A modern concept simplifying the interpretation of pumping tests
M. Stundner, G. Zangl & F. Komlosi
LISTEN AND TALK - Environmental Services, A-2500 Baden, Austria
E-mail: listen+talk@magnet.at

Abstract

A thorough analysis of hydrologic pumping tests requires the knowledge about the flow regimes in order to employ the right analysis method. A two step approach to the analysis process is recommended that offers the possibility of a precise definition of each flow regime. A qualitative surveying should precede the quantitative analysis of a pumping test using conventional straight line and type-curve matching techniques. This diagnostic step is based on the representation of the drawdown together with the derivative of the drawdown with respect to the natural logarithm of time. Within the petroleum sciences the practical use of this technique was mainly pioneered by D. Bourdet.

The theoretical principles and application of the derivative plot, its inherent advantages and practical considerations concerning its proper use will be demonstrated.

1 Introduction

The interpretation of pressure or drawdown data recorded during a pumping test aims to evaluate aquifer flow characteristics. The results of this analysis are important factors for decisions concerning groundwater management or countermeasurements in the case of groundwater contamination.

Especially the transient phase of a pumping test can reveal a lot of important aquifer characteristics. Therefore an abundance of different kinds of test procedures and various methods of analysis have been developed. These methods are based on the following principle. A particular known signal as a certain production or injection rate or a sequence of various rates is imposed on the unknown system groundwater reservoir. The response of the system to this signal is then measured in terms of drawdown versus time. These data are compared with well-defined theoretical aquifer models having the same input signal. In matching both the theoretical and real response of the aquifer the
interpreter tries to estimate specific reservoir parameter. Thus the interpretation of pumping tests relies very much on the use of the right theoretical model.

2 Conventional methods

The most widely used method is based on the semilog representation of the drawdown versus the time. This method was first introduced by Cooper and Jacob[5] for drawdown tests and extended by Theis[6] for recovery analysis. This method is in fact a more restricted version of the Theis well function, which is found under the symbol W(u) as well as -Ei(-u) in the literature. These formulations describe an ideal reservoir, which is horizontal, has a constant thickness, a lower and upper confining boundary and constant reservoir and fluid properties. Flow in such an ideal reservoir is not influenced by inner and outer boundaries. Hence this model exhibits an infinite acting radial homogeneous flow regime. Unfortunately real aquifer behavior and the influence of the wellbore cause several different flow regimes to appear within a single pumping test in the majority of the cases. Thus the mentioned straight line methods have to be used with caution regarding the fact that straight lines can easily be found on a semilog plot. The situation gets even more difficult if there is no sequence present showing infinite acting radial flow.

The procedure, which helps to overcome the shortcomings of the straight line methods and which have been used for decades, is type curve matching. This technique is also called "log-log analysis" because drawdown versus time is represented on a log-log scale. These type curves represent the drawdown behavior of theoretical aquifers. The big advantage of this technique now is that a sequence of several flow regimes can be represented on one plot. This ability enhances pumping test interpretation in two ways. First it facilitates quantitative analyses, when it is not possible to apply semilog methods, or it performs as a backup to the conventional analysis. And second the application of the type-curve analysis provide qualitative information since different aquifer behavior causes different shapes of the aquifer pressure response.

In this sense the Theis method is also a type-curve matching technique though it represents only a single flow regime. Other very well known methods are the following. Walton’s method and Hantush’s type curves[5] are both for evaluating leaky aquifers. The family of Neuman’s type curves[5] describes the behavior of an unconfined aquifer. Each of these three sets of curves shows at least two different flow regimes overlapping each other partially, which are induced by aquifer characteristics. On the contrary the type curve set of Agarwal, Al-Hussainy and Ramey[1] introduced in petroleum engineering represents a model composed of a radial infinite acting reservoir influenced by a wellbore with finite extension and a near well zone disturbing the pressure behavior of the reservoir. The widely used set of Gringarten et al.[3] are basically the same as the above ones. It is just the graphical representation which is different. All these techniques have one big problem in common that
severely restricts their performance in quantitative as well as in qualitative analysis. Over wide ranges of time the curves on one set are barely distinguishable and quite insensitive to changes of the governing variables.

3 The Pressure Derivative

3.1 Theory

In transient pressure analysis the variation of pressure or drawdown with time is the information which is used for identifying and estimating aquifer properties. This task is accomplished by making use of graphs of pressure versus time either with straight line methods or with the type curve matching technique. But one can also consider procedures which are based on the rate of change of pressure versus time (dp/dt). From a mathematical point of view the derivative of pressure with respect to time is even more fundamental than pressure itself as it is the pressure partial derivative appearing directly in the diffusivity equation, which describes the transient fluid flow in an aquifer system. Hence the derivative response is more sensitive even to small trends.

![Figure 1: Dimensionless drawdown and derivative (from Bourdet et al.[2])](image)

Though Tiab and Kumar[7] described the application of the pressure derivative for the first time, it was the set of type curves from Bourdet[2] which made this technique widely accepted. Figure 1 shows the common representation of the drawdown and its derivative in dimensionless variables.

This set of curves represents the theoretical response of a constant rate test in a well with finite extensions that completely penetrates a infinite radial
acting, homogeneous, isotropic, confined aquifer for various welbore storage and near well conditions. The dimensionless parameters used here are defined as follows:

\[ p_D = \frac{(2\pi T/Q)\Delta h}{4} \]  
\[ C_D e^{2s} = r_w^2 / 2(r_w^2 S) \]  
\[ t_D = \left( \frac{T}{2} \right) / \left( r_w^2 S \right) \]

where the meaning of the variables can be seen from nomenclature at the end of the paper. It is clearly visible that the appearance of the different flow regimes is much more pronounced on the derivative. The curves are generated by taking the derivative of pressure with respect to natural logarithm of time in the following way:

\[ \frac{dp_D}{d\ln(t_D/C_D)} = \frac{t_D}{C_D} \frac{dp_D}{d(t_D/C_D)} = \frac{t_D}{C_D} p_D' \]

The first regime clearly exhibited on the plot is the wellbore storage effect. This early time behavior is described by the dimensionless formula

\[ p_D = \frac{t_D}{C_D} \cdot \]

The derivation according to eqn. (4) yields

\[ \frac{t_D}{C_D} p_D' = \frac{t_D}{C_D} \cdot \]

Drawdown and its derivative show the same behavior at early time. During the infinite acting period induced by the aquifer the drawdown is given by:

\[ p_D = 0.5 \left[ \ln(t_D/C_D) + 0.80907 + \ln(C_D e^{2s}) \right] \]

As the semilog slope is constant in this flow regime the derivative yields the following result

\[ \frac{t_D}{C_D} p_D' = 0.5 \cdot \]

This characteristic of the derivative not only facilitates the type curve matching capabilities of this graphical method but also outlines the superior possibilities offered in qualitative analysis. Infinite acting radial homogeneous flow regime exhibits a very distinctive behavior.

### 3.2 Differentiation

One serious problem in the application of the derivative of pressure for analysis of pumping test data is the noise in the signal. This is also the reason for the late advent of this method. The high resolution pressure transducers necessary to get precise measurements have not been available before. So the noise was just too high for the sensitive process of differentiation to be successful.
Nevertheless some noise will always be present. The problem when differentiating real data is to remove as much of the noise as possible without altering the signal. The numerical differentiation recommended by Bourdet[2] works the following way. It chooses one point left and one point right of the point of interest for which the derivative is calculated. Then the first derivative of the pressure change with respect to the natural logarithm of the change of time is calculated between the left point and the point of interest and the right point and the point of interest. The weighted mean of these two derivatives is placed at the point of interest. The most simple approach is to use three consecutive points. This approach leads to a quite noisy derivative. The result is improved when choosing the left and right points sufficiently distant from the middle point. The distance between the abscissas of the respective points is expressed in terms of the appropriate time function of the particular representation. These distances can vary between 0.0 - 0.5. The higher this value the more pronounced is the smoothing effect. Generally it is recommended to try several spacings to get an idea of the influence of this number.

Care has also to be taken with early and late time data. The approach with the differentiation interval may flatten the early data in an excessive manner. Since this part of the data is not sensible to noise one can use quite short intervalls or even arithmetic differentiation. In the case of the late time portion of the data problems may occur when the abscissal distance to the last data point is less than the spacing. Then the distance of the respective point of interest to the point on its right side has to be fixed to the value of the distance to the last point.

4 Identification of flow regimes

4.1 Procedure of interpretation

The most important application of the derivative of the drawdown with respect to natural logarithm of time has become the diagnostic interpretation of the whole pumping test sequence. For this purpose all the test data should be represented in the proposed form at the start of the analysis process. As will be shown later it is not only the infinite acting radial behavior of a homogeneous aquifer which will exhibit a distinctive shape on the derivative plot. Nevertheless the qualitative analysis has to start with the separation of the main parts of the whole period into three distinct regions. First there will be the early time data which are strongly influenced by wellbore and near wellbore effects. This part of the data is then followed by the response of the aquifer. The late time region starts when boundary effects begin to influence the infinite acting of the aquifer. After this distinction of flow periods has been made a more detailed interpretation including further use of the derivative as well as straight line quantitative analysis methods can follow.
The thorough investigation of the early time region can reveal information about near well flow restriction due to the drilling and completion process. A quantification of this flow restriction is possible in the following quantitative interpretation. Furthermore, information can be gained about vertical permeability under certain circumstances.

The middle time region of the pumping test response can directly lead to the semilog analysis of a radial infinite acting behavior. But it is also possible that a more complex behavior of aquifer induced flow is present. This case will require a more detailed investigation of the shape of the derivative before specialised straight line methods are employed.

The precise definition of the beginning of the late time period will allow to estimate reservoir parameters with a high degree of accuracy. Furthermore, the use of superposition in space will provide information about the distance and type of boundaries.

### 4.2 Patterns of various flow regimes

On the figures (2-5) presented in this chapter are some of the patterns of the derivative in log-log scale of frequently encountered aquifer systems. These are theoretical curves generated from analytical models.

#### 4.2.1 Unconfined Aquifer

![Figure 2: Unconfined aquifer behavior, drawdown and derivative](image)

The derivative on Fig. 2 clearly shows the limits of the two periods of the response of an unconfined aquifer analyseable with the semilog straight line method. The period of delayed watertable response has a clear impact on the
shape of the derivative. This representation does not account for any wellbore effects.

### 4.2.2 No flow boundary

The following plot (Fig. 3) shows the aquifer response including wellbore as well as boundary effects. The late time behavior of both curves is typical for a drawdown test. Though the pressure trend also indicates this system behavior it is the derivative which separates the different flow regimes without any doubt. This is an absolute prerequisite in employing the semilog straight line method to the right portion of the response of the aquifer.

![Figure 3: Response of an aquifer with influence of no flow boundary](image)

### 4.2.3 Sealing fault

Figure 4 shows the behavior caused by the presence of a linear sealing fault in an otherwise radial infinite acting homogeneous aquifer. The influence of the wellbore is not included in this plot.

The first flat line on the derivative is indicative of the undisturbed aquifer behavior. When the influence of the fault is felt a transition period is established. This transition period develops into a second flat line, which is equivalent to the doubling of the slope on the semilog plot.
Figure 4: Drawdown and derivative pattern caused by a sealing fault

4.2.4 Heterogeneous behavior
An aquifer system which is heavily fissured and exhibits matrix to fissure flow shows the response on Figure 5. The drop and rise of the derivative is typical for a heterogeneous behavior. If such a pattern is visible on the derivative, one should continue further interpretation with care. Similar behavior can also be caused by a layered aquifer or distinctive areal changes in permeability. It is useful to use data from different sources in such a situation.

Figure 5: Dual porosity behavior
Conclusions

Employing the derivative of the drawdown in a pumping test for interpretation purposes improves quality of the analysis. The limits of the particular flow regimes are much better defined. Hence the quantitative estimation of aquifer properties gets more reliable.

This way of interpreting pumping tests considers all measured data with an improved sensitivity. This representation allows the identification of all existing flow regimes. This is the precondition for correct use of specific quantitative analysis methods. Even if there is no qualitative analysis possible the derivative may give some qualitative information about the aquifer which helps to design further investigations.

Though the quality of the measured data is usually sufficiently high nowadays, the differentiation process has to be conducted with care. Noise in the data should be smoothed without changing the reservoir response. The efficient algorithm described here has proved its applicability over a wide range of actual field data.

Nomenclature

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Units</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>CD</td>
<td>[-]</td>
<td>dimensionless wellbore storage coefficient</td>
</tr>
<tr>
<td>Δh</td>
<td>[m]</td>
<td>drawdown</td>
</tr>
<tr>
<td>p_D</td>
<td>[-]</td>
<td>dimensionless pressure</td>
</tr>
<tr>
<td>p_D'</td>
<td>[-]</td>
<td>derivative of dimensionless pressure</td>
</tr>
<tr>
<td>Q</td>
<td>[m³/s]</td>
<td>discharge rate</td>
</tr>
<tr>
<td>r_C</td>
<td>[m]</td>
<td>well casing radius</td>
</tr>
<tr>
<td>r_W</td>
<td>[m]</td>
<td>effective well radius in test intervall</td>
</tr>
<tr>
<td>s</td>
<td>[-]</td>
<td>skin factor</td>
</tr>
<tr>
<td>S</td>
<td>[-]</td>
<td>storativity</td>
</tr>
<tr>
<td>t</td>
<td>[s]</td>
<td>time since test started</td>
</tr>
<tr>
<td>T</td>
<td>[m²/s]</td>
<td>transmissivity</td>
</tr>
<tr>
<td>t_D</td>
<td>[-]</td>
<td>dimensionless time</td>
</tr>
</tbody>
</table>
REFERENCES


RECOMMENDED LITERATURE
