A comparative study of soil column experiments and inverse parameter optimization predicting the unsaturated hydraulic functions

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Abstract

Several experimental methods are employed to determine effective hydraulic functions of an unsaturated homogeneous sand. Classical one-step outflow and evaporation experiments were carried out on small soil columns, and a modified one-step outflow method was employed on a column with 150 cm length and a saturated-unsaturated water flow. Hydraulic properties of the quartz sand are assumed to be represented by van Genuchten’s closed-form expressions, the parameters $\alpha$ and $n$ given by this expressions are evaluated with an inverse optimization procedure. Only the large column experiment results allow to draw conclusions from parameter variation to pore size distribution and the hydraulic behaviour of the soil in a dynamic flow regime.

1 Introduction

Knowledge of soil hydraulic properties is an essential requirement for prediction of flow and transport through the unsaturated zone. There are many laboratory methods to determine these highly nonlinear functions. Usually column outflow or evaporation experiments are applied to obtain the relationships between water pressure, saturation and permeability. The problem of characterizing the soil hydraulic properties then reduces to estimating parameters of an appropriate constitutive model.

Besides the problems of uniqueness of the parameter estimations, an additional difficulty arise from the fact that different laboratory experiments...
provide various hydraulic characteristics for the same porous media. Dam van et al. [1] investigated a few laboratory and field methods and proposes effective parameters for the hydraulic functions of soils, representing the mean unsaturated soil moisture-water pressure relationships. The purpose of this paper is to compare three commonly used laboratory tests with a new concept concerning the onestep outflow method based on a synthetic porous medium. These methods are: (i) the one-step outflow method, Kool et al.[3], Kool & Parker [4]; (ii) the evaporation method, Dam van et al. [1]; (iii) the estimation of hydraulic properties from particle size distribution data, Mishra & Parker [5], and (iv) the modified one-step outflow method with a gravitational discharge of a saturated-unsaturated column, Nützmann et al. [7]. In this paper we consider at first the last method and their results followed by a discussion of all methods comparing optimized parameter values.

2 Materials and methods

The used synthetic porous medium for all evaluations is a quarz sand with particle sizes ranging from 0.30 to 0.80 mm and with a saturated hydraulic conductivity of $k_f = 2.92 \text{ cm/min}$. By means of the artificial package of the soil column a bulk density of $\rho_d = 1.5 \text{ g/cm}^3$ is achieved and the porosity is $n = 0.4$. This physical parameters were taken constant during the application of the estimation methods.

The conventional one-step outflow method (i) uses small standard cylindrical samples with a volume of 100 cm$^3$. During drainage the pressure head at the bottom of the column was increased erratically to $\psi = 0, 32, 63, 100, 316$ and 15000 cm. The evaporation method (ii) was carried out at standard cylindrical samples with a volume of 250 cm$^3$. The pressure head at two levels in the column and the total weight was recorded continuously during the experiment.

After determining the grain size distribution of the quarz sand the program SoilProp was applied to estimate unsaturated soil hydraulic properties, method (iii). The modified one-step outflow method (iv) uses a 150 cm soil column, where the initial fully saturated soil is only drained by a 'natural' groundwater table fluctuation, not by an 'artificial' pressure condition at the bottom of the column. For details of the experimental set-up see Nützmann et al. [7]. The evaluation of the parameters $\alpha$ and $n$, given by van Genuchten’s formulas, was done by inverse modeling.

3 Mathematical model

The transient saturated-unsaturated water flow in a vertical soil column is described by Richards-equation.
\[ C \frac{\partial h}{\partial t} + \frac{\partial}{\partial x} \left( K \frac{\partial h}{\partial x} \right) = 0. \]  

(1)

where \( h = \psi + x \) is the hydraulic head with \( \psi \) the pressure head and \( x \) the vertical distance, \( C = \frac{d\Theta}{d\psi} \) is the specific water capacity with \( \Theta \) the water saturation, \( K \) is the hydraulic conductivity and \( t \) is the time.

The solution of Eq. (1) with the appropriate initial and boundary conditions was obtained by a Galerkin finite element code SUNSOL, Nützmann [7]. Cumulative outflow is than calculated from discrete pressure head distribution at every time step using a spline-integration.

In this paper the hydraulic functions of the unsaturated sand are described by the commonly used Mualem/van Genuchten-model, Mualem [6], Genuchten van [2]

\[ \Theta = \frac{(\theta - \theta_r)}{(\theta_s - \theta_r)} = \left[ \frac{1}{1 + (\alpha \psi)^n} \right]^m, \ m = 1 - 1/n \]  

(2)

\[ K(\Theta) = K_s \Theta^{0.5} \left[ 1 - \left( 1 - \Theta^{1/m} \right)^m \right]^2 \]  

(3)

where \( \theta_s \) is the saturated water content, \( \theta_r \) is the residual water content, \( K_s \) is the saturated hydraulic conductivity, and \( \alpha \) and \( n \) are empirical parameters to be determined with an inverse optimization method of Levenberg-Marquardts type. In the case of measurements of the cumulative outflow values only, the objective function can be written as a \( \chi^2 \)-formulation. Necessary conditions for local identifiability could be satisfied, see Nützmann [8], and convergence of the Levenberg-Marquardt scheme is achieved because initial values of the parameters are pre-calculated with a simple gradient method.

4 Results and discussion

A series of outflow experiments according to method (iv) was carried out. Each experiment was driven due to a phreatic decline between the top of the column and the fixed groundwater level ranging from 0.20 m to 1.00 m. The minimization of the objective function (4) is now calculated as described above. As shown in Figure 1 the optimal parameters vary with the phreatic decline and converge to a constant value for gradients larger than 50 cm. This differs from previous results, Nützmann et al. [7], were a simple optimization routine was used.
The hydraulic functions of a homogeneous porous medium, which are considered commonly as unique expressions in unsaturated water flow simulation, seems to be dependent from the boundary condition and the length scale of the column. This dependence is also significant calculating the water retention function after Eq. (3a) as it is shown in Fig. 2. All curves for hydraulic gradients $\Delta H \geq 50$ cm have an identical plot, which could be expected. 

The behaviour for the final soil moisture distribution for each experiment was modeled and the consequences of gradient-dependent parameters are shown on the left side in Figure 3. In the upper region of the column a contradiction can be stated between the soil moisture distribution resulting from small gradient experiments ($\Delta H = 20, 30$ and $40$ cm) and these of larger ones. The water content at the top of the column does not show a reasonable behaviour, because for the three small-gradient experiments the upper soil is dryer than for those cases of deeper groundwater levels. Although the pressure heads result vice versa, which is caused by the estimated parameter values. To give an impression of the behaviour of a ‘homogeneous’ system, the mean values of the optimized parameters $\alpha$ an $n$ are calculated from the experiments with $\Delta H \geq 50$ cm to $\bar{\alpha} = 0.049$ and $\bar{n} = 2.36$, and the simulated final soil moisture distributions are depicted on the right side of Fig. 3.

The dependence of both parameters $\alpha$ and $n$ from the phreatic decline may be explained by an incomplete mathematical description of the hydraulic functions. This is due to the one-step outflow process at small gradients is principally driven by large active pores. Increasing the distance of the phreatic decline more and more small pores are activated for the drainage by increasing pressure heads. The convergence of the optimized parameter tuples shows, that

Figure 1: Optimized parameters $\alpha$ und $n$ by the method (iv).
Figure 2: Calculated water-retention functions.

Figure 3: Simulated soil moisture distribution.
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from gradients $\Delta H \geq 50 \text{ cm}$ all pores sizes contribute to the drainage process. In Fig. 4 the pore size distribution is illustrated by means of the water capacity versus pressure head. This is an additional evidence, that the range of pore sizes has reached a limit for gradients larger than 50 cm.

![Figure 4: Illustration of the pore size distribution.](image)

The question remains, whether it is possible to derive ‘representative’ soil hydraulic functions considering the different described experimental methods. Neglecting the results of the small-gradient experiments ($\Delta H = 20, 30$ and $40 \text{ cm}$), the method (iv) provides constant parameter tuples for the considered porous medium because the $\alpha$ and $n$ converge to the mean or effective values $\overline{\alpha}$ and $\overline{n}$ described above. This result and the parameters identified by the other methods are given in Tab. 1.

It is obvious, that the modified one-step outflow and the evaporation method produces almost identical results, while the classical one-step outflow method and the estimation from the particle size distribution differs significantly. In Figure 5 these differences are depicted by the corresponding water retention functions. The results indicate, that the length-scale of the column and/or
natural flow conditions of the experimental setup influence substantially the reliability of inverse parameter optimization.

Table 1: Optimized parameters.

<table>
<thead>
<tr>
<th>method</th>
<th>$\alpha$</th>
<th>$n$</th>
</tr>
</thead>
<tbody>
<tr>
<td>one-step-outflow (i)</td>
<td>0.103</td>
<td>2.90</td>
</tr>
<tr>
<td>evaporation (ii)</td>
<td>0.05</td>
<td>2.50</td>
</tr>
<tr>
<td>particle size distribution (iii)</td>
<td>0.089</td>
<td>4.73</td>
</tr>
<tr>
<td>modified one-step outflow (iv)</td>
<td>0.049</td>
<td>2.36</td>
</tr>
</tbody>
</table>

Figure 5: Water retention functions for the methods (i) - (iv).
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References


