PTV measurement of a three-dimensional particle motion in a bubbling jet mixing water vessel
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ABSTRACT
The present paper proposes a new stereo pair matching technique with correction of light refraction at water-air boundary using two video pictures and a new particle tracking algorithm for three-dimensional particle-tracking velocimetry (3D PTV) based on a cross-correlation method using two-consecutive-time-step binary pictures. The technique of 3D PTV is applied to measure three-dimensional velocity vectors of tracer particle motion in a cylindrical water vessel with a bottom blowing bubbling jet. The performance of stereo pair matching and the measurement results of the bubble-water two-phase flows are discussed, and as a result the proposed technique is shown to be useful in measuring the three-dimensional bubbling jet flow field.

INTRODUCTION
Particle-imaging velocimetry (PIV in abbreviation) using the techniques of flow visualization and digital image processing is a method for measuring fluid flow velocity indirectly by analyzing tracer particle motion at a time interval on the pictures. It has the advantages of whole field, contact-free and high speed measurement of instantaneous two- or three-dimensional (2D or 3D) flow, but it also has a disadvantage of low time resolution and relatively low spatial resolution, compared to the one-point measurement methods such as hot wire anemometer and LDV.
In the past ten-odd years, numerous papers on two-dimensional PIV have been published. One such method is the correlation method (Yano [1], Kimura and Takamori [2]) which computes the particle cloud displacement at a time interval and then the particle velocity vectors after finding the most similar distribution patterns of particle cloud density in two-consecutive pictures, using the cross-correlation coefficient to evaluate the pattern matching. Another is the so-called four-consecutive-time-step method (Kobayashi [3], Kaga and Yoshikawa [4], Kaga, Inoue and Yoshikawa [5], Chang, Watson and Tatterson [6]) which tracks each particle motion in four-consecutive pictures by finding the smoothest particle trajectory which gives the minimum value of deviation of the particle displacement and angle. The authors have proposed a new technique which is the so-called binary cross-correlation method (Yamamoto, Uemura, Koukawa, Itoh and Teranishi [7], Uemura, Yamamoto and Koukawa [8]). It also tracks the motion of each particle in two-consecutive pictures. The principle is based on the similarity of particle distribution patterns in the two-consecutive binarized pictures to identify each particle motion using the cross-correlation for the pattern matching. The technique has a high speed algorithm for the particle identification and takes a very short time to compute the velocity vectors. The authors have also proposed a double-check technique (Uemura, Yamamoto and Ohmi [9]) to replace some mismatched velocity vectors with the correct ones.

Recently the PIV has been applied to the measurement of turbulent flows, in which quantitative measurement of three-dimensional flow field is required. Three-dimensional PTV has been developed and applied to the unsteady flow measurement in rotating double circular cylinders to investigate the turbulent energy distribution of homogeneous turbulent flows in an agitated tank using the four-consecutive-time-step method (Nishino, Kasagi, Hirata and Sata [10]).

The present paper proposes a new stereo pair matching technique with correction of light refraction at water-air boundary using two or three video camera images and a new particle tracking algorithm for 3D PTV based on a cross-correlation method using two binary pictures. The technique of 3D PTV is applied to measure the three-dimensional velocity vectors of tracer particle motions in a cylindrical water vessel with a bottom blowing bubbling jet. The performance of stereo pair matching and the measurement results of the bubble-water two-phase flows are discussed, and as a result the proposed technique is shown to be useful in measuring the three-dimensional flow field.

**MAIN THREE ROUTINES FOR 3D PTV**

The 3D PTV consists of the following main three routines.
(1) First, flow field is visualized by putting small solid particles as tracers into fluid flow, and pictures of instantaneous particle motion in a fluid flow are taken...
consecutively using two or three CCD type video cameras. (2) Second, three-dimensional X, Y and Z coordinates of each particle are decided by a stereo pair matching method using the two or three pictures taken at the same time by cameras. (3) Third, each particle is tracked or identified between the first and the second spaces at a time interval $\Delta t$.

The following two sections describe a new stereo pair matching method and a new three-dimensional particle tracking technique based on a cross-correlation method using binary imaging data, omitting the explanation of techniques for flow visualization.

![Illustartion of stereo pair matching](image)

Fig. 1 Illustration of stereo pair matching
Stereo Pair Matching Method

The visualized flow field is photographed by two video cameras, whose perspective screens are fixed in contact with a spherical surface and whose visual-axes intersect at the central point of the sphere, $O_s$. A pair of screens are aligned so that either one of visual-axes on each plane is contained in one common plane of a large circle, on which the viewlines are co-planar. Pictures are obtained with equal scale factor.

Geometrical and camera parameters are shown in Fig. 1. The viewpoints of left and right cameras are denoted by $L$ and $R$, respectively. The length of the base line, $LR$, should be measured at a high accuracy because it influences the overall accuracy of the measurement results.

Suppose that a point, $P_*$, in Fig. 1 is a tracer particle in a three-dimensional space. For the left screen a line which connects a viewpoint, $L$, and an image point, $P_{ki}$, of a particle, $P_*$, is called a viewline. A line, $RP_{kr}$, is also called a viewline. A perspective projection of the line, $P_kL$, on the right screen is called an epipolar line (EP-line in abbreviation) on which the corresponding particle image should exist in the right screen. In reality, due to measurement uncertainty, the corresponding particle image may be found in neighborhood of the EP-line. One of some images that is nearest to the EP-line may be the most probable candidate for a stereo pair of the image, $P_{i}$. Then in the same manner, the second, third, and so on nearest candidates can be found.

A simpler method for calculating EP-line and the distance between a particle image and an EP-line might reduce the analysis time of the procedure. By noticing that every EP-line in one picture goes through a fixed point, that is, a perspective projection point, $R_o$, where base line $LR$ is crossing. A simple expression giving a gradient of an EP-line is derived from geometrical relationships as follows:

\[ \tan \alpha_{ril}' = \frac{\sin \theta_r}{\sin \beta_{ril}} \tan \alpha_{ril} \quad (1) \]

where $\alpha_{ril}$ is the elevation angle of a left viewline, $P_kL$, $\beta_{ril}$ is the included angles between $LR$ and the vertical projection of the viewlines, $P_{ril}$, on the base plane, and $\theta_r$ is the included angle between $LR$ and $R_oS_r$. The EP-line on the left screen can be calculated in the same manner. Since the gradient of an EP-line, $\tan \alpha_{ril}'$, is known in this step, it is easier to find a stereo pair of the closest particle image by calculating and comparing the gradient of a line, $\tan \alpha_{ril}$, which connects the fixed point, $R_o$, and a candidate particle image, $P_{ril}$, than by measuring the distance between EP-lines and candidate images.

The reconstruction of three-dimensional coordinates ($X_*$, $Y_*$, $Z_*$) of particles is illustrated in Fig. 2. A Cartesian right-handed coordinate system is defined, in which the origin, $O$, is taken at the middle point of the base line, $LR$, the $X$-
Fig. 2 Reconstruction of three-dimensional coordinates of a particle

axis along LR, the Y-axis on the base plane, and the Z-axis in the vertical direction. Since the two viewlines do not intersect in actual measurement, the particle, P*, is assumed to be located at the point formed by the following three planes. Two vertical planes containing the two viewlines and an average of the two planes defined by the base line, LR, and the viewlines. Thus a three-dimensional position of a particle, P*, is decided by the following equations:

\[
(X, Y, Z) = 
\left( \frac{LR \tan \beta_{rj}}{(\tan \beta_{ii} + \tan \beta_{rj})}, \left( x + \frac{LR}{2} \right) \tan \beta_{ii}, y \tan \alpha_0 \right)
\]

(2)

here \( \alpha_0 = \frac{1}{2} (\alpha_{0i} + \alpha_{0r}) \), \( \alpha_{0i} = \angle P_{i, i} \angle LH \), \( \alpha_{0r} = \angle P_{r, r} \angle RH \)

Correction of Light Refraction

Figure 3 shows an optical system of the arrangement of a camera and a measuring area in a water vessel. Suppose that a point, C, is a camera center, a point, P, is a particle in the water reservoir, and point, O, is a central point of the measuring area.

When a camera, whose view-axis, OC, is perpendicular to the wall of the water vessel, is installed at such a distance as the included angle, \( \alpha \), between the view-axis, OC, and a ray of light from a particle, PP'C, is small enough, the correction of light refraction can be made by the following simple technique.
An imaginary camera center is put at such a point, $C_{fix}$, as the following equation is satisfied:

$$L_{fix} = O_x C_{fix} = nL$$

where $n (=1.333)$ is the index of light refraction, and a point, $O_x$, and the length $L$ equals to $O_x C$ as shown in Fig. 3. Although a production of the line, $PP'$, intersects at a point, $C_{st}$, on the view-axis, $OC$, in reality, for the simplification of calculation an imaginary camera may be placed at the distance, $L_{fix}$, instead of $O_x C_{st}$ which should be decided for each particle, from the wall of water vessel. The measurement error caused by this simplification is thought to be negligible. An image of a particle in the real screen is denoted by $P$, on a real screen and the argument of the position vector, $O_c P'$, on the screen is denoted by $\theta$, which is given by the following equation.

$$\theta = \tan^{-1}\left(\frac{Y}{X}\right)$$
Length of a position vector is denoted by $e(=\sqrt{X^2 + Y^2})$. An imaginary point, $P_\text{i}'$, corresponding to $P_\text{i}$, is taken on an imaginary screen so that the argument of the position vector $O_\text{e} P_\text{i}'$ is $\theta$, and that the length of the vector is taken as $e'=e/n$ on the imaginary screen. The coordinates of the point $P_\text{i}'$, $X'$ and $Y'$ are given by the following equation.

$$\langle X', Y' \rangle = \langle e' \cos \theta, e' \sin \theta \rangle$$

(5)

Using such imaginary coordinates of $P_\text{i}'$ for both the left and the right cameras in the same way as mentioned above, the correction of light refraction can be made.

**Particle Tracking Algorithm**

Here it is assumed that the time interval between the two-consecutive pictures is short enough and fixed, and that flow velocities in the flow field do not change suddenly in time or space, that is, the similarity of flow patterns is preserved.

![Particle Tracking Algorithm](image)

**Fig. 4** Particle distribution in cubic identification subregion $I$ and $J$ when a reference particle $i$ is overlapped with a candidate particle $j$ at their centers.
The present problem is how to track each particle in the first space at time \( t \) with itself in the second space at time \( t + \Delta t \), in other words, how to identify each particle. The procedures are as follows (see Fig. 4):

1) A cubic search subregion with a side length \( S_3 \) is set up in the second space, whose center has the same coordinates as the center of a reference particle \( i \) in the first space. The number of particles contained in the search subregion is denoted by \( n \). All particles in the search subregion are taken as the candidate particles, \( j_m (m=1, 2, \ldots, n) \).

2) The origin of the coordinate system in the second space is successively moved to make the center of a candidate particle, \( j_m \), have the same coordinates as those of the reference particle \( i \).

3) One cubic identification subregion each with the same side length \( S_i \), is set up in both the first and the second space, whose center has the same coordinates as the center of the particle \( i \). The particle distribution patterns \( I \) and \( J \) are made up of the particles in the first and the second identification subregions, respectively. A candidate pattern \( J \) for the particle \( j_m \) is denoted by \( J_m \).

4) After the pattern \( I \) is overlapped with the pattern \( J_m \), the number of particles which are overlapped with each other in the overlapped patterns, \( N \), is counted. Here \( N \) is called the number of overlapped particle pairs.

If any two particles in the overlapped patterns meet the following conditions given by Eqs. (6) and (7), they are taken for an overlapped particle pair. Here, for simplicity of computation, the actual spherical shape of tracer particles is replaced with a sphere with diameter \( D \). Such a spherical particle is called an imaginary particle.

\[
\begin{align*}
\Delta X &= | X_{i,i} - X_{j,m} | \\
\Delta Y &= | Y_{i,i} - Y_{j,m} | \\
\Delta Z &= | Z_{i,i} - Z_{j,m} | \\
S &= \sqrt{ (\Delta X^2 + \Delta Y^2 + \Delta Z^2) } < D
\end{align*}
\]  

(6)  

where, \( X_{i,i}, Y_{i,i}, \text{ and } Z_{i,i} \) are coordinates of a particle with the number \( i \) in the pattern \( I \) except the central particle \( i \) in the first space; \( X_{j,m}, Y_{j,m}, \text{ and } Z_{j,m} \) are coordinates of a particle with the number \( m \) in the pattern \( J \) except the central particle \( j \) in the second space.

5) The value of the cross-correlation coefficient \( C_{i,j} \), defined by the following Eq. (8) is computed for the two patterns \( I \) and \( J \):

\[
C_{i,j} = \frac{\sum_{m=1}^{n} \left( 1 - \frac{3}{2} \frac{S_m}{S_i} + \frac{1}{2} \frac{S_m^3}{S_i^3} \right) }{\sqrt{N_i N_j}}
\]

(8)
where $N_i$ is the number of particles in the pattern $I$ except the central particle $i$, and $N_j$ is the number of particles in the pattern $J$ except the central particle $j$. $N$ and subscript * are the number and the order number of overlapped pairs, excluding the pair of particles $i$ and $j$, respectively. $S_i = S_i/D$, and $O_i S_i S_j$.

The above Eq.(8) can be derived from the calculation of total overlapped volumes of overlapped imaginary particle pairs in the two patterns, total volumes of particles in the pattern $I$ and total volumes of particles in the pattern $J$ using binary image data, based on the mathematical definition of the cross-correlation coefficient, as in the following form.

$$C_{ij} = \frac{\int f(x, y, z) \cdot g(x, y, z) \, dV}{\sqrt{\int [f(x, y, z)]^2 \, dV \cdot \int [g(x, y, z)]^2 \, dV}}$$

Here it is assumed that the pattern distribution functions $f(x, y, z)$ and $g(x, y, z)$ take binary data, that is, values of unity inside the particles and zero outside the particles and that the diameter of the imaginary particle is constant $D$.

The steps from 2) to 5) are repeated until both the values of $N$ and $C_{ij}$ are computed for all the candidate patterns. Which candidate pattern has the maximum value of $N_{max}$ can be determined after investigating the numbers of overlapped particle pairs for all candidate patterns. If there is only one pattern that takes the maximum value of $N$, $N_{max}$, the candidate particle can be thought to be correctly identified. If more than one pattern takes $N_{max}$, the candidate particle around which the pattern takes the maximum value of $C_{ij}$ is taken as the correctly identified one. If the maximum number $N_{max}$ is zero, the reference particle $i$ is excluded for the calculation of identification. The number of such excluded particles is denoted by $N_{**}$.

**EXPERIMENTAL RESULTS**

**Performance of Stereo Pair Matching**

In order to investigate performance of the new stereo matching method, small spots with a diameter of 0.5 mm are distributed on a model plate, which can be moved along Y axis by a precise driving system, as shown in Fig. 5, and are photographed by two cameras which are fixed on a large circle with a diameter of 910 mm. Two cases where the model plate is put in air or in a water vessel are investigated preliminarily for discussing the effects of refraction correction on the accuracy of three-dimensional coordinate reconstruction. The $X$, $Y$, and $Z$ coordinates of the spots are known at a high accuracy beforehand. Misalignments of the camera setting can be corrected and estimated statistically using confirmed stereo pairs accumulated in the stereo matching processes. The
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Fig. 5 Model plate for stereo pair matching performance

Fig. 6 Test results of stereo pair matching performance

(a) without correction of light refraction
(b) with correction of light refraction

coordinates of the spots are reconstructed by the stereo matching method as mentioned above.

Figure 6 (a) and (b) show the test results of stereo pair matching performance in the two cases without or with refraction correction. The average of deviations of the distances between the two test cases in air and in water, \( \Delta S \), are as follows;
AS = 1.84 mm when refraction correction was not made.
AS = 0.51 mm when refraction correction was made.
Besides the average value of Y coordinates was obtained as 909.96 mm and its standard deviation was 0.185 mm.
As a result, the satisfactory performance of three dimensional coordinate reconstruction was obtained by the present stereo matching method with the refraction correction.

Fig. 7 Outline of experimental apparatus
Velocity Measurement by 3D PTV

The 3D PTV based on cross-correlation method using binary data was applied to gas-water two-phase flows of bottom blowing bubbling jet in a cylindrical water vessel.

Figure 7 shows schematically the outline of the experimental apparatus. The cylindrical vessel is made of transparent acrylic pipe with a diameter of 200 mm and depth of 280 mm and is filled with water. A water jacket made of transparent acrylic plates is installed to avoid the effect of refraction on images. Air bubbles are ejected vertically through a nozzle installed at the bottom of the cylindrical bath. The air flow rate is 1.4 cc/s. The measuring volume is a 100 mm cubic and is illuminated using a light system for a slide projector with a power of 650 W. Nylon-12 balls with a diameter of 0.9 mm and the specific gravity of 1.02 are put as tracers for the flow visualization in the bath. The water contains some salt to have the same specific gravity as the particles so that the particles float at neutral positions in still water. The motion of particles are photographed using two CCD type video cameras and are analyzed by the 3D PTV.

Fig. 8 Visualization of flow field in a bubbling jet mixing water vessel
Figure 8 shows a visualized whole field of gas-water two-phase flow in the cylindrical vessel. The pathlines of particle tracers were photographed using a camera and illumination through a slit type of light source with the exposure time of 0.5 s. Strong vortex ring is observed in the upper part of the vessel, and the air bubbles agitate and mix the water in the vessel. Note that such pathlines were not employed for measurement of velocity by the present technique.

Fig. 9 Bubble deformation and tracer particle motion

Figure 9 shows a picture of the flow field near the nozzle which is installed at the bottom of the vessel. It was taken by a camera which was set closely to the wall of the water jacket. The observation area was illuminated by a slit type of light with a thickness of about 10 mm. The initial form of a bubble is spherical at the exit of the nozzle. It soon deforms to a mushroom shape of hemisphere in the upper part and horizontally flat disc in the lower part. Then the mushroom shape deforms to a wholly flat disc and the bubble goes up along a zigzag trajectory. Each trajectory of a bubble is different from
another. According to the observation by a high speed video picture, particles which are far from bubbles move at much lower velocity than the bubbles, but some particles which are very ahead near bubbles move at the nearly same high velocity as the bubbles.

![Fig. 10 Three-dimensional velocity vectors of tracer particles in a bubbling jet mixing water vessel](image)

The authors [11, 12] have investigated the flow field using the 2D PTV by the binary cross-correlation method, and Hassan, Blanchat, Seeley and Canaan [13] measured the both components of a two-phase flow using a point-by-point digital cross-correlation analysis algorithm for the 2D PTV. Both of the studies described the velocity fluctuation of the water induced by the rising bubble motion. In present study, the three-dimensional velocity vector diagram was obtained as shown in Fig. 10. Although the number of the velocity vectors is not so large, it is observed that the water in front of bubbles moves upward at very high speed and that it enrolls following the bubble motion.

CONCLUSION

The present paper proposed a new stereo pair matching method with correction of light refraction at water-air boundary and a new three-dimensional cross-correlation method for particle tracking velocimetry using binary image data. The new 3D PTV were applied to measure velocity field of water-air two-phase flow of bubbling jet in a cylindrical vessel. As a result, the water motion around bubbles is explained clearly.
REFERENCES


