Development of a 3D non cohesive suspended sediment transport model for free surface flows
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ABSTRACT
The mathematical model for simulating three-dimensional (3D) suspended sediment (SS) transport under non-equilibrium situations is described. The model consists of a refined hydrodynamic model and a scalar transport model involving bed boundary conditions for either the sediment flux to or from the bed, or an estimated concentration level. It may be applied with the rigid-lid assumption or with an effective water-depth calculation so as to take the free surface effects into account. A test case for a situation with net erosion flux from the bed is presented, and the predicted development of the concentration profiles is compared with measurements.

INTRODUCTION
The prediction of sediment transport with laboratory experiments is costly, and mathematical models are needed to simulate these processes. The mathematical models available are mostly restricted to transport under equilibrium conditions, but in morphological processes, the sediment transport often takes place under non-equilibrium condition. A continuous adjustment of the transport discharge to the transport capacity is observed in natural rivers, settling basins, estuaries,... In this paper, the authors present a 3D non cohesive SS transport model for free surface flows and its ability to reproduce observed phenomena. They compare the computed results with experimental data presented by Van Rijn [1] in a previous paper.

HYDRODYNAMICS MODELLING
A turbulent and 3D incompressible flow can be described by the Reynolds equations resolution, in combination with an efficient turbulent closure. The Reynolds equations are established under the Boussinesq's assumption. In this paper, they are coupled with the $k - \varepsilon$...
eddy–viscosity turbulence model. A description of the entire mathematical model is to be found in Rodi [2]. The free surface boundary conditions proposed by Celik and Rodi [3] for the $k$ and $\epsilon$ equations have been employed.

**SUSPENDED SEDIMENT TRANSPORT MODELLING**

**General equation**

The evolution of each SS granulometric class can be described from its own mass–balance equation:

$$\frac{\partial \overline{C_i}}{\partial t} + \frac{\partial \overline{U_i} \overline{C_i}}{\partial x_i} + \frac{\partial \overline{U_{ei}} \overline{C_i}}{\partial x_i} = \frac{\partial}{\partial x_i}(-\overline{u'_{ei} c'})$$  \hspace{1cm} i = 1, 2, 3

where $\overline{C_i}$: mean local concentration, $c'$: concentration fluctuation, $x_i$: directional axe, $U_i$: fluid velocity component, $u'_{ei}$: fluid velocity fluctuation component, $U_{ei}$: slip velocity component (In most models, $U_{ei}$ reduces to $-\overline{w_{ei}^s}$, $w_{ei}^s$ being the settling velocity)

The fluid velocity components are calculated with the hydrodynamic model. The turbulent sediment flux are modelled as follows:

$$u'_{ei} c' = -\Gamma_{ei} \frac{\partial \overline{C_i}}{\partial x_i}$$

and need to determine the turbulent diffusivity coefficient $\Gamma_{ei}$.

**Sediment mixing coefficient**

In most models, the turbulent sediment mixing coefficient $\Gamma_{ei}$—or at least its vertical component—is assumed to be proportional on each point to the eddy viscosity $\nu_t$ resulting from the hydrodynamic model. $\sigma_c$ being the turbulent Schmidt number, it reads:

$$\Gamma_{ei} = \frac{\nu_t}{\sigma_c}$$

Following the literature, the best correlations between experiments and numerical results are obtained for values of $\sigma_c$ in range 0.5–1.0: Launder [4] recommends $\sigma_c = 0.7$ for all free surface flows; Van Rijn [5], in a panel of tests, obtained the best agreement with a mean value of $\sigma_c$ about 0.6; Celik et Rodi [6] recommend $\sigma_c = 0.5$ in open channel flows. In fact, $\sigma_c$ is more realistic when connected with the local Richardson number.

**Boundary conditions**

At surface, the boundary condition is:

$$-\overline{w_{ei} c'} + \overline{w_{ei}^s \overline{C_i}} = 0$$

Two ways may be used to determine the bed boundary conditions.

1. The first one assumes the near–bed concentration $\overline{C_{a,e}}$, at a reference level $z_a$, to be equal to its value under equilibrium conditions $\overline{C_{a,e}}$. Yalin [7], Yang and Molinas [8] proposed to estimate the reference concentration from the bed load discharge.
Under this assumption, Van Rijn [5] established the following empirical proposal:

\[
\overline{C}_{a,e} = 0.015 \frac{D_{50}}{(z_a - z_b)} \left( \frac{T}{D_s^{0.3}} \right)^{1.5}
\]  

(5)

with

\[
D_s = D_{50} \left[ \frac{\rho_s}{\rho - 1} \right]^{1/3} \left( \frac{\nu^2}{\gamma} \right) : \text{adimensional particle diameter}
\]

\[
T = \left( \tau_w - \tau_{w,cr} \right) / \tau_{w,cr} : \text{adimensional bed-shear stress}
\]

\[
\frac{D_{50}}{\tau_w} / \frac{D_{50}}{\tau_{w,cr}} : \text{median bed-particle diameter}
\]

\[
\tau_{w} / \tau_{w,cr} : \text{effective bed-shear stress}
\]

\[
\tau_{w,cr} : \text{critical bed-shear stress according to Shields}
\]

Equation (5) may be applied to a reference level \( z_a \) equal either to half the bed-form height or to the equivalent sand roughness height \( k_z \), in respect with the minimum possible value of \( (z_a - z_b) \) : 0.01 \( (z_a - z_b) \).

When the characteristics of the sediment are known, the most sensible unknown value in equation (5) is the effective bed-shear stress \( \overline{\tau}_w \). Celik and Rodi [9] recommend the following formulation:

\[
\overline{\tau}_w = \left[ 1 - \left( \frac{k_s}{H} \right)^n \right] \overline{\tau}_w
\]

(6)

\( n \) being an empirical constant (about 0.06) and \( k_s \) being the equivalent roughness height, whereas Van Rijn [5] proposes:

\[
\overline{\tau}_w = \mu \overline{\tau}_w
\]

(7)

with

\[
\mu = \left( \frac{C_h}{C_{h'}} \right)^2 : \text{bed-form factor or efficiency factor}
\]

\[
C_{h'} = 18 \log \left( \frac{12h}{3D_{50}} \right) : \text{particle Chézy coefficient}
\]

\[
C_h = 18 \log \left( \frac{12h}{k_s} \right) : \text{bed Chézy coefficient}
\]

The second is based on the bed flux modelling:

\[
-w'c' + \overline{w_s}C = \Phi_d - \Phi_e
\]

(8)

at \( z = z_a \), where \( \Phi_e \) (resp. \( \Phi_d \)) is the particule erosion flux (resp. particule deposition flux) per unit time and per unit area. The flow is supposed to erode as many particles as it can transport. Then the erosion flux is the same as under equilibrium conditions, when \( \Phi_{d,e} - \Phi_{e,e} = 0 \). The deposit flux is that of the near bed settling particules. Over an erodible bed, it comes (Celik and Rodi [6]):

\[
\Phi_e = \overline{w_s} \cdot \overline{C}_{a,e}
\]

(9)

\[
\Phi_d = \overline{w_s} \cdot \overline{C}_a
\]

(10)

As expressed by Celik et Rodi [6], equation (9) is to be changed over a fixed bed as follows:

\[
\Phi_e = \min \left( \overline{w_s} \cdot \overline{C}_{a,e} \right)
\]

(11)
NON-EQUILIBRIUM FLOW WITH NET EROSION

The first sample problem describes the vertical concentration profile evolution in a steady flow, initially free of sediment, over an erodible bed (see fig.1). The experimental data were collected at the Delft Hydraulics Laboratory and presented by Van Rijn [1].

The mean velocity was 0.67 m/s, the water depth being 0.25 m. The flume characteristics were: length = 30 m, width = 0.5 m. The bed material was sand with $D_{50} = 0.230 \text{ mm}$, $D_{90} = 0.320 \text{ mm}$. The suspended sediment had a representative diameter about 0.200 mm and a settling velocity about 0.022 m/s. The samples were collected simultaneously in 5 sections, in 4 points in each profile (at levels: 1.5 cm, 2.5 cm, 5.0 cm and 10.0 cm above bed). The measuring period was as short as possible to reduce the scouring depth downstream of the rigid bed to a minimum.

![Figure 1 Sediment pick-up in flow without initial sediment load (Van Rijn [1])](image)

The hydrodynamic and SS transport models are resolved with a finite-volume procedure using the SIMPLE algorithm. The SS transport equation computation is uncoupled from the hydrodynamic model. The roughness height of the rigid bed is $k_g=0.0005 \text{ m}$ then $k_s=0.01 \text{ m}$ downstream. The bedform height is about 0.015 m. The numerical parameters are $\Delta x=0.2 \text{ m}$ and 10 grid points in vertical direction. At the first calculation level which is 0.005 m above bed, the authors assume that the concentration is immediately equal to its equilibrium value. Equation (5) gives:

$$C_{a,e} = 1.136 \cdot T^{1.5}$$ (12)

$T$ can be estimated in two ways: experimentally, with $\tau = \tau_{measured} = 2.3 \cdot P\alpha$, or theoretically, with $\tau = \tau_{computed} = 3.1 \cdot P\alpha \cdot \tau^*$ may also be calculated in two ways, $\tau$ being known, using either the Celik and Rodi formula (6) or the Van Rijn proposal (7). Four $C_{a,e}$ values can so be obtained (see Table 1). One can note that, for a same $\tau$ value, the $C_{a,e}$ values change about 1 to 6, and the extreme values are in ratio 1 to 11!

Computation tests were carried out, with the four $C_{a,e}$ values, and with Schmidt number ranging from 0.5 to 1. Its has been noticed that:
- With the two maxima $C_{a,e}$ values the computed concentration profiles are not in agreement with the experiments.
- With $C_{a,e} = 7.2 \text{ g/l}$ and $\sigma_e = 1.0$, for $x/h > 5.0$, the computed concentration values are in good agreement only with the experimental values at $z = 1.5 \text{ cm}$, the above calculated
values being smaller than the experimental values. The test with the same \( \overline{C_{ae}} \) value with \( \sigma_e = 0.6 \) gives a good profile form, but the numerical values are higher than the experimental ones.

- Using \( \overline{C_{ae}} = 3.9 \text{ g/l} \) and \( \sigma_e = 0.6 \), the computed concentration profile is accurate for \( x/h = 40 \).

<table>
<thead>
<tr>
<th>Table 1 ( \overline{C_{ae}} ) estimations</th>
<th>Employment of ( \overline{\tau} = \overline{\tau}_{measured} = 2.3 Pa )</th>
<th>Employment of ( \overline{\tau} = \overline{\tau}_{computed} = 3.1 Pa )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \overline{\tau}_{cr} ) (Pa), estimated from ( \tau ) on Shields' curve</td>
<td>0.123</td>
<td>0.123</td>
</tr>
<tr>
<td>Application of equation ( (6) ) from Celik and Rodi</td>
<td>( \overline{\tau'} ) (Pa)</td>
<td>0.404</td>
</tr>
<tr>
<td></td>
<td>( \overline{C_{ae}} ) (g/l)</td>
<td>3.9</td>
</tr>
<tr>
<td>Application of equation ( (7) ) from Van Rijn</td>
<td>( \overline{\tau'} ) (Pa)</td>
<td>1.15</td>
</tr>
<tr>
<td></td>
<td>( \overline{C_{ae}} ) (g/l)</td>
<td>27.4</td>
</tr>
</tbody>
</table>

Figure 2 shows the computed and experimental concentration values in terms of \( x/h \), with \( \overline{C_{ae}} = 3.9 \text{ g/l} \) and \( \sigma_e = 0.6 \). Except for \( x \leq 5h \), the results are in good agreement. The observed difference for \( x \leq 5h \) may be explained because of a small 2cm height scour existing just after the bed change, which has been ignored in the computation test.
NON-EQUILIBRIUM FLOW WITH NET DEPOSITION

Jobson and Sayre [10] carried out experiments to study SS transfers in sediment-laden open channel flows under non-equilibrium conditions. The authors have simulated one of their experiment, with uniform fine sand particles being discharged continuously from a line source located near the water surface into fully developed flow. The channel has a fixed bed covered with large rectangular wooden cleats as roughness elements; its width is approximately 6 times the water depth. Other experimental data are reported on Table 2.

Table 2 Calculated and measured quantities for simulated cases (Jobson and Sayre [10])

<table>
<thead>
<tr>
<th>Original run number</th>
<th>h (cm)</th>
<th>$U_m$ (cm/s)</th>
<th>$u^*$ (cm/s)</th>
<th>$k_s/h$</th>
<th>$p_s/\rho$</th>
<th>$D_{50}$ (mm)</th>
<th>$W_s$ (cm/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>FS11,FS11A</td>
<td>40.7</td>
<td>60.0</td>
<td>8.66</td>
<td>6.03</td>
<td>0.25</td>
<td>2.42</td>
<td>0.123</td>
</tr>
</tbody>
</table>

The hydrodynamic simulations have been carried out using the $k-\varepsilon$ model. Grid independence was ensured by taking a sufficient number of grid points (19) in the vertical direction. The bottom element corresponds to $\Delta z = 0.1 \, h$ and the longitudinal step $\Delta x$ to 0.5 $h$. The equivalent roughness height $k_s$ entering the calculation was not known from the experiments. The authors have adopted the value chosen by Celik and Rodi [6] when simulating the same problem. They had estimated $k_s$ from the comparison between the use of the log law near the bed and the measured velocity components.

For the measured profiles, as well as the predicted ones, which are reproduced in figure 3, $C$ is made dimensionless with the initial discharge concentration $C_{\text{ref}}$. The case corresponds to the discharge of fine sediments with moderate velocity. The velocity was not high enough to carry all the discharged sediments so that net deposition ($\text{Deposition} \geq \text{Entrainment}$) occurred until the suspended load was reduced to the carrying capacity, far downstream from the sand source ($x/h \approx 100.0$).

Calculation were started five water depths downstream of the sand discharge with the measured profile as an upstream condition. The downstream condition, for $x/h=100.0$, is $\partial C/\partial x = 0$. As bed boundary conditions, the authors have assumed that the deposit flux of particles could be estimated everywhere at bottom from the gravitational flux expressed by equation (10). For low $x/h$ values, the concentration level at $z=a$ near bed ($\overline{C}_a$) is smaller than the value which could be measured under equilibrium ($\overline{C}_{a,e}$). Then, under gravitational forces and so settling of particles, $\overline{C}_a$ reaches $\overline{C}_{a,e}$ for a peculiar $x/h$ value (about 10.0), and even overshoots $\overline{C}_{a,e}$ downstream, until equilibrium. Therefore the erosion flux expressed by equation (11) has been imposed everywhere at bottom. Under these assumptions, until $x/h$ equals about 10.0, all particles reaching bed are re–entrained in flow, whereas for greater values of $x/h$, the erosion flux is limited by its value under equilibrium, which is smaller than the deposit flux.
The choice for $C_{a,e}$ was made from tests, so as to simulate as accurate as possible bed concentration far from discharge ($x/h=67.5$). The chosen value ($C_{a,e}=1.34 \ C_{ref}$) is the same as that adopted by Celik and Rodi [6]. Several tests were also made on $\sigma_c$ so as to obtain the best concentration profile far downstream ($x/h=67.5$). $\sigma_c = 1.0$ was retained.

Figure 3 Development of concentration profiles. Experiments of Jobson and Sayre [10] with net deposition of fine sediments over a fixed bed. $C_{ref}$ (ppm) $\times$ 146 $\triangle$ 168 $C_{b,e} = 1.34 \ C_{ref}$ $\sigma_c = 1.0$

Figure 3 compares the downstream development of the concentration with measurements. As can be seen from the figure, the downstream development of the concentration profile is simulated very well by the model. The greater difference between $C_{measured}$ and $C_{computed}$ is observed near bed for $x/h=12.0$. Its relative value is smaller than 20%. This difference seems to be due to a light overestimation of near-bed velocities in the computations, which rise the advective transport of particles and so limits the relative weight of the settling flux on their motion (see [11]). This light limit of the simulation accuracy seems to be due more to the hydrodynamic model over very rough bottom than to the SS transport model. The noticed difference is quite acceptable in comparison with other numerical predictions from
litterature. It is not only the shapes of the profiles that are well predicted but also the level of concentration. After Celik and Rodi’s tests, that indicates that the deposition and entrainment processes are simulated realistically by the model. More results are to be found in Ouillon [11].

CONCLUSION

The suspended sediment transport model enhanced by the authors reproduce with good accuracy the non-equilibrium flows with net erosion or net deposition. The computed results point out an extreme sensitivity of the model both to the Schmidt turbulent \( \alpha_c \) number and to the \( \bar{C}_{a,e} \) concentration value statement, each of them being required to calibrate the model.

KEY WORDS

Suspended sediment, erosion, deposition, free surface flow.

REFERENCES