Two-dimensional mathematical modelling of debris flows with practical applications

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Abstract

The paper deals with mathematical modelling of landslide-debris flow. One-dimensional models can be used in upper parts of the affected regions where debris flow is in a steep narrow canyon, while at relatively flat downstream areas near villages two-dimensional models are necessary. Two mathematical models are described into more detail: one-dimensional model DEBRIF1D and two-dimensional model PCFLOW2D, both developed at the Faculty of Civil and Geodetic Engineering of the University of Ljubljana. Basic continuity and momentum conservation equations were accomplished with the non-Newtonian Bingham plastic fluid model proposed by O’Brien and Julien. As a case study, numerical simulations of the landslide-debris flow, which occurred in November 2000 and destroyed a part of the village Log pod Mangartom, Slovenia, was performed. Hydrographs of the debris flow discharge at the upstream end of the affected village were determined by the model DEBRIF1D and then used as the upstream boundary condition for the simulations by two two-dimensional models: PCFLOW2D and a commercial model FLO-2D. The three models were calibrated by field measurements and then used efficiently to determine the necessary measures for the future protection of the village. A special feature of the DEBRIF1D model enables direct computation of the initial hydrograph. Validity of the quadratic equation expressing resistance was roughly confirmed by field measurements even for changing flow regimes.
1 Introduction

Landslide induced mud/debris flows are often the cause of significant loss of property and casualties in mountainous regions. In Slovenia, approximately one third of the country is covered by slopes, which are conditionally stable due to their unfavorable geological composition, and are often threatened by landslides, caused e.g. by earthquakes or heavy rainfalls [7]. Scientists are trying to develop reliable numerical models, able to predict inundation limits for potential future landslides and to determine the effect of different protective measures.

The phenomenon of mud/debris flow is similar to unsteady flow of water, in particular to the dam-break flow, and also to snow avalanche dynamics [12]. In all these cases the flow is governed by the physical laws of conservation of mass and of momentum. However, it is well known that the resistance to flow movement is more complex in debris flows and snow avalanches than in pure water flows [8]. The rheological behavior of the flow depends to a great extent on the debris flow sediment/water concentration. This can range from nearly dry landslides to debris flows, mud flood and pure water flows [1]. The concentration often changes along the debris path, as additional sediment is scavenged into the flow and/or additional water streams join the debris flow, which makes the simulations even more difficult.

One- and two-dimensional mathematical models of debris flows that are currently used in Slovenia are presented. The development and practical applications of these models were intensified after the event of a debris flow of relatively large extent (initial landslide mass $1.2 \times 10^6 \text{ m}^3$), causing in November 2000 the loss of seven lives and a great loss of property in the village Log [6]. The paper also includes the description of this tragic event, the calibration and the validation of mathematical models and finally also debris flow simulations to propose protective measures for the village.

2 Mathematical models

2.1 One-dimensional model DEBRID1D

In 1972 a one-dimensional model LAXDAM [13] for the simulation of a dam break flow was developed at the Faculty of Civil and Geodetic Engineering of the University of Ljubljana. Later on, this model was used as a basis for developing a numerical model for the simulation of snow avalanche dynamics SNOWDYN [12], [11]. In 2000 this model was extended to a debris flow model DEBRID1D. All three models solve one-dimensional St. Venant equations in what is called the »conservation« form.

The main difference between the three phenomena is in the formulation of flow resistance and in the formulation of initial and boundary conditions. The DEBRID1D model uses the formulation for resistance slope $S_r$, adapted from [9]:
\[ S_f = \frac{\tau_y}{\gamma_m h} + \frac{KV\eta}{8\gamma_m h^2} + \frac{n_g^2 V^2}{h^{4/3}} + S_e + S_b \]  

where \( \tau_y \) = yield shear stress of the sediment-water mixture; \( \gamma_m \) = specific weight of the water-sediment mixture; \( h \) = water depth; \( K \) is the non-dimensional friction parameter; we used the value of \( K = 2285 \), proposed by O'Brien [3] for natural streams with highly irregular terrain configuration.; \( \eta \) = dynamic viscosity of the mixture; \( V \) = flow velocity; \( n_g \) = Manning roughness coefficient; \( S_e \) = resistance slope due to eddy losses at sharp channel expansions, equal to the difference of the values of kinetic terms in two subsequent cross-sections [13]; \( S_b \) = resistance slope due to bends; a simple equation, known from steady state river hydraulics [13] was used.

Explicit numerical Lax-Wendroff method is used for the solution. Since the momentum equation is used in conservative form and solved by the appropriate finite difference numerical method, the model can partly take into account the energy losses at the wave front. This enables direct simulation of the wave front without special wavefront tracking technique [13].

One important feature of the DEBRIF1D model is that it includes the computation of the initial flow hydrograph \( Q(t) \) at the downstream end of the initial debris mass. The procedure is taken from the dam-break flow model, where at the first instant after the dam collapse (\( t = 0 \)) the water level and the velocity at the dam site are calculated using the momentum and continuity equations and the equation of the forward characteristic [13]. The geometry of the landslide mass before the displacement must be known. At \( t > 0 \) the computation runs continuously from the upstream end of the moving landslide mass, to the front of the debris flow (moving boundary), and the hydrograph at the “dam site” is calculated implicitly. Most debris flow models, among them also PCFLOW2D and FLO-2D, demand the determination of the initial hydrograph using approximate methods, based on the initial debris mass.

2.2 Two-dimensional model PCFLOW2D

The continuity (2) and momentum equations (3) and (4), describing two-dimensional, incompressible unsteady depth-averaged debris flow, are written in the following form:

\[ \frac{\partial h}{\partial t} + \frac{\partial (hu)}{\partial x} + \frac{\partial (hv)}{\partial y} = 0 \]  

\[ \frac{\partial (hu)}{\partial t} + \frac{(hu^2)}{\partial x} + \frac{\partial (huv)}{\partial y} = -gh \frac{\partial h}{\partial x} - gh \frac{\partial z_b}{\partial x} - ghS_{f_x} \]  

\[ + \frac{\partial}{\partial x} (h v_{ef} \frac{\partial u}{\partial x}) + \frac{\partial}{\partial y} (h v_{ef} \frac{\partial u}{\partial y}) \]
where \( t \) = time, \( h \) = water depth, \( u \) and \( v \) = depth-averaged velocity components in the x and y directions, \( z_b \) = bottom level, \( g \) = gravity constant and \( \nu_e \) = effective coefficient of viscosity. The terms \( S_{fx} \) and \( S_{fy} \) express energy line slopes in the x and y directions and can be analogous to the one-dimensional flow formulation (Eq. 1) written as follows:

\[
S_{fx} = \frac{\tau_y}{\gamma_m h} + \frac{K u \eta}{8 \gamma_m h^2} + \frac{n_g^2 u \sqrt{u^2 + v^2}}{h^{4/3}}
\]

\[
S_{fy} = \frac{\tau_y}{\gamma_m h} + \frac{K v \eta}{8 \gamma_m h^2} + \frac{n_g^2 v \sqrt{u^2 + v^2}}{h^{4/3}}
\]
3 Case study

4.1 Description of debris flow below the Stože landslide

On November 15th, 2000, a landslide – debris flow glided down the slope of Stože, NW Slovenia (Fig. 1). The mass mostly stopped at Bridge I, just some percent of the mass flew over the bridge (Reach A, Fig. 1). During the next two days the mass was moistened by heavy rain and by the inflow of the Mangart Creek. Just after midnight of November 17th, a smaller additional landslide glided down the slope and triggered the very wet mass of the first landslide into a debris flow. The combined mass overflew the bridge, completely erasing it, and flew down the narrow canyon of the Predelica Torrent (Reach B, Fig. 1). In about 4 minutes (front velocity about 15 m/s) it reached the village Gorenji Log (Reach C, Fig. 1), where it killed 7 people in their homes, destroyed 6 and severely damaged 23 residential or farm buildings. Fig. 3 shows the bottom view of the devastated Reach C, together with the envelope of maximum levels from numerical simulation of the real event.

Figure 1: Situation of the region and computational reaches.
AB1, AB2 = concrete dams; Z1, Z2 = earth dams.

In Reach C, nearby the village Gorenji Log, the debris flow received additional water discharge from the Koritnica River, and it continued its movement more slowly, partly depositing the mass along its path. Most of the
debris mass was deposited upstream of the very narrow section of the Koritnica Creek, at the end of Reach D, and a smaller part of it was transported further down to the Soča River.

In total a volume of 1 200 000 m³ was displaced from its original location on the Stožec slope (Fig. 1). The volume of the first debris-landslide (on November 15th), deposited along Reach A was estimated to about 600 000 m³. On November 17th about 950 000 m³ moved across the section of Bridge II downstream. A volume of about 800 000 m³ reached the village Gorenji Log, as 150 000 m³ were deposited along the path.

The flow along Reach A was simulated by DEBRID and FLO-2D models, the flow in Reach B was typically one-dimensional and thus simulated by the DEBRID model only. Along Reach C (village Gorenji Log) the flow was two-dimensional and, as this is the most important region (safety of inhabitants, loss of property), we simulated this reach with both PCFLOW2D and FLO-2D models. Reach D was simulated by one-dimensional model, along the upper part (village Spodnji Log, Fig. 1) also with the FLO-2D model. Connections between subsequent reaches were done with calculated hydrographs at the end of each upstream reach.

3.2 Calibration of mathematical models

Calibration of both one- and two-dimensional models was made by finding proper values of the three rheological parameters in Eqs. (1), (5) and (6): \( \tau_f \), \( \eta \), and \( n_g \). We used three possible methods for determining these values: (a) values available in the literature; (b) results of geo-mechanical laboratory measurements [9]; and (c) values obtained by the calibration of the model – comparison with the results of measurements.

Landslide A (November 15th) was relatively dry and it stopped on a slope of 16%. The 1D model simulated this phenomenon with the values of \( \tau_f = 2 000 \text{ N/m}^2 \) and \( \eta = 156 \text{ Pa.s} \). Very wet debris flow B, along Reach B, was calibrated with the values \( \tau_f = 20 \text{ N/m}^2 \) and \( \eta = 40 \text{ Pa.s} \). Since Reach B is a very steep, narrow canyon, with the average slope of 17%, and maximum slope of 119%, the Manning coefficient changed in broad limits from 0.03 to 0.35 \( \text{sm}^{-1/3} \). The greatest values were found along the steepest parts of Reach B, where the flow must have been similar to a waterfall. Fig. 2 shows the longitudinal profile of simulated and measured debris flow surface elevations along Reach B. We can consider the agreement as satisfactory.

Similar accuracy was obtained also for Reaches A, C and D. Along Reach C, the comparison of measured and simulated inundation limits obtained by two-dimensional models PCFLOW2D and FLO-2D is shown in Fig. 3. Calibrated debris flow values along Reach C were \( \tau_f = 20 \text{ N/m}^2 \), \( \eta = 10 \text{ Pa.s} \) and \( n_g = 0.05 \) to \( 0.065 \text{ sm}^{-1/3} \).
Figure 2: Comparison of measured and computed debris flow elevations along the upstream part of Reach B.

Figure 3: Comparison of measured and computed inundation limits of the debris flow along Reach C.
3.3 Simulations of possible future debris landslides

Based on detailed geological, hydrologic, and geo-mechanical analyses, it was determined that above the region of the last landslide on the Stože slope, there are potentially unstable masses and that in the future there is a potential danger of further landslides of the same or even greater extent as that in November 2000. Among professionals it was agreed that the village should be protected against the same magnitude of landslide mass as that of November 2000, when the total displaced volume was 1 200 000 m$^3$. In the original landslide event, about 400 000 m$^3$ were deposited along Reaches A and B, while in the future only a deposition of 200 000 m$^3$ can be expected, as the bed is partially filled with the material of previous landslides and thus a volume of 1 000 000 m$^3$ would reach the village. For this case, Figs. 4 and 5 show the computed velocity vectors and the free surface of the debris flow along Reach C 70 seconds after the transition of wave front through the cross-section of Bridge II. To estimate the potential danger, other simulations were also done for possible initial volumes of 2.0 mio, 1.6 mio, and 0.8 mio m$^3$. A series of simulations was also done with the initial volumes reduced by 600 000 m$^3$. This would be approximately the retention volume of four dams. Several variations of the terrain excavation along Reach C were taken into account in the simulations, to determine the most cost-effective solution to protect both villages.

Figure 4: Computed velocity vectors along Reach C 70 seconds after the transition of wave front through the cross-section of Bridge II.
4 Conclusions

The one- and especially two-dimensional depth-averaged mathematical models with adopted flow resistance formulations for non-Newtonian fluids can serve as useful tools for debris flow simulations. They can help to determine the hazard zone plans of potentially endangered areas and to propose possible measures for the protection of lives and property.

In the case of debris flow below Stož, the combination of one-dimensional model DEBRIF1D and two two-dimensional models PCFLOW2D and FLO-2D proved to be the right methodology for the simulations of possible future disastrous events. The verification of the three models has proved satisfactory reliability of the model results. The quadratic law of debris flow resistance, proposed by O'Brien [9], gives acceptable results, even for flows along longer reaches, where the flow regimes change. FLO-2D shows a certain smoothing of the computed levels in comparison with PCFLOW2D, but on average the agreement of the results of both models is acceptable and the accuracy is comparable.

As a final result, the simulations have shown the following possible technical measures for the protection of the village Log: (1) Construction of two earth dams and two concrete dams to retain a part of the debris mass and to diminish the debris flow energy (Fig. 1); (2) Removal of the debris flow deposits from the
events in 2000, thus increasing the conductivity, especially along Reach C, and enabling safe transition or deposition of any future debris flows; (3) Construction of a longitudinal deflector (wall) at the upstream part of the village to deflect the debris flow from the village center.

References


