharged by submarine outfall. Initial dilution is calculated using parametric models. Results of the model of initial dilution such as value of initial dilution and characteristics of the polluted plume were used as input data for 2-D model of secondary dilution. Model of secondary dilution consists of two submodels: the flow field model and the model of transport and transformation of bacterial pollution. Mathematical base for the flow field model is equation of potential flow. Mathematical base for the model of transport and transformation of bacterial pollution is 2-D form of the equation of convection and diffusion with adequately adapted boundary conditions. Equations are solved numerically using the finite elements method. Model was used to determine the possible bacterial pollution of sea water due to the construction of planned submarine outfall of the Trogir-Kaštela collection system and it proved to be very useful for finding out engineering solution.

1. Introduction

One of aspects of utilization of coastal sea water is as a recipient of waste water. Waste water is usually disposed into the sea through submarine outfalls. A main characteristic of waste water is high organic and bacterial pollution. Discharge of organic pollution from small settlements (less than 100000 inhabitants) usually doesn’t cause significant ecological changes of sea and therefore waste water is mechanically treated prior to discharge from the outfall. Therefore for purposes of submarine outfall design, bacterial pollution is detrimental criteria for determination of outfall and diffuser characteristics.

Problem of discharge of waste water is in the Republic of Croatia regulated by legal acts. Croatian standards are of a recipient type. Sea water is in accordance with its purpose divided into four categories and for each
category maximum allowable concentrations of bacterial pollution are regulated. Selected characteristics of the outfall must fulfill the requirement that concentrations of bacteria at the outfall location and in coastal area are below those regulated by law.

2. Overview of previous researches

Nowadays, various types of mathematical simulation models are commonly used to determine pollution of sea water discharged by submarine outfalls. Process is very complex and therefore models are built of one or more submodels. The most commonly used ones are submodels of initial and secondary dilution.

Models of initial dilution describe the phase of wastefield dilution immediately after discharge from the submarine outfall orifice, when wastefield motion is under dominant influence of initial momentum that wastewater had at the outfall orifice and the difference between density of wastewater and recipient water. Initial dilution is usually and easily modeled using parametric models (e.g. [3], [4], [5], [6], [7]) Main disadvantage of parametric models is that their application is connected with fulfillment of certain conditions concerning recipient and outfall that are usually not completely fulfilled in practice. Initial dilution can be obtained using complex numerical models derived from basic conservation laws. According to [1], for calm recipient where influence of streams can be disregarded, it is recommended to use Euler type models (e.g. [8]), while for recipient with pronounced horizontal streams better results are obtained using Lagrange type models (e.g. [9]).

Models of secondary dilution are used for modeling a second phase of dilution that begins after the density of wastefield becomes equal to the density of recipient and the influence of initial momentum can be disregarded. Two types of submodels are usually used to model secondary dilution: flow field model (hydrodynamic model) and model of transport and transformation of substances in the sea (sea quality model).

Hydrodynamics of sea masses is usually mathematically described formulating dynamic differential equations and equations of continuity. If e.g. streams were caused by wind Ekman theory (e.g. [10]) should be used. For streams caused by tide cycle Euler or Navier-Stokes equations (e.g. [11]) are used. For residual sea currents e.g. procedure of minimization given in [12] can be used. Derived equations are then, in forms and with boundary conditions that depend on particular problem, solved using numerical methods.

Transport and transformation of parameters can be described using models of Euler and Lagrange type. Models of Lagrange type can be deterministic (e.g. [13]) or stochastic (e.g. [14]). However, models of Euler type are more
frequently used. Mathematical base of these models is various types of convection-diffusion equation, which are solved using different numeric methods for set boundary conditions.

In general, all previously mentioned models can find their application in modeling of bacterial pollution. However, in distinction from other parameters that are not degradable or their degradation lasts for days, time of bacterial decay in maritime environment is very short and after few hours their number decreases significantly. As a consequence, modeling of bacterial pollution in maritime environment has its specificities which to a certain extent facilitate the process of modeling:

- already after few hours of expansion through maritime environment the number of bacteria decreases for more than ten times,
- there is no possibility of significant accumulation of pollution in certain locations by return of wastefield to that location because concentrations within returned field are more than ten times smaller,
- it is enough to model only the aquatorium around the location of discharge, and coastal strip from the shore to the place of discharge,
- it is necessary to analyze only the most unfavorable alternatives of the flow field, such streams have dominant direction toward the shore and mainly simple form,
- the assumption of stationary flow field can be adopted.

Nowadays, mainly because of previously mentioned reason, the process of determination of bacterial pollution using Brooks models of initial and secondary dilution is still in use in engineering practice. Disadvantage of Brooks model of initial dilution is the fact that it doesn’t take in account the influence of streams on the value of initial dilution. Disadvantage of the model of secondary dilution is the assumption of equal intensity and direction along the entire area of expansion of the wastefield. Consequence is overdesigned outfall and diffuser.

In order to obtain more realistic picture of expansion of bacterial pollution, more complex models must be used. One of such models is CDB-2D developed at Faculty of Civil Engineering in Split.

3. Description of the Model

CDB-2D Model consists of two submodels: model of initial dilution and model of secondary dilution. Model of secondary dilution is 2-D model that integrates the flow field model and the model of transport and bacterial decay.
3.1 Model of initial dilution

The initial dilution is calculated using parametric models based on researches conducted by Roberts [3], [4], [5], [6] and Lee [7]. Models are used for line or round plumes and jets in unstratified or linearly stratified media with or without horizontal currents in the area of upwelling of wastefield. All unknown parameters of initial dilution shown in Figure 1 can be calculated using these models.

Figure 1 Parameters of initial dilution

Results of the model of initial dilution, that is a value of initial dilution and characteristics of the polluted plume, are used as input data for the model of secondary dilution.

3.2 Model of secondary dilution

Model of secondary dilution is 2-D model, used to model the layer of recipient of height \( h_e \) (Figure 1) assuming that a layer is uniformly mixed along the height and wastefield is held within it during the entire process of secondary dilution. Bacterial pollution is introduced into the model of secondary dilution through the nodes located in the wastefield (at the distance \( x_i \) from the diffuser, Figure 1). The calculated value of initial dilution at the outfall location is obtained iteratively by redistribution of total flux in particular nodes.

3.2.1 Flow field model

Previously described specificities of modeling of bacteriological pollution enable the use of simpler stationary flow field models. The mathematical base of the model is 2-D equation of potential flow:

\[
\frac{\partial^2 \Phi}{\partial x^2} + \frac{\partial^2 \Phi}{\partial y^2} = 0 ; \quad v_x = \frac{\partial \Phi}{\partial x}, \quad v_y = \frac{\partial \Phi}{\partial y}
\]  

\( (1) \)
where $v_x$ and $v_y$ are components of velocity vector of the velocity field in $x$ and $y$ direction. The solution is sought over an area $\Omega$ surrounded by boundary $\Gamma$ where $\Gamma_l \subset \Gamma$ is an open boundary. Boundary conditions on an open boundary $\Gamma^l_1 \subset \Gamma^l_i$ through which the flow exists can be set as known values of the potential:

$$\Phi = F(x, y) ; (x, y) \in \Gamma^l_i$$

(2)

or as known value of flux through the boundary surface:

$$v_x n_x + v_y n_y = V(x, y) ; (x, y) \in \Gamma^{l'*}_i$$

(3)

where $\Gamma^{l'*}_i \cup \Gamma^{l'*}_i \equiv \Gamma^l_i$ and $n_x$ and $n_y$ are components of normal vector on the boundary. The rest of the open boundary $\Gamma^2 = \Gamma^l_1 \backslash \Gamma^l_i$ is considered to be parallel to the streamlines and therefore there is no flow through it.

Standard Galerkin method and finite element method are used for numerical calculation [2].

Intention is to obtain a flow field that is the most similar to the one measured at measuring stations by selection of adequate discretisation and boundary conditions. In the process of adaptation of flow field, beside discretisation and boundary conditions, there is also a possibility to assign the value of velocity vector to particular elements.

### 3.2.2 Model of transport and bacterial decay

Mathematical base of the model of transport and decay of bacterial pollution is 2-D stationary form of convection-diffusion equation:

$$\frac{d c}{d t} = \frac{\partial}{\partial x} \left( D_x \frac{\partial c}{\partial x} \right) + \frac{\partial}{\partial y} \left( D_y \frac{\partial c}{\partial y} \right) - K c$$

(4)

where: $c$ is bacterial concentration, $D_x$ and $D_y$ is turbulent diffusion in $x$ and $y$ direction, $K$ is a coefficient of bacterial decay. Solution of equation (4) is sought over the area $\Omega$ with boundary $\Gamma$ for constraint and natural boundary conditions on the open boundary $\Gamma_i$ formed as:

$$c = C(x, y) ; (x, y) \in \Gamma^l_i$$

(5)

$$\left( D_x \frac{\partial c}{\partial x} \right) n_x + \left( D_y \frac{\partial c}{\partial y} \right) n_y = 0 ; (x, y) \in \Gamma^{l'*}_i$$

(6)

where $\Gamma^l_i \cup \Gamma^{l'*}_i = \Gamma^l_i$. Using such boundary conditions on the open boundary, application of basic lemma of calculus of variation and Gauss-Green-Ostrogradski theorem into diffuse member of the expression (4), the weak formulation can be obtained:

$$\int_{\Omega} \left( v_x \frac{\partial c}{\partial x} + v_y \frac{\partial c}{\partial y} \right) w d\Omega + \int_{\Omega} \left( D_x \frac{\partial c}{\partial x} \frac{\partial w}{\partial x} + D_y \frac{\partial c}{\partial y} \frac{\partial w}{\partial y} \right) d\Omega + \int_{\Omega} K c w d\Omega = 0$$

(7)

where $w$ is a test function - a variation of solution.

Problem (7) is then solved using the finite elements method. Solution is sought in form of linear combination of localized basic functions:
\[ c = \sum_{r=1}^{n} c_r \varphi_r \]  
\( (8) \)

where \( \varphi_r \) is localized basic function for node \( r \) of finite elements net, \( c_r \) is value of solution in node \( r \), and \( n \) is total number of nodes in the net. Space discontinuous Galerkin method (SDGM) [15] is used to find the solution, and test functions are selected in the following form:

\[ w_s = \varphi_s + \delta v_x \frac{\partial \varphi_x}{\partial x} + \delta v_y \frac{\partial \varphi_y}{\partial y} ; \quad s = 1,2,\ldots,n \]  
\( (9) \)

\[ \delta = \frac{|v_x| \Delta x + |v_y| \Delta y}{(v_x^2 + v_y^2)^{1/2}} (1 - e^{-Pec}) \]

\[ Pec = \frac{v_x \Delta x + v_y \Delta y}{D_x + D_y} \] - Peclet number

where \( \Delta x \) and \( \Delta y \) are characteristic lengths of finite elements in \( x \) and \( y \) direction.

Selected SDGM method considerably decreases occurrence of oscillations of the solution for high values of Peclet number. Uniformity of solution is secured by constraint boundary conditions.

Selected weak formulation (7) gives certain flexibility in setting of boundary conditions because it allows omission of boundary conditions through open boundaries, assuming there is only convective component through these boundaries. Possible error due to this assumption is several times smaller than possible error due to arbitrary defined constraint boundary conditions through discharge boundaries.

4. Example of application of the model

Model was used to determine the possible bacterial pollution of sea water due to the construction of planned submarine outfall of the Trogir-Kaštel collection system. Outfall is located on the south side of the Čiovo island in front of Mavaršica Bay in Split Channel (Figure 2). One of analyzed alternatives will be described.

Analyzed alternative of submarine outfall is situated around 1600 m from the shore, diffuser length is \( L=400 \) m and it is laid in NS direction. Waste water quantity is \( 650 \) l/s. Density of waste water is \( \rho_w=1000 \) kg/m\(^3\). Concentration of coliform bacteria in waste water is \( c_0=10^7 \) MPN/100ml. Depth of discharge is \( H=52 \) m. Picnocline position is at depth of 15 m. Below the picnocline density is approximately constant and mean density of sea is around \( \rho_a=1028 \) kg/m\(^3\). Coefficient of coliform bacterial decay is \( K=12.3 \) 1/day (\( T_{90}=4.5 \) hours). Velocity in the area of upwelling of polluted plume is \( v_u=15 \) cm/s and it has NW (\( \theta=45^\circ \)) direction. Coefficient of turbulent diffusion was
selected in accordance with Brooks model of secondary dilution and its value is $D_x = D_y = 1\text{m}^2/\text{s}$.

Area covered with the model is shown in figure 2. Modeled area was discreted with the net consisted of 943 finite elements (boundary elements were also included). Nine-nodal 2-D elements were selected, therefore the net consisted of 3359 nodes.

Results of modeling are shown in figure 3. Maximum concentration of bacteria at the outfall location is $c_m = 3910 \text{MPN/100ml}$ that is far less than legally required $20000 \text{MPN/100ml}$. Height to top of wastefield is $z_e = 37 \text{m}$. Results of secondary dilution are shown as isolines with $50 \text{MPN/100m}$ step. Maximum concentration of bacteria in coastal strip is around $400 \text{MPN/100ml}$, which has not exceeded maximum allowable concentration for the coastal area of $500 \text{MPN/100ml}$.

As it can be observed model didn’t show significant numeric oscillations of the solution, which means that the selected finite element net was dense enough in all parts. Some numeric oscillations still exist in solutions which can be observed by slightly “winding” isolines. Oscillations are the most distinct at the outfall location where there are rapid changes in the value of solution and values of Peclet number $Pe_g$ are also slightly higher.
Application of Brooks model for design of submarine outfalls leads to unnecessary overdesign of outfall and diffuser, since the assumption of constant direction and intensity of flow field along the entire area is not realistic. Such modeling requires more complex models.

Model presented in this paper was used to calculate the initial dilution using parametric models that resulted from researches [3], [4], [5], [6] and [7].
Since bacterial decay in the sea is relatively fast, for analyses of expansion of bacterial pollution it is necessary to model only the smaller area in the vicinity of the outfall and directly endangered coastal zone. In accordance with that fact, the flow field is generated using the assumption of stationary potential flow. Intention is to obtain a flow field that is the most similar to the one measured at measuring stations by selection of adequate discretisation and boundary conditions. Additional possibility is to define the value of velocity vector in particular elements of the net.

Submodel of transport and bacterial decay is adapted to the size of modeled area by selection of appropriate boundary conditions. Selected weak formulation (7) allows the omittance of boundary conditions through open boundaries assuming there is no diffusion through these boundaries. Error derived from this assumption is several times smaller than possible error due to arbitrary defined constraint boundary conditions. Selected SDGM method considerably decreases a possibility of oscillations of the solution.

Model CDB-2D obtained satisfactory results that were very useful for analyses of expansion of bacterial pollution and especially for analyses of influence of certain parameters of submarine outfall and diffuser on the value of initial and secondary dilution. Obtained results can have significant application for design of submarine outfall and selection of the most favorable alternative. Designed model was used to analyze, in relatively short time, a great number of alternative solutions of the outfall and diffuser upon which the most optimal solution was selected after technical and economic analyses were carried out. Model was successfully used for sensitive analyses of selected solution in relation to fulfillment of sanitary protection of coastal sea.

References


