Multimodal network design problems

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Abstract

In this paper we study the Multimodal Network Design Problem (MNDP); this problem arises when a decision maker (e.g. traffic authority) can operate on different transportation modes simultaneously. For this problem we formulate a general multimodal network design model from which descend multimodal and monomodal network design submodels. Possible solution approaches and research perspectives are identified. This paper, therefore, provides a general framework on multimodal and monomodal network design problems and their modeling.

1 Introduction

The monomodal network design problems are widely studied in literature, as for private transportation system (e.g. Billheimer & Gray [2], Chen & Alfa [6], Davis [7], Le Blanc & Boyce [12], Cascetta et al. [4], Gallo [11], Marcotte [13], Sheffy & Powell [16], Yang & Yagar [17], Cantarella & Vitetta [3]) as for the public one (e.g. Florian & Constantine [8], Furth & Wilson [10], Montella et al. [14], Baaj & Mahmassani [1], Ceder & Wilson [5]). Less interest was instead raised by multimodal network design problems (MNDPs).

In this paper we provide a general framework on network design problem proposing a general multimodal model from which almost all monomodal and multimodal network design models descend.

In section 2 the MNDP will be defined from a general point of view; in
section 3 a general model for the MNDP is formulated and possible submodels that descend from it will be illustrated; in section 4 possible solution methods will be summarized; research perspectives on the argument will be analyzed in section 5 and in section 6 conclusions will be summarized.

2 Multimodal network design problem

The MNDP arises when a decision maker (e.g. traffic authority) can operate on different modes simultaneously; moreover a multimodal approach is necessary whenever the transportation demand has to be assumed elastic on modal choice level. In urban transportation systems the assumption of elastic demand should be stated, in theory, for any network design problem, because a change on supply configuration influences the modal choice.

In urban transportation systems three main modes can be identified: pedestrian, transit (bus, metro, etc.) and auto (private car); in general, changes on supply configuration concern only motorized modes.

As for any network design problem we can determine two kinds of variables: decisional, on which a decision maker can directly operate, and descriptive, that depending on decisional variable values.

In the MNDP we can identify many decisional variables:
- topological auto (\( y_a \) vector): open or closed road, width, lane number, etc.;
- parking (\( s_a \) vector): parking, no parking, disposition of parking lots, etc.;
- topological transit (\( y_b \) vector): bus routes, priority lanes, etc.;
- signal settings (\( g_a \) vector): cycle length, phase plan, green times, offset, etc.;
- transit frequencies (\( q_b \) vector) and vehicle sizes (\( C_{ap_b} \) vector);
- road (or park) pricing (\( t_a \) vector) and transit fares (\( t_b \) vector).

The descriptive variables are the traffic flows: on road network (\( f_a \) vector), on transit network (\( f_b \) vector) and on pedestrian network (\( f_p \) vector); having fixed the decisional variables, the values of descriptive variables result from applications of a modal choice model and monomodal assignment models. The use of two kinds of models could be overcome with a multimodal assignment model (Montella et al. [15]).

The formal relations between problem variables are reported in figure 1; in it we differentiate: design relations (gray lines), performance relations (black lines) and assignment relations (bold type lines). Dashed lines indicate the relations that are negligible in most practical problems.

Formally, design relations represent the direct or indirect influence that some (decisional or descriptive) variables have on decision maker choices on other (decisional) variables; in MNDP these relations are:

\[
y_a = y_a (f_a^*, s_a, y_b) \tag{1}
\]

\[
s_a = s_a (f_a^*, y_a) \tag{2}
\]

\[
y_b = y_b (f_b^*, y_a) \tag{3}
\]
\[ g_a = g_a (f_a^*, f_b^*, f_p^*) \]  
\[ \varphi_b = \varphi_b (f_a^*, f_b^*, \text{Cap}_b) \]  
\[ \text{Cap}_b = \text{Cap}_b (\varphi_b) \]  
\[ t_a = t_a (C_{\text{system}}, t_b) \]  
\[ t_b = t_b (C_{\text{system}}, t_a) \]

where:
- \( f_m^* \) is the (equilibrium) flow vector for mode \( m \);
- \( C_{\text{system}} \) is the system cost, useful to calculate the optimal transportation prices.

The performance relations link performance parameters of the network to variables of the problem; for the MNDP these relations are:

\[ C_a = C_a (f_a^*, f_b^*, f_p^*, \gamma_a, s_a, g_a, \text{Cap}_a, t_a) \]  
\[ C_b = C_b (f_a^*, f_b^*, f_p^*, \gamma_b, g_a, \varphi_b, \text{Cap}_b, t_b) \]  
\[ C_p = C_p (f_p^*, g_a) \]

where:
- \( C_m \) is the route cost vector for mode \( m \).

The assignment relations, instead, represent some constraints of the problem: users' choice modes and paths so as to minimize one's own perceived costs (user optimal assignment) and not to minimize total system costs. The assignment relation in the MNDP can be formulated as:

\[ (f_a^*, f_b^*, f_p^*) = \Lambda [C_a(...), C_b(...), C_p(...)] \]

where:
- \( \Lambda \) is the multimodal assignment function that relates descriptive variables to decisional ones in function of the travel costs;
- \( C_m(...) \) represent the mathematical relation for mode \( m \).

In the follow we introduce these concise notations: \( Y \) is the decisional variables vector, \( F^* \) is the descriptive variable vector and \( C \) is the user cost vector. With these notations the eqn (12), taking into account eqns (9), (10), (11), becomes:

\[ F^* = F^* [C (Y, F^*)] \]

that synthetically represents the assignment constraint.
Figure 1: MNDP general framework.
3 Model formulation

The general model for the MNDP can be formulated as:

\[
\begin{align*}
\{y_a^\wedge, s_a^\wedge, y_b^\wedge, g_a^\wedge, \varphi_b^\wedge, \text{Cap}_b^\wedge, t_a^\wedge, t_b^\wedge\} &= \\
= \arg\min_{y_a \in S_{y_a}, s_a \in S_{s_a}, y_b \in S_{y_b}, g_a \in S_{g_a}, \varphi_b \in S_{\varphi_b}, Cap_b \in S_{Cap_b}, t_a \in S_{t_a}, t_b \in S_{t_b}} Z(y_a, s_a, y_b, g_a, \varphi_b, Cap_b, t_a, t_b, f_a^*, f_b^*, f_p^*) \\
\text{subject to:} & \\
(f_a^*, f_b^*, f_p^*) = \Lambda[C_a(...) , C_b (...) , C_p(...)]
\end{align*}
\]

where:
- \(Z\) is the objective function;
- \(v^\wedge\) indicates the optimal value for decisional variable vector \(v\);
- \(S_v\) is the feasible set for decisional variable vector \(v\).

Using concise notation the general model can be written as:

\[
\begin{align*}
Y^\wedge &= \arg\min_{Y \in S_Y} Z(Y, F^*) \\
\text{subject to:} \quad & \\
F^* &= F^* [C(Y, F^*)]
\end{align*}
\]

Different objective functions can be used, e.g. total travel time minimization, pollutant emission minimization, social cost minimization, road safety maximization, etc. It is evident that different objective functions lead to different MNDPs, with different mathematical and design relations between variables.

As concerns the theoretical properties of the model it is possible to demonstrate the existence of optimal solution. In fact once fixed a supply configuration (\(Y\) vector) the corresponding \(F^*\) vector exists and is unique under some assumptions: the modal choice model leads to a unique modal split pattern and the assignment models for the different three modes give a unique solution fixed the supply configuration. Therefore among all feasible supply configurations (\(Y \in S_Y\)) the optimal solution will be the one(s) to which the optimal value of objective function corresponds, calculable known \(F^*\); only if \(S_Y\) is an empty set an optimal solution doesn't exist.

The proposed model can be ascribed to the Equilibrium Network Design (END) models, taking into account the constraint (15) that, in general, represents an equilibrium assignment relation.
3.1 Submodels

Different simpler submodels can be obtained by general model in function of the decisional variables composing Y vector; it is possible to identify two main network design submodel classes:

- Multimodal network design submodels: the decisional variables regard more transportation modes simultaneously and/or the demand is elastic on mode choice level.
- Monomodal network design submodels: the decisional variables regard only a transportation mode and the demand is rigid.

In tables 1-5 are summarized the main network design models according to the used decisional variables; we indicate by "x" the direct design variables, by "o" the indirect design variables that vary in order to respect geometrical or other constraints and by "-" the non used variables. Moreover we indicate with (r.d.) models assuming a rigid demand and with (e.d.) models assuming an elastic demand at mode choice level. Other models can be obtained combining in different ways the decisional variables each other but they are, in general, not common in practical applications.

### Table 1: General multimodal network design models.

<table>
<thead>
<tr>
<th>Model</th>
<th>( y_x )</th>
<th>( s_x )</th>
<th>( y_b )</th>
<th>( g_a )</th>
<th>( q_b )</th>
<th>( \text{Cap}_b )</th>
<th>( t_a )</th>
<th>( t_b )</th>
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</thead>
<tbody>
<tr>
<td>General multimodal network design (e.d.)</td>
<td>x</td>
<td>x</td>
<td>x</td>
<td>x</td>
<td>x</td>
<td>x</td>
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</tbody>
</table>

### Table 2: Monomodal urban private network design models.

<table>
<thead>
<tr>
<th>Model</th>
<th>( y_x )</th>
<th>( s_x )</th>
<th>( y_b )</th>
<th>( g_a )</th>
<th>( q_b )</th>
<th>( \text{Cap}_b )</th>
<th>( t_a )</th>
<th>( t_b )</th>
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</thead>
<tbody>
<tr>
<td>Road (or park) prices, topological and signal settings network design (r.d.)</td>
<td>x</td>
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<tr>
<td>Topological and signal settings network design (r.d.)</td>
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<td>Road (or park) prices and signal settings design (r.d.)</td>
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<td>-</td>
<td>x</td>
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<td>Signal settings design (r.d.)</td>
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### Table 3: Monomodal extra-urban private network design models.

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<thead>
<tr>
<th>Model</th>
<th>( y_x )</th>
<th>( s_x )</th>
<th>( y_b )</th>
<th>( g_a )</th>
<th>( q_b )</th>
<th>( \text{Cap}_b )</th>
<th>( t_a )</th>
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<tbody>
<tr>
<td>Tolls and topological network design (r.d.)</td>
<td>x</td>
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<td>Topological network design (r.d.)</td>
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<td>Tolls design (r.d.)</td>
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### Table 4: Monomodal transit network design models.

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<thead>
<tr>
<th>Model</th>
<th>( y_x )</th>
<th>( s_x )</th>
<th>( y_b )</th>
<th>( g_a )</th>
<th>( q_b )</th>
<th>( \text{Cap}_b )</th>
<th>( t_a )</th>
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<tbody>
<tr>
<td>Transit network design (r.d.)</td>
<td>-</td>
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<td>x</td>
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<tr>
<td>Transit routes and frequencies network design (r.d.)</td>
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<td>Transit fares and frequencies network design (r.d.)</td>
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<tr>
<td>Transit frequencies design (r.d.)</td>
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Table 5: Urban multimodal network design models.

<table>
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<tr>
<th>Model</th>
<th>( y_a )</th>
<th>( s_b )</th>
<th>( y_b )</th>
<th>( g_a )</th>
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<td>Transit and private car network design (e.d.)</td>
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<td>Transit and private car network design with road (or park) pricing</td>
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<td>Transit and private car network design with transit fares optimization</td>
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<td>Road (or park) prices, topological and signal settings design (e.d.)</td>
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<td>transit fares optimization and bus priority lanes</td>
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4 Solution methods

Several solution methods were proposed in literature in order to solve some monomodal network design problems; a concise, and necessarily incomplete, state of art can be organized as follows:

- (extraurban) topological network design problem: Billheimer & Gray [2], Chen & Alfa [6], Davis [7], Le Blanc & Boyce [12];
- (urban) signal settings design: Cascetta et al. [4], Gallo [11], Marcotte [13], Sheffy & Powell [16], Yang & Yagar [17];
- (urban) topological and signal settings network design: Cantarella & Vitetta [3], Gallo [11];
- frequencies transit design: Florian & Constantine [8], Furth & Wilson [10];
- routes and frequencies transit network design: Montella et al. [14], Baaj & Mahmassani [1], Ceder & Wilson [5].

For a more complete bibliography we remand to references of single papers.

The high complexity of MNDPs suggests, in a first analysis, to use heuristic solution methods or simulation based methods.

The first ones are approximate algorithms that starting from a feasible solution generate a better one until a convergence test is verified; these algorithms can lead to a good, but not optimal, network configuration. Exact methods that lead to global system optimum cannot be proposed, because the objective function isn’t linear neither convex for real problems, the constraints
are nonlinear and some variables are continuous and other discrete.

Simulation based methods, or "what if" methods, are another way to help decision maker to find a "good" solution of a MNDP; in this case some supply configuration are simulated and the simulation results are used to propose a new (and better) supply configuration.

5 Research opportunities

Research opportunities in the field of network design problems are considerable (see also Friesz [9]) and regard models and algorithms; the attention given by transportation researchers to these problem was absolutely less than other research fields as transportation demand and assignment models.

As regards models can be identified, among others, these research opportunities:
- to include an elastic demand in network design models;
- to propose multimodal assignment algorithms;
- to define the accessibility concept in objective function or in the constraints;
- to model the relations between variables;
- to propose joint model of network design and location.

As regards algorithms the main research fields are:
- to develop efficient algorithms;
- to study new heuristic techniques for the MNDP (taboo search, simulated annealing, genetic algorithms, etc.).

6 Conclusions

The Multimodal Network Design Problem (MNDP) is very difficult to model and to solve because the variables are numerous and no homogeneous, the constraints are non linear and the objective function isn’t linear neither convex.

In this paper we tried to provide a general framework on the subject, examining the problem, formulating a concise model and suggesting some opportunities in this research field.

This paper should be seen only as a first step towards the solution of MNDPs; the next steps will be: the formulation and detailed study of some multimodal network design submodels (see table 5), starting from the simpler ones; the proposal of solution methods for the single submodel; the application of models and algorithms on test (and real) networks.
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References


8. Florian, M. & Constantine, I., Optimizing frequencies in a transit network: a nonlinear programming approach, *Crt, 914*, University of Montreal, Canada, 1993


