Laboratory experiments related to the injection of buoyant fluid layers in stratified flows

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ABSTRACT

An experimental analysis on the injection of buoyant fluid in stratified flows is presented in this study. A numerical model of the buoyant fluid layer development in the stratified flow was developed and calibrated with the resulting experimental data. Results of numerical simulations agree quite well with experimental data for different stratification conditions. This model may be successfully employed to predict the trajectory of a laminar buoyant jet injected into a single or a double-diffusion (D-D) stratified environment.

INTRODUCTION

The disposal of liquid waste, resulting from industrial, municipal, agricultural and domestic activities, is commonly performed by discharging such wastes into natural water bodies. These water bodies are usually subject to temperature and salt variations along their depth. The environmental impact of the discharged pollutants depends on the efficiency of the mixing processes; these processes are characterized by the size and stratification of the receiving water body, by the buoyancy and momentum of the injected flow, etc. Liquid waste releases are often performed through multiport diffusers, which offer a high degree of initial dilution; however, the details of the three-dimensional jets issuing from the individual nozzles can be often neglected, and the pollutant discharge can be modeled as
a two-dimensional plane jet in a stratified environment [3]. Engineering predictions of such jets commonly use the integral method, which assumes that the profiles of velocity and density deficit are self-similar after the zone of flow establishment [6]. Being most environmental flows turbulent, several theoretical and experimental studies have been performed in the past on turbulent buoyant jets in stratified fluids [e.g. 1]; the laminar buoyant jets, instead, were not studied sufficiently. However, there is a certain similarity between the patterns of laminar and turbulent jets; fine-scale turbulence provides additional mixing on a small scale and behaves like an added eddy viscosity on the mean and large scale motions [2].

In this study we simulated the spread of the liquid waste in the vicinity of the injection ports by performing experimental and numerical investigations on the injection of buoyant fluid layers, under laminar conditions, in a D–D environment.

THE EXPERIMENTAL SET-UP

A laboratory set-up was constructed for the experiments related to the injection procedure. It consists of a flume whose length, width and depth are 320 cm, 60 cm and 100 cm, respectively, as shown in Figure 1. It is made of concrete, and one side of the flume is made of glass for flow visualization.

Fig. 1: Scheme of the experimental set-up
At the upstream side of the flume the entrance unit is located. The middle outlets were used to create the parallel jet flow of maximal thickness up to 6 cm (in the case of three jets). The upper and lower outlets are installed to circulate water at the bottom and the top parts of the flume in order to support constant temperature at the boundaries. Three outlets are located at the downstream part of the flume. The upper and lower outlets withdraw water to recirculate through the heat exchangers. The middle outlet removes water from the stratified zone and maintains the constant depth inside the flume. A system of tanks, connected with tubes and equipped by valves and flowmeters, provides the water storage needed for the gradient zone (GZ) creation and the discharge of water at the required salinity and temperature.

A special procedure was developed to enable filling the tank with the desirable density profile during a short time. Basically it is a modified Oster scheme [4], which suggests filling the pond by layered sections. This scheme works perfect for the salinity (S) and temperature-salinity (TS) stratified GZ, but it cannot be applied to create a temperature (T) stratified GZ, because of the continuous loss of heat from the surface during the filling process. The T stratified GZ was build-up through a preheating of the homogeneous temperature layer by hot water filled above it.

The density gradient was measured by slowly withdrawing fluid at a particular height and measuring its specific gravity by a hydrometer. The experimental uncertainty in the density measurements was ±2.0 kgm⁻³. The injection and filling discharges were measured by flowmeters of 1.4 – 33.0 cm³sec⁻¹ for small discharge values and 14 – 240 cm³sec⁻¹ for higher values and accuracy of 2.0%. Temperature profiles were measured by a PCLD – 889 Amplifier/Multiplexer Board which works in couple with the PCL – 711 PC MultiLab Card. This system was connected directly to a PC and could process 16 different inputs from the thermocouples which were fastened on the fiberglass bar with different intervals. Dye injected into the jet provided flow visualization and allowed observation of the jet behaviour. Crystals of potassium permanganate were also dropped into the tank along its length to make visible not only the jet, but also the flow patterns below and above the jet.

EXPERIMENTAL RESULTS

The experiments described in this study, performed under different conditions, may be classified as follows

- single jet injection into the T-stratified environment (T-series);
- single jet injection into the S-stratified environment (S-series);
- experiments with a single D–D jet injection into the D-D stratified environment (TS-series).
The major objective of this study was to analyze the influence of density difference on the jet deviation from the horizontal level. According to Zangrando [7] the deviation of a turbulent jet injected horizontally depends on the injection velocity and on the dimensions of the outlets and may be characterized by the Froude number; in the present set of experiments the Froude number is less than 1.0. The problem of turbulent jet propagation was successfully analyzed by means of dimensional analyses using the characteristic length scales defined as follows [e.g. 2, 5]:

\[
l_Q = \frac{Q}{M^{1/2}}, \quad l_M = \frac{M^{3/4}}{B^{1/2}}, \quad l_N = \frac{M^{1/4}}{N^{1/2}}
\]  

(1a–c)

where \( Q \) is volume flux, \( M \) is momentum flux, \( B \) is buoyancy flux and \( N \) is buoyancy frequency. Computation of the relative magnitudes of these length scales can help in unravelling the dominant mechanisms in particular situations. Roberts & Matthews [5] expressed the vertical jet deviation \( \Delta Z \) from the horizontal level as follows:

\[
\frac{\Delta Z}{l_N} = f\left(\frac{l_Q}{l_N}, \frac{l_M}{l_N}, \frac{M^{1/2}}{\nu}\right)
\]

(2)

\( l_N \) is chosen as the normalizing length scale as this will always be important for the present problem, while \( l_Q \) and \( l_M \) may or may not be important. The experimental results are plotted in the form of eq. (2) in Figure 2.

![Figure 2: Jets with negative, positive and neutral buoyancy](attachment://figure2.png)

*Fig. 2: Jets with negative, positive and neutral buoyancy* [Δ, ⊕, □ refer to \( \frac{\Delta Z}{l_N} = f(\frac{l_Q}{l_N}) \); ●, *, ◊ refer to \( \frac{\Delta Z}{l_N} = f(\frac{l_M}{l_N}) \)]
It is evident from Figure 2 that there is no influence of the $l_Q/l_N$ ratio on the jet deviation and there is a slight dependence on $l_M/l_N$ ratio. Only the buoyancy forces were significantly changed during the experiments, while the velocity at the outlet was kept almost constant; therefore, there was no possibility to assess the influence of the $M^{1/2}/\nu$ ratio on the jet deviation.

The mechanism of the jet deviation should be generally influenced by the initial density difference between the jet and the ambient fluid, $\rho_j - \rho_a$, the density gradient in the receiving environment, $\Delta \rho_a/\Delta z$, the buoyancy frequency $N$ and the jet initial velocity $||v_j||$. Following a dimensional analysis, the amplitude of the jet vertical displacement $\Delta Z$ may be expressed as

$$\frac{\Delta Z}{l_N} = f \left( (\rho_j - \rho_a) \left( \frac{\Delta \rho_a}{\Delta z} \right)^{-1} ||v_j||^{-1} N \right) \quad (3)$$

The experimental measurements are plotted in the form of eq. (3) in Figure 3.

![Figure 3: Dependence of $\Delta Z/l_N$ on the parameter $(\rho_j - \rho_a)(\Delta \rho_a/\Delta z)^{-1}||v_j||^{-1}N$; jets with positive (□) and negative (☆) buoyancy](image)

It is evident from Figure 3 that the value of $\Delta Z/l_N$ does not depend linearly on the RHS of eq. (3). Therefore, as long as values of the density gradient, of the velocity and of the buoyancy frequency in eq. (3) are not zero, a small difference between ambient and jet densities leads to big values of $\Delta Z/l_N$. The higher the density difference, the higher is turbulent mixing and therefore entrainment into the jet and as a consequence the jet rises or drops until the level where the ambient fluid density is equal to the mixed jet density.
NUMERICAL RESULTS

A mathematical model was developed for the analysis of the experiments; it essentially routes the jet through the domain of interest by solving the continuity, momentum and transport equations. Pressure variations from hydrostatic are assumed negligible. Entrainment, diffusion and dissipation terms are also included in the mathematical model. We consider a two-dimensional flow geometry, as sketched in Figure 4; the jet is discharged at vertical position $Y_j$, with jet thickness $d_j$, initial velocity $v_j$, temperature $T_j$ and solute concentration $C_j$. The jet flows into an ambient environment with linear temperature and solute concentration gradients over a depth $Y_G$. The equations governing the buoyant jet flow are the:

**equation of state**

$$\rho = \rho_o(1 - \beta_T \Delta T_o + \beta_C \Delta C_o) \quad (4)$$

where $\rho_o$ is the reference density, $\beta_T$ and $\beta_C$ are the thermal and solutal expansion coefficients, respectively, $\Delta T_o$ and $\Delta C_o$ are variations from reference property values;

**equation of mass continuity**

$$\frac{d}{d\zeta}(\rho d|v|) = \rho_a q_e = 2\alpha \rho_a (|v| - u_\zeta) \quad (5)$$

where $\chi = (\zeta, \eta)$ is an orthogonal reference system oriented in the direction of the jet velocity vector $v$, $q_e$ is the entrainment discharge, $\rho$ is the jet density, $d$ is the jet width, $u = (u_x, u_y) = (u_\zeta, u_\eta)$ is the ambient velocity vector, $\alpha$ is the entrainment coefficient, the subscripts ($\alpha$) refer to ambient characteristics;

**equations of momentum**

$$\frac{d}{d\zeta}(\rho d|v|(v_x - u_x)) \succeq 0$$

$$\frac{d}{d\zeta}(\rho d|v|(v_y - u_y)) = -\rho_m 2C_D (|v| - u_\zeta)^2 \frac{v_y}{|v|} + \rho_m d g \frac{\rho_a - \rho_m}{\rho_a}$$

where $C_D$ is a drag coefficient; the subscripts ($\alpha$) refer to average characteristics between $\zeta$ and $\zeta + d\zeta$, respectively;

**equations of heat and solute transport**

$$\frac{d}{d\zeta}(\rho d|v|T) = \rho_a q_e T_a - 2\rho_a \kappa_T \frac{(T - T_a)}{d} \quad (7 a - b)$$

$$\frac{d}{d\zeta}(\rho d|v|C) = \rho_a q_e C_a - 2\rho_a \kappa_C \frac{(C - C_a)}{d}$$

where $T$ is the temperature, $C$ is the solute concentration, $\kappa_T$ and $\kappa_C$ are the thermal and solute diffusivities, respectively. Eqs. (4–7) represent a
nonlinear system of six equations in the six unknowns ρ, v_x, v_y, d, T and C.

Fig. 4: Two-dimensional flow geometry

Numerical simulations were performed by integrating numerically eqs. (4-7). In our calculations we used ΔT_o = T - 4°C, ΔC_o = C, β_T = 5 \times 10^{-4} °C^{-1}, β_C = 8 \times 10^{-3} %^{-1}, g = 981 cm/s², κ_T = 1.4 \times 10^{-3} cm²/s⁻¹, κ_C = 1.5 \times 10^{-5} cm²/s⁻¹, ρ_o = 1 g/cm³ and (u_x, u_y) = (0, 0).

Following the analysis of Jirka & Akar [1991], we can define the point of jet collapse as the point where the effect of density stratification begins to become important, the jet motion terminates and is transformed in a density current at the terminal level. Figure 5 shows the measured and computed deviations; since velocities were very low, we assumed C_D = 2 and α = 0.1 in the initial zone. To account for the turbulence created by the fluid injection, heat and salt diffusivities were increased by two orders of magnitude in the initial zone. After the point of jet collapse, the entrainment coefficient was reduced by a factor of ten. Figure 5 shows that, with the exception of few cases, the jet behavior could be correctly predicted. It seems that negatively buoyant jets are the hardest to simulate numerically, while positively buoyant jets show a better agreement between numerical and experimental results. However, additional simulations are needed to find optimal values of C_D and α which will reduce disagreement with the experiments.

CONCLUSIONS

The present set of experiments on buoyant jets under laminar flow conditions showed that the buoyancy force is the primary mechanism which influence the laminar jet deviation from the horizontal level; instead, momentum and volume fluxes were found to be of limited importance. Moreover, the amplitude of the jet deviation did not depend linearly on the density difference between the jet and the receiving fluid. A mathematical model based on a Lagrangian approach was developed to simulate the injection of jets into single or D–D stratified environments. The model was calibrated with the experimental results and a good agreement between the measured and computed jet vertical displacement was observed.
Fig. 5: Comparison between computed and measured jet deviations

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