A nonlinear numerical model for surface and internal waves around various permeable breakwaters

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Abstract

The effect of different types of permeable structures on surface and internal waves has been studied considering the nonlinear properties of fluid motion. The governing equations are a set of nonlinear equations representing a multilayer fluid system in porous media. The Lagrangian of each fluid layer was vertically integrated satisfying nonlinear boundary conditions on the interfaces, after which the variational principle was applied to yield a set of time-dependent, fully nonlinear equations taking into account a continuous distribution of porosity. Any structure shape described as a porous body can be treated. Several numerical computations were carried out to investigate the deformation of surface/internal long waves around different permeable breakwaters including submerged breakwaters, breakwaters with an opening, and submerged horizontal plates. Water particle trajectories were also simulated.

1 Introduction

Permeable or transmissible structures that allow waves to partially pass through are receiving research attention from the standpoint of coastal structure development, as they can positively affect seawater-purity control. These structures include a breakwater having openings (watercourses) for water exchange, a porous breakwater, a perforated-caisson breakwater which has holes or slits in its front and rear walls, a breakwater composed of several flaps, a submerged breakwater, a submerged plate, or a floating
structure. All serve to maximize removal of disturbance factors of water circulation, and some control waves and currents to not only conserve but also improve water quality.

The effects of these structures on surface waves and currents have been studied via hydraulic-basin experiments and numerical calculations. Using numerical methods, Gao et al. [1] calculated coexisting fields of broken waves and turbulent flows around various kinds of submerged breakwaters applying \( \sigma \) coordinates. Mizutani et al. [2] and Liu and Saki [3] used BEM-FEM combined analyses to investigate nonlinear surface waves around submerged porous breakwaters and beneath flexible floating structures, respectively, while Kakinuma [4] calculated interaction of surface and internal waves with very large floating/submerged structures.

The density stratification in a coastal environment may well play a key role. Water temperature and salinity physically appear as a difference in mass density, and fluid motion in stratified waters is different from that in a single-density case, thereby resulting in different forces on structures. Water stratification, for instance, occurs in a bay in summer where heavy seawater, whose temperature has become low in winter, stagnates near the bottom, or in an estuary where fresh water is supplied from a river into the sea. Such stratification becomes especially important when internal waves, e.g. internal tides, propagate.

Here, both transformation of surface/internal waves and trajectories of water particles are studied by considering nonlinear properties of fluid motion around various types of permeable structures. Specifically, a numerical model is described that represents two-layer fluid systems in a vertical section, where (i) the governing equations are a set of nonlinear equations for a multilayer fluid system in porous media and (ii) nonlinear boundary conditions on the interfaces are satisfied. The structures are described as porous bodies whose porosity is assumed to change rapidly but smoothly in the vicinity of the structure edges. Accordingly, the whole domain can be considered including permeable structures such as submerged breakwaters, breakwaters with an opening, and submerged horizontal plates. Water particle trajectories are also simulated in the upper and lower layers.

2 Derivation of a set of fully nonlinear equations for surface and internal waves in porous media

2.1 Multilayer fluids in porous media

As shown in Fig. 1, inviscid and incompressible fluids in porous media are assumed to be stable in still water, where the fluids are represented as \( i \) \( (i = 1, 2, \ldots, I) \) from the top to the bottom. The \( i \)-layer thickness in still water is denoted by \( h_i(x)(\sum_{i=1}^{I} h_i = h(x)) \). None of the fluids mix even in
motion, and the density \( \rho_i \) \( (\rho_1 < \rho_2 < \cdots < \rho_l) \) is constant in each layer. At each point in the \( i \)-layer, the horizontal vector component and vertical scalar component of seepage velocity (actual velocity of the fluids) are represented by \( u_i \) and \( w_i \), respectively.

Fluid motion is assumed to be irrotational in the seepage velocity field, resulting in existence of the velocity potential \( \phi_i \) defined by

\[
\mathbf{u}_i = \nabla \phi_i, \quad w_i = \frac{\partial \phi_i}{\partial z},
\]

where \( \nabla = (\partial/\partial x, \partial/\partial y) \) is a partial differential operator in the horizontal plane.

Porosity of the porous media is represented by \( \varepsilon(x,z) \) \( (0 < \varepsilon \leq 1) \), and when fluids move in these media the discharge velocity (apparent velocity of the fluids) becomes \( (\varepsilon u_i, \varepsilon w_i) \).

The \( i \)-layer has two interfaces: the lower interface \( z = \eta_{i-1,0}(x,t) \) and upper interface \( z = \eta_{i,1}(x,t) \). Any profile of surface waves, internal waves and the seabed can be expressed by \( z = \eta_{i,j}(x,t) \) \( (i = 1, 2, \cdots, I; \ j = 0, 1) \). It is assumed that no unstable phenomenon, such as vortex generation or wave breaking, occurs on any interface, and hence \( \eta_{i,j} \) is a single-valued function of \( x \). The pressure on \( z = \eta_{i,0} \), i.e., on the lower interface of the \( i \)-layer is written by \( p_i(x,t) \). The effect of surface tension on the interfaces is neglected compared with that of gravity.

### 2.2 Functional for the variational problem

Porosity \( \varepsilon \) is introduced and the set of fully nonlinear equations derived by Kakinuma [5] for surface and internal waves is extended to a model for waves in porous media. In the \( i \)-layer, if both the elevation of one interface, \( z = \eta_{i,1-j}(x,t) \) \( (j = 0 \ or \ 1) \), and the pressure on the other interface, \( p_{i-j}(x,t) \), are known, then the unknown variables are the velocity potential \( \phi_i(x,z,t) \) and the interface elevation \( \eta_{i,j}(x,t) \) such that the functional for the variational problem in the \( i \)-layer, \( \mathcal{F}_i \), can be determined using...
\[
\mathcal{F}_i[\phi_i, \eta_{i,j}] = \int_{t_0}^{t_1} \int_A \int_{n_{i,0}}^{n_{i,1}} \frac{\partial \phi_i}{\partial t} + \frac{1}{2} (\nabla \phi_i)^2 + \frac{1}{2} \left( \frac{\partial \phi_i}{\partial z} \right)^2 + g z + \frac{P_{i-j} + P_i}{\rho_i} \, dz \, dA \, dt,
\]

where \( g \) is the gravitational acceleration, \( P_i = \sum_{k=1}^{i-1} (\rho_i - \rho_k)gh_k \) is constant in each layer, \( (\nabla \phi_i)^2 \equiv |\nabla \phi_i|^2 \), and the plane \( A \) which is the orthogonal projection of the object domain on to the \( x-y \) plane is assumed to be independent of time.

When \( \delta \mathcal{F}_i \) is equal to zero for any small value of \( \delta \phi_i \) and \( \delta \eta_{i,j} \), Euler's equation of continuity for irrotational motion in the \( i \)-layer and the kinematic and dynamic boundary conditions on the interfaces of this layer are satisfied under appropriate initial and lateral-boundary conditions.

### 2.3 Vertical distribution functions

In order to derive a set of horizontally two-dimensional type equations, vertical integration is analytically performed. In a manner similar to that of Isobe [6], the velocity potential \( \phi_i \) is expanded into a series in terms of a given set of vertical distribution functions \( Z_{i,\alpha} \) multiplied by their weight factors \( f_{i,\alpha}, \) i.e.,

\[
\phi_i(x, z, t) = \sum_{\alpha=0}^{N-1} \{Z_{i,\alpha}(z) \cdot f_{i,\alpha}(x, t)\} = Z_{i,\alpha} f_{i,\alpha}.
\]

### 2.4 Euler-Lagrange equations under the variational principle

We substitute eqn (3) into eqn (2), integrate the Lagrangian vertically, and then obtain functional \( \mathcal{F}_i[f_i, \eta_{i,j}] \) which is an integral of another functional \( \mathcal{L}_i[f_i, \eta_{i,j}] \) in a horizontally two-dimensional form, i.e.,

\[
\mathcal{F}_i[\phi_i, \eta_{i,j}] = \mathcal{F}_i[f_i, \eta_{i,j}] = \int_{t_0}^{t_1} \int_A \int_{n_{i,0}}^{n_{i,1}} \mathcal{L}_i[f_i, \eta_{i,j}] \, dA \, dt,
\]

where \( f_{i,0}, f_{i,1}, f_{i,2}, \ldots, f_{i,N-1} \) are simplified into \( f_i. \)

Euler-Lagrange equations on \( f_{i,\alpha} \) and \( \eta_{i,j} \) are derived by

\[
(L_i) f_{i,\alpha} - \nabla (L_i) \nabla f_{i,\alpha} - \frac{\partial}{\partial t} (L_i) \frac{\partial f_{i,\alpha}}{\partial t} = 0, \quad (L_i) \eta_{i,j} = 0.
\]

Thus, a set of fully nonlinear equations for surface and internal waves is obtained as

\[
\varepsilon^{n_{i,1}} Z_{i,\alpha}^{n_{i,1}} \frac{\partial \eta_{i,1}}{\partial t} - \varepsilon^{n_{i,0}} Z_{i,\alpha}^{n_{i,0}} \frac{\partial \eta_{i,0}}{\partial t} + \nabla \left( \int_{n_{i,0}}^{n_{i,1}} \varepsilon Z_{i,\alpha} Z_{i,\beta} \, dz \nabla f_{i,\beta} \right) - \int_{n_{i,0}}^{n_{i,1}} \varepsilon \frac{\partial Z_{i,\alpha}}{\partial z} \frac{\partial Z_{i,\beta}}{\partial z} \, dz f_{i,\beta} = 0.
\]
2.5 Transition area of porosity

One model feature is to express all structures as bodies composed of porous media having porosity $\varepsilon$. At a point where $\varepsilon \ll 1.0$, there exists an object which strongly inhibits penetration of fluid. No structure exists where $\varepsilon = 1.0$. The porosity distribution is assumed continuous in space. For this reason, use of a vertically integrated type model for fluid motion from the seabed or bottom of porous bodies to the water surface allows treating several structures having arbitrary shapes in three dimensions.

A transition area where the porosity changes smoothly but rapidly in space is installed in the vicinity of the surface of a structure; e.g., in the case of Structure C-1 (Fig. 4 (a)), on the straight line where $x/h = 45$ in the vertical section ($x - z$ plane), $\varepsilon$ is defined as follows (Fig. 5):

$$\varepsilon(z) = \left\{ \frac{1 + \varepsilon_0}{2} - \frac{1 - \varepsilon_0}{2} \tanh \left( c + \frac{2c z + 0.4}{d} \right) \right\} + \left\{ \frac{1 - \varepsilon_0}{2} + \frac{1 - \varepsilon_0}{2} \tanh \left( c + \frac{2c z + 0.2}{d} \right) \right\},$$

where $c$ and $d$ are positive constants. In the present calculation, it is assumed that $c = 4.0$ and $d = 0.1$.

3 Numerical calculation

3.1 Long wave assumption

The governing equations are simplified as follows for nonlinear, long wave equations in very shallow water:

$$\varepsilon^{n_{i,1}} \frac{\partial \eta_i}{\partial t} - \varepsilon^{n_{i,0}} \frac{\partial \eta_i}{\partial t} + \nabla \left( \int_{\eta_i,0}^{\eta_i,1} \varepsilon \, dz \nabla f_i \right) = 0,$$

$$\frac{\partial f_i}{\partial t} + \frac{1}{2} (\nabla f_i)^2 + g \eta_{i,j} + (p_{i-j} + P_i)/\rho_i = 0,$$

where only one term is used for the vertical distribution function: $N = 1$; $Z_{i,\alpha} f_{i,\alpha} = Z_{i,0} f_{i,0} = 1 \cdot f_{i,0} = f_i$.

Note that resistance and energy dissipation of fluids moving through porous media are not taken into account, although their effects on wave profiles have been considered by Kakinuma [7].
3.2 Permeable structures

A two-layer fluid system is studied in the vertical section. Figures 2-4 show the three types of structures treated here. They are set up in coexistence fields of surface and internal waves.

Structure A-0 (Fig. 2) is a porous breakwater, while A-1 and A-2 are submerged porous breakwaters with different heights. Structures B-1 and B-2 (Fig. 3) have a part where the porosity equals 1.0, i.e., it is an opening for water exchange. Structures C-1 and C-2 (Fig. 4) are submerged horizontal plates fixed inside the upper or lower layer, respectively. Although openings or submerged plates may have other features if they incline back and forth, only the horizontal type is considered here.

3.3 Numerical calculation of surface/internal waves and currents

(1) Propagation of surface waves

The positions of the surface and interface are defined by $z = \zeta(x, t)$ and $z = \eta(x, t)$, respectively. Equations (9) and (10) are solved using a similar finite difference scheme to that of Kakinuma [5]. The density ratio $\rho_2/\rho_1$ equals 1.025 and depth ratio $h_1/h_2$ is 1.5. The porosity of a structure, $\varepsilon_0$, is equal to 0.2 outside the transition areas. The lateral boundaries of the calculation domain are smoothly connected to each other. In the initial state, a profile of a crest-type solitary wave is given on the surface or interface and fluid velocity is set at zero everywhere.

The initial profiles of the surface and interface are first taken to be $\zeta(x, 0) = 0.1h/(10^{s} + 10^{-s})$ ($s = 0.1x/h$) and $\eta(x, 0) = -h_1$, respectively. Figure 6 (a) shows calculation results for the propagation of surface waves, where the cases shown indicate different celerity, with Structure A-0 being most effective in decreasing both celerity and wave height. In Fig. 6 (b), B-0 reduces wave height more than B-1 and B-2, and in Fig. 6 (c) the celerity is reduced almost equally for C-1 and C-2.

(2) Propagation of internal waves

Initial profiles are next taken as $\zeta(x, 0) = 0$ and $\eta(x, 0) = 0.1h/(10^{s} + 10^{-s}) - h_1$ ($s = 0.1x/h$), with Fig. 7 showing the propagation of internal waves. In Fig. 7 (a), note that A-2 does not produce as much reflection of internal waves compared to the others, while in Fig. 7 (b) the celerity is different between B-1 and B-2, although the celerity of surface waves is almost same (Fig. 6 (b)). In Fig. 7 (c), the celerity is different between C-1 and C-2, but not so different from C-0 (no-structure case) as that for surface waves.
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Figure 2: Submerged breakwaters (Structures A).

(a) A-0: The top is at the still water level. inside the upper layer. inside the lower layer.
(b) A-1: The top is the still water level. inside the upper layer. inside the lower layer.
(c) A-2: The top is inside the upper layer. inside the lower layer.

Figure 3: Breakwaters with an opening (Structures B).

(a) B-0: This has no opening.
(b) B-1: The opening is inside the upper layer. inside the lower layer.
(c) B-2: The opening is inside the upper layer. inside the lower layer.

Figure 4: Submerged horizontal plates (Structures C).

(a) C-1: The plate is inside the upper layer. inside the lower layer.
(b) C-2: The plate is inside the upper layer. inside the lower layer.

Figure 5: Vertical distribution of porosity for Structure C-1 ($x/h = 45$).

(3) Trajectories of water particles

Marker particles are arranged in the calculation domain and then chased, moving at the seepage velocity of the fluid. Figures 8–10 show water particle trajectories calculated under the same conditions as in Fig. 7. Roughly speaking, water particles inside the lower layer move in the same direction as the internal waves, opposite inside the upper layer. We define an absolute value as $\ell = \int_0^\tau U_M \, dt$ where $\tau = 400/\sqrt{g^*/h}$, $g/g^* = 9.8$ and $U_M$ is seepage velocity of an identical marker particle M.
Figure 6: Surface wave profiles (In Figures 6–10, $g / g^* = 9.8$, $\rho_2 / \rho_1 = 1.025$, $h_1 / h = 0.6$ and $\varepsilon_0 = 0.2$).

Figure 7: Internal wave profiles.

Figure 8 shows that the upper layer $\ell$ inside Structure A–0 is larger than that outside. This indicates that water particles stay near the front face where $x/h$ is smaller inside A–0 before another internal wave comes, i.e., flow fields inside and outside a breakwater lead to water exchange. On the other hand, in the lower layer the trajectories for A–0 and A–1 are similar and the amplitude of particle movement for them is relatively large inside the breakwaters due to reflection of internal waves. For A–2, however, little reflection occurs; hence, water particles easily travel through the breakwater towards the rear side where $x/h$ is larger.

As shown in Fig. 9, although the trajectories for B–0 and B–2 are similar in the upper layer, those for B–0 and B–1 are similar in the lower layer and the amplitude of these trajectories is larger than that for B–2.

Figure 10 indicates that in the lower layer the trajectories for C–1 are similar to those in the no-structure case, and $\ell$ is larger inside C–2. In the upper layer $\ell$ is not so different among the three cases.
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4 Conclusions

Nonlinear numerical simulation was performed in the vertical section for several long wave and current fields around and inside different types of
coastal permeable structures existing in a stratified field, where the governing equations are a set of nonlinear equations for a multilayer fluid system in porous media. To allow for wave propagation and water exchange, permeable breakwaters have relatively complicated shapes that can be described in the present model using continuously distributed porosity.

In future work the proposed numerical model will be extended to be horizontally two dimensional which will allow us to consider higher order terms of nonlinearity such that we can investigate the mechanism of water exchange around permeable breakwaters and man-made gathering places for fish in harbors or estuaries connected to outside regions.

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References


