Optimal dimensioning of district heating networks by use of the Lagrange Multiplier Method

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Abstract

In this paper we present the process for optimal dimensioning of pipe networks for the transport of incompressible fluids. On the basis of mathematical modelling the non-linear objective function and a system of hydraulic restriction inequalities are obtained. The solution of the mathematical model is achieved by using the Lagrange Multiplier Method. The cost analysis is made according to the Method of Present Value which includes the investment, operating and maintenance costs, taking into account the corresponding interest rate.

1 Introduction

Decrease of energy supplies and consequently the requirements for energy savings demand the introduction of new technical and scientific know-how in the field of thermal technique, i.e. the use of different mathematical models for optimal design and dimensioning of district pipe heating. The pipe network should be optimally dimensioned, as the transport of fluid induces costs which are, among others, dependent also on dimensions of the pipe network.
Transport and distribution of hot water under pressure in district heating networks could be done in various types of networks: tree, loop, combined, etc. Each variant of network has to be researched and dimensioned separately for quality technical and economic dimensioning of entire network. The dimensioning can be either conventional or operational research based. With operational research dimensioning, the lawfulness of events in selected pipe network is described mathematically with the objective function and inequalities of hydraulic limitations.

The methods for determining the minimum and maximum of the objective function have been known for quite a time. Nevertheless, they have not substantially influenced on the dimensioning processes. The dimensioning of pipe networks have undergone substantial change with the introduction of computers and operational research. For optimal design and dimensioning of pipe networks beside the presented Optimal Dimensioning by the Use of Lagrange Multiplier Method the following models has been made:

- mathematical model for optimal design of pipe networks with consideration of the future extension of the pipe network and increase of heat consumption respectively e.g. Krope[10]. The solution is based on the use of the Fibonacci’s method.
- mathematical model for optimal pump location in the pipe networks of different designs e.g. Goricanec[7]. The solution is based on the use of the Mixed Integer Non-Linear Programming.

2 Flow – pressure analysis

In fluid transport through pipe networks we are often faced with the incorrectness which is caused by wrong choice of elements or devices respectively, or their wrong connection based on unknown relationships: flow, pressure, friction, pump efficiency, etc. This is the reason why we must perform flow - pressure analysis during the process of optimal dimensioning of pipe networks e.g. Krope[1, 2], Goricanec[3]. Pressure drops in the incompressible fluid flow from node i to node j are function of the pipe diameter, flow, pipe roughness, physical properties of fluid, local friction, and are determined by the non-linear Darcy - Weisbach equation e.g. Krope[4], Goricanec[5].

In planning the flow - pressure model we must consider:

- continuity of nodes flow (the sum of flows which flow into a node must be equal to the sum of flows which flow out of a node).
- preservation of energy around closed loop (the pressure drops sum in any closed loop of the pipe network must be equal to zero).
- continuity of the pipe network flow (the sum of flows into the pipe network must be equal to the sum of flows out of the pipe network).

3 Economic criteria

It is necessary to find the optimal relation between the fixed and variable costs by the planning of district heating networks. The investment costs are a function of pipe diameter and thickness of insulation, and the variable costs are a function of for pump operation used electrical energy and heat losses.

The cost analysis is made according to the Method of Present Value e.g. Kurtz[8], which is based on cognition that the money which will be gained in the future is worth less than the money which we have at our disposal in the moment of decision. The Method of Present Value discounts estimated future costs and profits respectively in present value. In the calculation of present value (PV) eqn. (1) we must consider all costs \( C_n \) which arise with investment, operation and maintenance in the period of system operation (N) and discount them with the corresponding interest rate \( r \).

\[
PV = \sum_{n=0}^{N} \frac{C_n}{(1 + r)^n} \tag{1}
\]

3.1 Pipe network expenditure

The costs of pipes per a running meter of the pipe eqn. (2) are a function of the pipe diameter \( d \) and can be defined with pipe costs polynomial second grade e.g. Peters[6], Krope[9, 11].

\[
C_1 = A + Bd + Cd^2 \tag{2}
\]

3.2 Insulation expenditure

The costs of insulation eqn. (3) depend on insulation thickness \( E \).

\[
C_2 = A_1 E^{B_1} \tag{3}
\]

The constants \( A, B, C \) and \( A_1, B_1 \) are obtained by the least squares method approximation on the basis of individual prices of pipes and insulation.
3.3 Pump investment expenditure

The pump costs eqn. (4) increase with increasing necessary pump power (P) and depend on pump price/W (C_p), flow volume (q_v), pressure drop (∆p) and pump efficiency (η) e.g. Krope[9, 11].

\[ C_3 = C_p \cdot P = C_p \cdot \frac{q_v \cdot ∆p}{η} \]  

(4)

3.4 Electrical energy expenditure

The pumping costs eqn. (5) depend on the price of electrical energy (C_e), flow volume (q_v), pressure drop (∆p), pump efficiency (η) and operating time (t) e.g. Krope[9, 11].

\[ C_4 = C_e \cdot P \cdot t = C_e \cdot \frac{q_v \cdot ∆p}{η} \cdot t \]  

(5)

3.5 Heat losses expenditure

Heat losses eqn. (6) depend on pipe diameter (d), thickness of insulation (E), difference of temperature (T_N - T_Z), pipe length (L), operating time (t) and thermal conductivity of insulation (λ_iz).

\[ Q = \frac{2πλ_iz(T_N - T_Z)}{ln(1 + \frac{2E}{d})} \cdot L \cdot t \]  

(6)

The heat losses costs eqn. (7) depend on price of heat energy (C_t) and heat losses (Q).

\[ C_5 = C_t \cdot Q \]  

(7)

3.6 Total expenditures

The costs of fluid transport through the pipe network presented in second level headings from 3.1 to 3.5 are discounted by the Method of Present Value and summed up. So obtained costs represent objective function (C) which is minimised with the use of Lagrange Multiplier Method. Diameter and thickness of insulation which minimise the objective function are obtained with the derivation of the total expenditures and equalising the derivative by zero eqn. (8).

\[ \frac{∂C}{∂d} = 0, \quad \frac{∂C}{∂E} = 0 \]  

(8)
4 Lagrange Multiplier Method

Pressure drop in each branch of pipe network ($\Delta p$) is given by eqn. (9).

$$p_{iz} - \sum_{j=1}^{k} \Delta p_j = p_j$$  \hspace{1cm} (9).

Lagrange function $\phi (d, E, \lambda)$ has the following form eqn. (10).

$$\phi (d, E, \lambda) = C + \sum_{j=1}^{M} \lambda_j p_j$$  \hspace{1cm} (10).

The optimal solution is obtained so that the partial derivations are equalised by zero eqn. (11).

$$\frac{\partial \phi}{\partial d_i} = 0 \text{(N)}, \quad \frac{\partial \phi}{\partial E_i} = 0 \text{(N)} \quad \frac{\partial \phi}{\partial \lambda_j} = 0 \text{(M)}$$  \hspace{1cm} (11).

We obtain the system of $2N+M$ equations with $2N+M$ unknowns which is solved by the use of numerical iterative method.

5 Example

The optimisation of pipe diameters and thickness of pipe insulation is performed for the pipe network in Fig. 1. The pipe network is composed of 8 nodes and 12 pipe sections. The fluid, in treating the example of hot water, enters the pipe network in node TO and comes out in the remaining nodes. Known data are shown in tables 1, 2 and 3, and results of optimisation are given in table 4.

![Figure 1: Scheme of pipe network](image-url)

Table 1: Pipe length

<table>
<thead>
<tr>
<th>pipe</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>L (m)</td>
<td>330</td>
<td>179</td>
<td>180</td>
<td>100</td>
<td>170</td>
<td>500</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>pipe</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
</tr>
</thead>
<tbody>
<tr>
<td>L (m)</td>
<td>340</td>
<td>430</td>
<td>310</td>
<td>250</td>
<td>400</td>
<td>520</td>
</tr>
</tbody>
</table>
Table 2: Flow and pressure in nodes

<table>
<thead>
<tr>
<th>node</th>
<th>flow (m$^3$/s)</th>
<th>pressure (kPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.002</td>
<td>50</td>
</tr>
<tr>
<td>2</td>
<td>0.002</td>
<td>50</td>
</tr>
<tr>
<td>3</td>
<td>0.005</td>
<td>50</td>
</tr>
<tr>
<td>4</td>
<td>0.004</td>
<td>50</td>
</tr>
<tr>
<td>5</td>
<td>0.011</td>
<td>50</td>
</tr>
<tr>
<td>6</td>
<td>0.020</td>
<td>50</td>
</tr>
<tr>
<td>7</td>
<td>0.040</td>
<td>50</td>
</tr>
</tbody>
</table>

Table 3: Input data

<table>
<thead>
<tr>
<th>number of pipes</th>
<th>number of nodes</th>
<th>amortisation period</th>
<th>interest rate</th>
<th>operating time</th>
<th>pump efficiency</th>
<th>pressure in node TO</th>
<th>density of medium</th>
<th>temp. of medium</th>
<th>temp. of sorround.</th>
<th>thermal conductivity</th>
<th>pipe roughness</th>
</tr>
</thead>
<tbody>
<tr>
<td>12</td>
<td>7</td>
<td>20 a</td>
<td>10 %</td>
<td>8760 h/a</td>
<td>75 %</td>
<td>700 kPa</td>
<td>958.4 kg/m$^3$</td>
<td>373 K</td>
<td>285 K</td>
<td>0.036 W/mK</td>
<td>0.0004 m</td>
</tr>
</tbody>
</table>

Table 4: Results of optimal dimensioning

<table>
<thead>
<tr>
<th>pipe</th>
<th>d (m)</th>
<th>E (m)</th>
<th>local friction</th>
<th>flow (m$^3$/s)</th>
<th>velocity (m/s)</th>
<th>press. drop (kPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.254</td>
<td>0.171</td>
<td>6.8</td>
<td>0.084</td>
<td>1.65</td>
<td>46</td>
</tr>
<tr>
<td>2</td>
<td>0.039</td>
<td>0.072</td>
<td>2.4</td>
<td>0.002</td>
<td>1.65</td>
<td>210</td>
</tr>
<tr>
<td>3</td>
<td>0.232</td>
<td>0.163</td>
<td>0.5</td>
<td>0.082</td>
<td>1.94</td>
<td>32</td>
</tr>
<tr>
<td>4</td>
<td>0.039</td>
<td>0.072</td>
<td>2.5</td>
<td>0.002</td>
<td>1.65</td>
<td>119</td>
</tr>
<tr>
<td>5</td>
<td>0.232</td>
<td>0.163</td>
<td>0.5</td>
<td>0.080</td>
<td>1.89</td>
<td>29</td>
</tr>
<tr>
<td>6</td>
<td>0.064</td>
<td>0.089</td>
<td>2.5</td>
<td>0.005</td>
<td>1.54</td>
<td>278</td>
</tr>
<tr>
<td>7</td>
<td>0.232</td>
<td>0.163</td>
<td>6.5</td>
<td>0.075</td>
<td>1.78</td>
<td>59</td>
</tr>
<tr>
<td>8</td>
<td>0.058</td>
<td>0.085</td>
<td>2.5</td>
<td>0.004</td>
<td>1.53</td>
<td>268</td>
</tr>
<tr>
<td>9</td>
<td>0.168</td>
<td>0.140</td>
<td>2.5</td>
<td>0.040</td>
<td>1.81</td>
<td>74</td>
</tr>
<tr>
<td>10</td>
<td>0.150</td>
<td>0.133</td>
<td>0.5</td>
<td>0.031</td>
<td>1.75</td>
<td>62</td>
</tr>
<tr>
<td>11</td>
<td>0.094</td>
<td>0.107</td>
<td>2.5</td>
<td>0.011</td>
<td>1.57</td>
<td>144</td>
</tr>
<tr>
<td>12</td>
<td>0.125</td>
<td>0.122</td>
<td>3.0</td>
<td>0.020</td>
<td>1.63</td>
<td>142</td>
</tr>
</tbody>
</table>
Conclusion

The solution of the mathematical model for optimal dimensioning of district heating pipe networks is realised by the Lagrange Multiplier Method, and the cost analysis by the Method of Present Value. The computer program enables quick determination of optimal standard pipe diameter and optimal thickness of insulation what reduce the investment and operating costs of fluid transport through the pipe network considerably. The estimation of the influences of changes in the economic and hydraulic conditions is also possible.

References


