A Stochastic Network Loading model for ITS urban transit networks

U. Crisalli

Department of Civil Engineering, “Tor Vergata” University of Rome, via di Tor Vergata 110, 00133 Rome, Italy
Email: crisalli@mingus.civ.utovrm.it

Abstract

In this paper a Stochastic Network Loading model that uses a dynamic approach to the simulation of transit services is presented. The model allows to consider the within day time-dependencies of demand and supply services for high frequency transit systems; it takes also into account the presence of ITS (Intelligent Transport Systems) that reflects on regularity/punctuality of services and on user behaviour at stops due to the presence of information at stops. The paper includes the description of the used supply network model, the specified path choice model and the behavioural hypotheses on which it is based, as well as a Dynamic Network Loading (DNL) procedure and an application to a real size network.

1 Introduction

The dynamic assignment models allow to obtain the time-dependence of flows and travel times (Cantarella and Cascetta [2]). In the case of transit systems, they allow to achieve on-board flows of each run and not only the average load of run sets having the same path (line), usually obtained through a static approach.

In this paper we refer to the transit dynamic assignment approach proposed by Nuzzolo and Russo [7,9] and we deal with a Network Loading model for urban transit networks that uses a dynamic path choice
model of stochastic type. The assignment model allows to consider the within day time-dependencies of demand and supplied services for high frequency transit systems. Such a model is described for regular services (obtained through APTS, Advanced Public Transportation Systems) and for presence of information to users at stops (obtained through ATIS, Advanced Traveller Information Systems). An updated state of the art on APTS systems already operative is presented by Federal Transit Administration [4].

The following sections will describe the used spatio-temporal (diachronic) supply model, the specification of the path choice model in relation to the user behavioural assumption on which it is based, a Dynamic Network Loading (DNL) procedure and an application example.

2 The spatio-temporal supply model

The supply model, which has to be applied to dynamic assignment models, is a spatio-temporal (diachronic) network that allows to define service runs both in space and in time; it is an update of the well known diachronic network model proposed by Nuzzolo and Russo [7] for extraurban transit systems.

If we consider a simulation period T (e.g. 7.30-8.30 a.m.) and the transit services (runs) relative to this period, the spatio-temporal graph is made of three subgraph combination (see figure 1 for an example): the first concerning services; the second relative to the demand temporal structure; the third concerning access/egress (A/E) networks to the transit system.

The diachronic subgraph representing services is made considering ideal temporal axes at each stop; the link representing the generic run r, from stop A to stop B, is located both spatially, between stop A and stop B, and temporally with initial and final nodes respectively in departure ($n_A^r$) and arrival ($n_B^r$) times.

The subgraph relative to temporal demand structure is composed by different subgraphs, at least one for centroid, in relation to O/D matrix structure characterised also for departure time. Generally time distribution can be discretised according to a certain number of time intervals in which the demand distribution, connected to the departure time, may be thought to be constant and located in the interval middle point, called “temporal centroids”; this point set, defined for each spatial centroid, constitutes the temporal demand subgraph (see figure 1).
The subgraph relative to access/egress allows to simulate pedestrian paths among different stops to carry out transfers and to connect the origin and destination to the transit services. For further details on network models see Nuzzolo and Russo [8].

![Figure 1. The spatio-temporal (diachronic) supply model.](diagram)

### 3 User behavioural hypotheses and path choice model

Considering the generic O/D pair, we assume the choice set $I_{ods}(t)$ is made of runs connecting directly or indirectly the stop $s$ with the destination one, leaving after the user arrival time $t$ at stop, satisfying some given rules (e.g. constrains on travel time components, as waiting time, on-board time, transfer time) and not being dominated, i.e. it is impossible to have two runs of which one, leaving after, arrives earlier.

In the following we deal with high frequency services (i.e. the cumulated frequency $\phi_{iods}$ of runs belonging to choice set $I_{ods}(t)$ is greater than 5 runs/hour) with regular runs (i.e. for any stop $s$ of the path, the differences, in absolute value, between the scheduled arrival time at stop $s$ and the real one result to be smaller than a pre-fixed value) and information to users at stops.
Considering the information to users, we will suppose that:

- at origin, user is completely informed about the scheduled times;
- at stop, the user is informed by ATIS about waiting times of arriving runs.

These service characteristics are strictly related to the user behavioural hypotheses to be taken into account in the specification of the path choice model.

### 3.1 User behavioural hypotheses

In order to define the user behaviour hypotheses, we refer to the random utility theory (Ben-Akiva and Lerman [1]) assuming the user as a rational decision-maker, i.e. he chooses an alternative (path) minimising his disutility among a set of paths linking the O/D pair. We associate a disutility $U_p$ to each path $p$ which is made of a set of components as access and egress time ($t_a$, $t_e$), on-board time ($t_b$), transfer time ($t_t$), number of transfers ($N_t$), waiting time ($t_w$), and so on.

Besides these general hypotheses other specific ones have to be considered in relation to the stop and run (or sequence of runs) choice mechanism and to the user arrival at stops.

In the following we assume the user as regular, i.e. he knows the real system functioning based on previous experiences.

Considering the *stop and run choice mechanism*, in literature (Nguyen and Pallottino [6]; Cascetta and Nuzzolo [3]; Spiess and Florian [10]) two types of choice are considered: preventive choice (when the user chooses, before leaving, on the basis of the available information about path alternatives, e.g. scheduled times and real ones experimented in previous trips) and adaptive choice (when the user chooses considering service variations that happen during the trip and eventual further available information, in addition to previous ones).

In the sphere of the service characteristics reported before, we assume a preventive access stop choice at origin and an adaptive run choice within $I_{ods}(t)$ at stop, with user arrival time $t$ not related to the run scheduled time. The adaptive choice is of intelligent type, i.e. when a run belonging to the choice set $I_{ods}(t)$ arrives at stop $s$, the user gets on the vehicle if the perceived disutility connected to that run is smaller than disutilities of the other runs (including the residual waiting time), which have not arrived yet.

For what concern the *user arrival* at stops, in the case of high frequency services, we can assume that:
due to frequency services, the user has an origin departure time $t_D$ (not related to specific scheduled runs) and leaves around $t_D$ in an interval $\Delta D = [t_D-\Delta, t_D+\Delta]$;
- the user will arrive at stop in a temporal interval $\Delta D_s = [t_{Ds}-\Delta, t_{Ds}+\Delta]$ with $t_{Ds}$ equal to $t_D$ plus $t_{as}$ (average access time from origin to stop $s$)

### 3.2 The path choice model

In the sphere of high frequency regular systems with user information, the path choice model is described considering a fixed O/D pair, hence in the following we omit the “od” index. We define a path as a combination of origin/access stop /run(or sequence of runs)/egress stop/destination. In the following we assume the user has a fixed origin departure time and so we do not take into account the choice of departure time.

The joint choice probability of stop $s$ and run $r$, conditioned to the origin departure time $t_D$, can be expressed as:

$$p(r,s| t_D) = p(s| t_D) \cdot p(r|s, \Delta D_s)$$

where:

- $p(s| t_D)$ is the probability of choosing stop $s$ conditioned to the origin departure time $t_D$;
- $p(r|s, \Delta D_s)$ is the probability of choosing run $r$ conditioned to the use of stop $s$ and to the average arrival time in $\Delta D_s$ at stop $s$.

#### 3.2.1 Stop choice model

For what concern the stop choice, we hypothesise that the perceived utility $U_s$ associated to stop $s$ is function of: a vector of specific stop attributes $X_s$ (as access time from origin, shops, news-stands presence and so on), an utility $H_{AD_s}$ (named “run inclusive utility”) relative to accessible runs from the stop $s$ in relation to origin departure time $t_D$, a random term $\eta_s$. It can be written as

$$U_s(t_D) = V_s(t_D) + \eta_s = \beta_{ja} \cdot X_s + \beta_H \cdot H_{AD_s} + \eta_s$$

in the case that $\eta_s$ is defined according to Gumbel distribution, the stop $s$ choice probability can be calculated using a multinomial logit model as
where $V_s(t_D)$ is the systematic utility of stop $s$ and $S_{od}$ is the accessible stop set to reach $d$ from $o$ (where accessible stop means, for example, a stop inside a given range of maximum distance from $o$).

Indicating with $H_s(t)$ the expected utility (satisfaction) that users with arrival time $t$ associate to choice set $I_s(t)$, if the residuals are defined according to Gumbel distribution, we can write:

$$H_s(t) = \log \sum_{r \in I_s(t)} \exp(V_r(t))$$

So the "run inclusive utility" $H_{\Delta_D s}$ for users that arrive in $\Delta_D s$ can be expressed (Nuzzolo and Russo [9]) as

$$H_{\Delta_D s} = E[H_s(t)] = \int_{D_s - \Delta}^{D_s + \Delta} \log \sum_{r \in I_s(t)} \exp(V_r(t)) \cdot f(t) dt$$

where $f(t)$ is the probability law of user arrival at stop $s$.

### 3.2.2 Run choice model

Considering the user arrival time $t$ at the stop $s$, the user can choose the run inside the choice set $I_s(t)$ previously defined.

The perceived utility $U_r(t)$ of the generic run $r$, conditioned to user arrival time $t$, can be written as:

$$U_r(t) = V_r(t) + \eta_r = \alpha_w \cdot T_{wr}(t) + \alpha_b \cdot T_{br} + \alpha_t \cdot T_{tr} + \alpha_{nt} \cdot N_{tr} + \eta_r$$

where: $T_{br}$ is the on-board time, $T_{tr}$ is the transfer time, $N_{tr}$ is the number of transfers, $T_{wr}(t)$ is the waiting time from arrival time $t$ at stop to the run $r$ departure time (known by ATIS) and $\eta_r$ is the random term. In the case of $\eta_r$ defined according to Gumbel distribution, the choice probability of run $r$ conditioned to arrival time $t$ can be expressed using a multinomial logit model as

$$p(r | t) = \frac{\exp(V_r(t))}{\sum_{r' \in I_s(t)} \exp(V_{r'}(t))}$$
where \( V_r(t) \) is the systematic utility of run \( r \) conditioned to user arrival time \( t \). So the probability \( p(r|\Delta_{Ds}) \) to use run \( r \) considering all possible arrival time \( t \) in the interval \( \Delta_{Ds}=[t_{Ds}-\Delta, t_{Ds}+\Delta] \) can be expressed through:

\[
p(r|\Delta_{Ds}) = \int_{t_{Ds}-\Delta}^{t_{Ds}+\Delta} p(r|t) \cdot f(t) \, dt
\]

where \( f(t) \) represents the probability law of user arrival at stop \( s \).

4 A Dynamic Network Loading procedure

The path choice model reported in section 3 can be used in transit assignment models to calculate the get on flow \( f(r,s|t_D) \) for the generic run \( r \) at stop \( s \).

Referring to link costs that are not flow-dependent, we can deal with Stochastic Network Loading models. In the following a computation procedure for such a model is described. It is based on a supply representation with a spatio-temporal (diachronic) network model, an explicit path enumeration and an O/D demand matrix also defined for origin departure times.

Given a generic O/D pair, all the possible paths connecting the O/D pair are generated; then they are analysed in order to eliminate paths that are: dominated, with access and egress distance greater than a pre-fixed value, with number of transfers greater than a pre-fixed value, with transfer time outside a pre-fixed range, with travel time greater than an \( \alpha\% \) compared to the minimum one.

Such a “useful” path choice set is used to obtain the choice set \( I_s(t) \), conditioned to the origin departure time \( t_D \), inside which path alternatives are considered.

In order to set up the algorithm, it is suitable to discretising the possible arrival time \( t\in\Delta_{Ds}=[t_{Ds}-\Delta, t_{Ds}+\Delta] \) in \( n \) points, associating to each point \( t_i \) a probability \( p(t_i) \) to arrive at time \( t_i \) and so, from eqn (8), it is possible to compute the average run choice probability in relation to the \( n \) arrival times as

\[
p(r|\Delta_{Ds}) = \sum_{i=1}^{n} p(r|t_i) \cdot p(t_i)
\]

In the same way, from eqn (5), we can calculate the run inclusive utility related to all \( n \) considered arrival times as
in order to estimate the probability \( p(s|t_D) \) through eqn (3).
Hence given the demand \( d_{od}(t_D) \) relative to the departure time \( t_D \), it is possible to calculate the get on flow on the generic run \( r \) at the stop \( s \) as

\[
f(r, s | t_D) = d_{od}(t_D) \cdot p(s | t_D) \cdot p(r | \Delta_{Ds})
\]

5 An application to a real size network

The assignment model has been applied to a real size transit network in order to show the goodness of the proposed approach. The transit network of a medium-size town in the south Italy (Salerno) has been considered. It is made of 58 service lines with 237 runs in the referring period (7:30-8:30 a.m.). The study area, characterised by 160,000 inhabitants and about 7,200 transit users in the peak hour, has been divided in 62 traffic zones.

The considered supplied services allow to hypothesise high frequency services and random user arrival at stops. For the application test we hypothesise regular user and, through the introduction of ITS systems, we also assume regular services working and presence of information to users at stops. In such conditions we can refer to a preventive stop choice at origin and an intelligent adaptive run choice at stop. The Stochastic Network Loading model, as described in section 4, uses stop and run choice models of logit type and a spatio-temporal (diachronic) network model, which is made of 13,801 nodes and 24,992 links. As regards the stop and run choice, the utility functions are those reported in eqns (2) and (6). The model parameters have been assumed according to preliminary results of a research currently in-progress (see LaPST [5]).

In order to evaluate only the implication of time-dependence supply, in the application the demand level has been held constant during the simulated period, using time slices of 2 minutes each and an arrival interval \( \Delta_{Ds} \) equal to 4 minutes. The test has been carried out on a standard PC (Pentium 300MHz processor and 64Mb RAM) that takes about 10 seconds to assign each O/D pair to the service network.

As described before, the assignment model allows to obtain on-board flows for each run of each service line. Figure 2 shows the on-board flows of each run belonging to a service line on a generic section between two stops and the average value.
The average on-board flow is the most detailed result that can be obtained using a traditional static assignment model and, as we can see, it can be quite different from the single run ones, obtained through the proposed approach. In order to analyse the dispersion of flows and times, table 1 reports the average, the minimum and the maximum values of the variation coefficient relative to:

- on-board flows $C_v(f_{lj})$, calculated considering on-board flows of runs for each section $j$ and each line $l$;
- O/D travel time $C_v(t_{od})$, considering average travel time of runs belonging to choice sets characterised by O/D pair and possible departure time.

<table>
<thead>
<tr>
<th></th>
<th>avg</th>
<th>min</th>
<th>max</th>
</tr>
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<tbody>
<tr>
<td>$C_v(f_{lj})$</td>
<td>0.50</td>
<td>0.01</td>
<td>1.215</td>
</tr>
<tr>
<td>$C_v(t_{od})$</td>
<td>0.18</td>
<td>0.06</td>
<td>0.37</td>
</tr>
</tbody>
</table>

The results show the goodness of the dynamic approach to give detailed information about system functioning.

6 Conclusions

In this work a Network Loading assignment model for urban transit services has been presented. The used dynamic approach is more complex and time-consuming than the traditional static one, but allows to obtain more precise and detailed outputs, in particular the time-
dependence of run loads and relative travel times. It is suitable to be used as support tool for operative planning or when it is necessary to evaluate the modifications on transit system, due to service characteristics changes (e.g. the introduction of ITS systems).

Further developments are in progress; they mainly regard the specifications and the adjustment of Dynamic Network Loading procedures for different service characteristics, as irregular high frequency services with and without user information.

References


