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2-D numerical modelling of pollution in estuaries with application to the Bay of Santander in Spain

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Abstract

The mathematical model for coastal zones management requires the elaboration of two numerical models. The first of them (circulation model) allows us to describe the state of currents and tide in the estuary, providing the data necessary for the second model application (dispersion model). This mathematical model will provide the concentrations of the polluting particles, dissolved in water.
In this paper a numerical model has been developed, based on the finite elements method, to forecast water movement and pollution in bays and estuaries. The response of the system is obtained as the solution of the continuity, moment and dispersion equations.
The developed model is based on a Galerkin formulation of the 2-D equations of the hydrodynamics and dispersion of the estuaries, with spatial discretization through linear triangular. The time integration scheme is in finite differences. In the circulation model an implicit scheme of two steps with 2nd order precision is followed; in the dispersion model an iterative method is carried out.
The model has been applied to the resolution of several tests, verifying its efficiency. Finally it has been applied to the Bay of Santander to obtain the space and time evolution of the distribution of the pollutants in different cases.

1 Basic equations.

For the circulation model the shallow water equations are used and for the dispersion model, the mass transfer equation which includes transportation, diffusion and reaction processes is used [1] [5].

$$\frac{\partial H}{\partial t} + \frac{\partial (HU)}{\partial x} + \frac{\partial (HV)}{\partial y} = 0 \tag{1}$$

$$\frac{\partial U}{\partial t} + U\frac{\partial U}{\partial x} + V\frac{\partial U}{\partial y} - fV + g\frac{\partial \zeta}{\partial x} = 0 \tag{2}$$

$$\frac{\partial V}{\partial t} + U\frac{\partial V}{\partial x} + V\frac{\partial V}{\partial y} + fU + g\frac{\partial \zeta}{\partial y} = 0 \tag{3}$$

$$\frac{\partial \Theta}{\partial t} + U\frac{\partial \Theta}{\partial x} + V\frac{\partial \Theta}{\partial y} - \frac{1}{H}\left[\frac{\partial}{\partial x}\left(HD_x\frac{\partial \Theta}{\partial x}\right) + \frac{\partial}{\partial y}\left(HD_y\frac{\partial \Theta}{\partial y}\right)\right] - F + \alpha\Theta - \sum_j Q_j\delta_j = 0 \tag{4}$$

To solve this equations system, the contour conditions need to be established: At fixed borders (coast, islands or land) $v_n = \bar{v}_n = 0$ and at open borders $\zeta = \bar{\zeta} = A \sin\left(\frac{2\pi}{T}t\right)$, being $H = \zeta + z$. At the points where the concentration is imposed $\theta = \bar{\theta}$; in the part of the border where the flow is imposed $q_n = -\rho D\frac{\partial \theta}{\partial n} = \bar{q}_n$, and in the part where the border is not reflexive or absorbent: $q_n = \gamma(\theta - \theta_e)$. The initial conditions should also be specified: $\zeta(0) = \bar{\zeta}_0$; $U(0) = \bar{U}_0$; $V(0) = \bar{V}_0$ and $\Theta(0) = \bar{\Theta}_0$.

2 Finite Element Approximation.

To obtain an approximate solution, a weak formulation of Galerkin is used. The unknowns are developed through the expressions:

$$\zeta \approx \sum_{j=1}^{M} \zeta_j(t).\Phi_j(x,y)\ ; \quad U \approx \sum_{j=1}^{M} U_j(t).\Phi_j(x,y)$$
$$V \approx \sum_{j=1}^{M} V_j(t).\Phi_j(x,y)\ ; \quad \Theta = \sum_{j=1}^{N} \Theta_j(t).\Phi_j(x,y) \tag{5}$$

with similar equations for the remaining parameters. For the spatial discretization the linear triangular element has been elected. Approximating the solutions of eqns (1), (2), (3) and (4) by eqns (5) and assembling them, we obtain the nonlinear system

$$[\,M\,]\{\,\dot{\zeta}\,\} - [\,A\,]\{\,H\,\} = -[\,R\,]\{\,H\,\} \tag{6}$$

$$[\,M\,]\{\,\dot{U}\,\} - [\,B\,]\{\,U\,\} - [\,N\,]\{\,V\,\} + g\,[\,P\,]\{\,\zeta\,\} = 0 \tag{7}$$

$$[\,M\,]\{\,\dot{V}\,\} - [\,B\,]\{\,V\,\} + [\,N\,]\{\,U\,\} + g\,[\,Q\,]\{\,\zeta\,\} = 0 \tag{8}$$

$$[\,M\,]\{\,\dot{\Theta}\,\} = \{\,E_{\Theta}\,\} \tag{9}$$

where matrices [A], [B] are functions of U, V [6].

3 Time integration.

The differential equations system of the circulation model (6), (7) and (8) is solved using finite differences with an implicit scheme of two steps [3]. In the dispersion model equation (9), the time integration scheme, in finite differences, is solved iteratively, using the known values of $\{\Theta\}_t$ as well as a valuation of $\{\Theta\}_{t+\Delta t}$, until a degree of specified tolerance is obtained [4], [2].

4 Numerical experimentation.

The case of a channel is studied, represented in the figure 1 and in which a polluting substance is injected throughout the right $x = 0{,}75\ m$.
The data used are: $U = 0.1\ m/sec$, $V = 0$, $D_x = 0.01\ m^2/sec$, $D_y = 0$, $H = 1.0\ m$ and $Q_k = 0.5\ sec^{-1}$.

4.1 Conservative pollutant.

The analytical solution for the case of a conservative substance, when the stationary regime has been reached, is:

$$\theta(x) = \begin{cases} 0.0221\, e^{10x} & x < 0.75 \\ 40.0 & x \geq 0.75 \end{cases} \tag{10}$$

Figure 2 represents the pollutant concentration as a function of the distance, corresponding both to the analytical solution and to the obtained FE solution.

4.2 Non conservative pollutant.

For the case of a non conservative pollutant with $\alpha = 0.01\ sec^{-1}$, the analytical solution is:

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$$\theta(x) = \begin{cases} 0.0201\ e^{10.0990x} & x < 0.75 \\ 42.2469\ e^{-0.0990x} & x \geq 0.75 \end{cases} \tag{11}$$

The approximate solution obtained by the FE method is represented in figure 3 where it is compared with the exact solution.

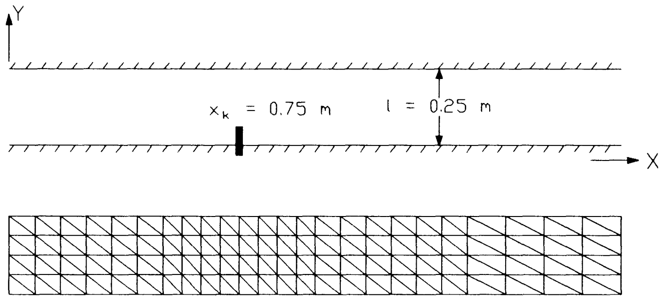

Figure 1: Channel geometry and discretization.

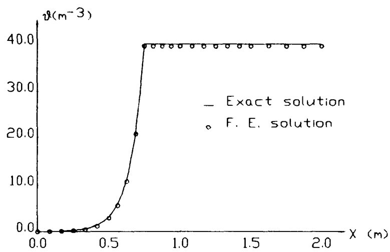

Figure 2: Conservative pollutant

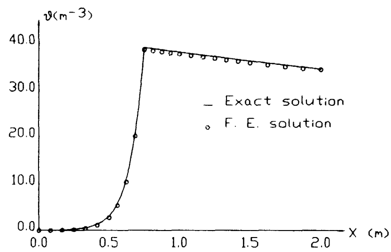

Figure 3: Non conservative pollutant

5 Application to the bay of Santander.

The Bay of Santander is located in the north of Spain. Its geometry is shown in figure 4. The Bay's surface is discreted by means of 130 elements and 89 nodes. The contour hydrodynamic conditions imposed are: At nodes of the oceanic border $\zeta = 2.0 \sin \frac{2\pi}{44640} t$, at nodes of the southern border $v = 0$ and at land borders $v_n = 0$.

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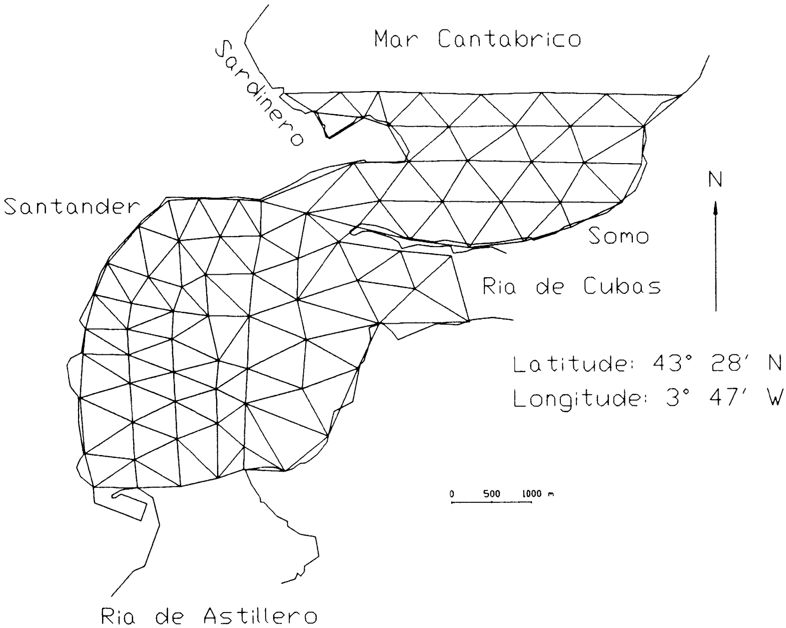

Figure 4: Geometry and discretization of the Bay of Santader

5.1 Conservative pollutant.

The effect of the entry of a conservative substance, with concentration unity, at the southern border has been modelled. A concentration zero in nodes of the oceanic border is imposed. $D_x = D_y = 10^3$ m^2/sec and $\Delta t = 15$ sec are data. The computed results are shown in figure 5 where the constant concentration isolines in the corresponding instants to high and low tide respectively are represented.

5.2 Stationary discharge at a point.

An effluent that discharges a population of bacteria in a continuous manner is supposed at the indicated point, and its distribution in the bay at various instants is studied. It is considered that the concentration on the southern border, where a river enters, and in the open sea, is null. For this study the following data are taken:

Discharge wealth:	$Q = 10^3$ l/sec
Density of the effluent:	$\theta_e = 10^3$ $bac/100$ ml
Decay coefficient:	$\alpha = 10^{-5}$ sec^{-1}
Dispersion coefficient	$D_x = D_y = 10^3$ m^2/sec
Time interval:	$\Delta t = 15$ sec

Oceanic and south border: $\theta = 0$
Figures 6 show the results obtained by the model. The constant concentration isolines for different states of the tide are represented therein. The instants that correspond to flood and ebb tide respectively have been taken.

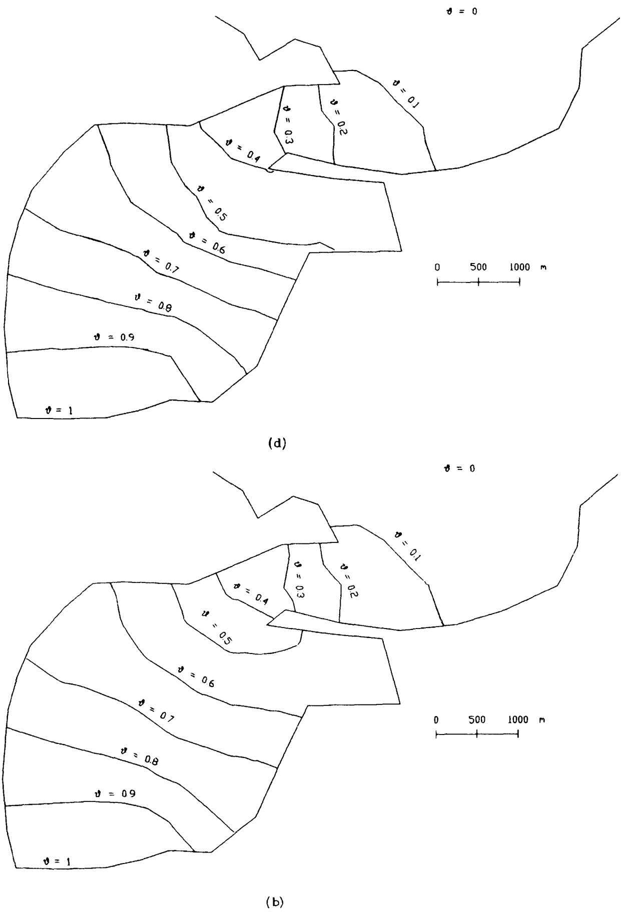

Figure 5: Distribution of conservative polluting in the Bay of Santander. (a) high tide. (b) low tide.

Conclusions.

The modular scheme used allows us to modify the boundary conditions and the source terms easily in order to simulate different processes. The model admits non-uniformly distributed sources and variable sources with time.

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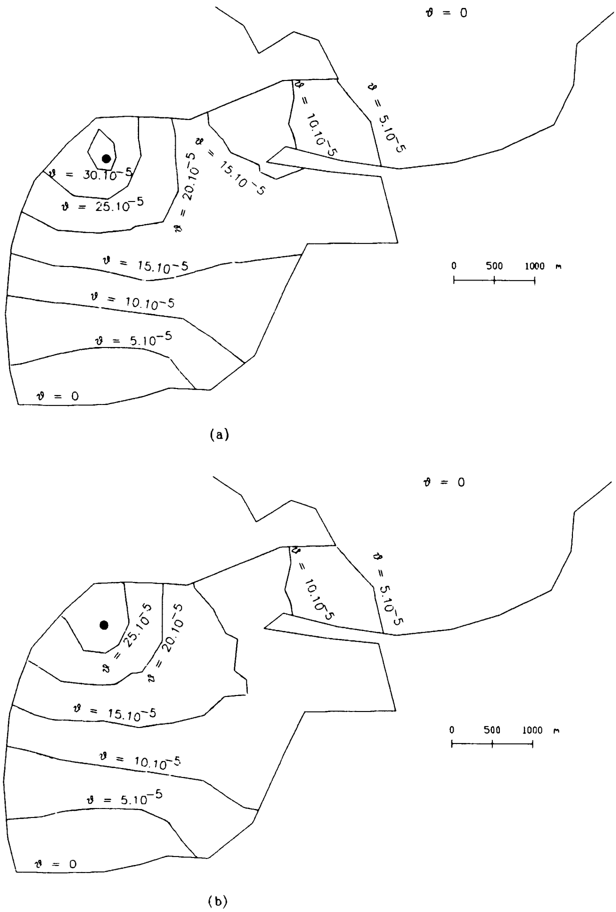

Figure 6: Stationary discharge at a point. (a) flood tide. (b) ebb tide.

For the discretization of the domain linear triangular elements has been used. The time integration of the equations of the hydrodynamics has been effected through an explicit method of two steps, and the equation of the dispersion through an implicit iterative scheme to minimize the computer report required. These methods cause a great speed to obtain the solution. For a time interval of $\Delta t = 15$ *sec*, with a mesh of NN = 89 nodes, NE = 130 elements and bandwidth NBAN = 22, a time of 12 minutes 20 secs. is invested in solving the two tide cycles in an 80486 / 66 MHz processor.
One of the prevailing factors for an adequate approximation level, is the definition of the finite elements mesh. It has been proven that the generation of the mesh

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requires prior knowledge of the position of the different sources, increasing the node density in zones adjacent to where strong gradients of the dependent variable are presented. Besides, a relationship among the areas of the adjacent elements superior to 2:1 produces oscillations in the solution and numerical instability. To assure the stability of the model, the time interval must verify $\Delta t \leq 0.3(\Delta x)^2/2D$. It has been proven a greater precision in the nodes of the finite elements mesh and a greater error in the interior of the element. It has also been proven that the quadratic middle error reduces with the square of the number of elements used. Analysing the results obtained in some studied cases, it has been confirmed that the nodes of the optimum mesh are located throughout the constant velocity and concentration isolines.
The sensibility of the parameters has been studied . The application of transient and accidental discharges of pollutant, provokes meaningful values in the concentration at all points, with a delay that depends on the dispersion coefficient.

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