Train regulation in single-track transit systems
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Abstract

This paper resumes the first part of a study which aims at assessing the performance of single-track transit systems and improving it by means of appropriate regulation techniques of train running. Firstly, theoretical performance is evaluated by assuming normal undisturbed operation; this analysis aims at providing a planning-level method to verify whether single-tracking is suitable for a particular application. Then, effects of train delays on system performance are investigated and train regulation criteria are discussed. Both an analytic model of single-track transit system operation and computer simulation are considered.

1 Introduction

In several European urban areas, the possibility of reusing some sections of railway lines closed or under-utilised in order to build new light rail transit systems is currently considered. As, in many cases, single-track sections are involved, it is necessary to face the problem of ensuring adequate traffic capacity within the constraints imposed by single-tracking. The same problem must be faced when the possibility of adopting a single-track layout rather than a double-track one in building new sections of a transit system is considered because of physical obstacles that reduce the available right-of-way width or when a single-track layout is regarded as a cost-effective solution. In all these cases, it must be verified that a single-track layout is consistent with the required traffic capacity.

Such problems may be approached on two levels: on a planning-level, and on an operation-level. Within the design of a transit system, free variables that
may be fixed in order to achieve the above-mentioned goals are the position of stations and signals, the length of passing tracks, the composition of trains. On the operation-level, the problems can be formulated by assuming target speeds and departure times as free variables.

The limits of single-track operation must be carefully analysed for a particular application as train delays can increase and become unacceptable when the frequency of services is relatively high. It must be also considered that disturbance propagation causes a transit system to be unstable under particular assumptions, that is the deviation of actual train running diagrams from scheduled running diagrams tends to increase because a train which is delayed has to collect more and more passengers in the subsequent stations [1].

2 An analytic model of single-track line operation

Let \( d \) be the length of a single-track line and suppose that there are no differences in traffic flow by direction. If we assume that there are sidings (that is, passing tracks) in every station and that the stations are equally spaced, then the traffic capacity \( P \) of the line, expressed in trains per hour for one running direction, is

\[
P = \frac{3600}{2WL + t_m}
\]

where:
- \( W \) = expected transit time including delays due to trains meeting [s]
- \( L \) = distance between stations [m]
- \( t_m \) = dead time related to train dispatching [s].

To evaluate the mean running time for trains on a single-track railway we can use the analytic model developed by Petersen [2] that can be adapted to reproduce the operation of a transit system by assuming that all the trains have the same priority. To this end, let us introduce:
- \( s \) = free average commercial running speed, without interference between trains [m/s]
- \( v \) = actual average commercial running speed [m/s].

The theoretical transit time \( T \) necessary to cover the whole line when no interference between trains occurs is given by

\[
T = \frac{d}{s}
\]

while the expected transit time \( W \) including the delays due to trains meeting is

\[
W = \frac{d}{v}
\]

If we introduce:
D = average delay incurred by a train when a meet takes place
M = expected number of meets along the whole line.
then, the expected transit time \( W \) can be expressed as
\[
W = T + DM
\]  \( (4) \)
where the expected number of meets \( M \) can be calculated as
\[
M = 2NW
\]  \( (5) \)
\( N \) being the number of trains in the unit time [trains/second]. Substituting equations (2), (3), and (5) into (4), we have
\[
\frac{d}{v} = \frac{d}{s} + 2ND \frac{d}{v}
\]  \( (6) \)
and then
\[
v = s(1 - 2ND)
\]  \( (7) \)
Finally, the value for \( W \) necessary to evaluate the traffic capacity through equation (1) is derived substituting equation (7) into (3):
\[
W = \frac{T}{1 - 2ND}
\]  \( (8) \)
To evaluate the average delay per meet \( D \), let us define \( x \) as the distance from the projected interference point to the station where the meet actually takes place (see Fig. 1). According to Petersen's model, the meet occurs in station B if \( 0 \leq x \leq L/2 \), otherwise it occurs in station C.

![Figure 1: Trajectories of meeting trains on a space-vs-time diagram.](image)

Let us introduce the following hypotheses:
i) two trains cannot enter the same station from opposite ends simultaneously;
ii) every station has a main track and one or more sidings, and a speed restriction is imposed on a train when it is platformed on a siding;
iii) the first train that enters a station where a meet is to occur is platformed on the siding while the other train takes the main track.
Within these hypotheses, the delays per meet \( d_1(x) \) and \( d_2(x) \) respectively incurred by a downroad train and an uproad train due to a meet can be expressed as:
where:
\( d_s' \) = distance between a station and its home signal

\( s_r \) = maximum speed along a siding

\( ST_0 \) = delay due to the fact that a train platformed on a siding must maintain the reduced speed \( s_r \) until it reaches the exit point.

The switching time \( ST_0 \) is given by

\[
ST_0 = \frac{m}{2s_r} - \frac{s_r}{2a} \quad (11)
\]

where \( m \) is the length of a station, intended as the distance between the entry points and the exit points at the opposite ends of the same station, and \( a \) is train acceleration. If the departure times for trains are uniformly distributed, then the average value \( D_1 \) of \( d_1(x) \) is

\[
D_1 = \frac{1}{L} \int_0^L d_1(x) dx = \frac{ST_0}{2} + \frac{L}{4s} - \frac{d_s'^2}{(2Ls_r^2)} \quad (12)
\]

and \( D=D_1=D_2 \) with the usual hypotheses. This value for \( D \) is to be substituted in equation (8) in order to calculate the line traffic capacity \( P \) through formula (1).
3 Traffic capacity of a single-track line

To analyse how the factors described so far affect traffic capacity of single-track lines, we have considered a single-track light rail transit system as a case study, having the characteristics reported in Table 1.

Table 1: Characteristics of a LRT lines considered as a case study

<table>
<thead>
<tr>
<th>Characteristics</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total length</td>
<td>( d = 14 \text{ km} )</td>
</tr>
<tr>
<td>Distance between stations</td>
<td>( L = 2000 \text{ m} )</td>
</tr>
<tr>
<td>Maximum line speed</td>
<td>( s_m = 75 \text{ km/h} )</td>
</tr>
<tr>
<td>Free average commercial speed</td>
<td>( s = 42.7 \text{ km/h} )</td>
</tr>
<tr>
<td>Maximum speed along sidings</td>
<td>( s_r = 30 \text{ km/h} )</td>
</tr>
<tr>
<td>Dead time</td>
<td>( t_m = 3 \text{ s} )</td>
</tr>
<tr>
<td>Average acceleration and deceleration of trains</td>
<td>( a = 0.55 \text{ m/s}^2 )</td>
</tr>
<tr>
<td>Distance between a station and its home signal</td>
<td>( d's = 400 \text{ m} )</td>
</tr>
<tr>
<td>Length of a siding</td>
<td>( m = 300 \text{ m} )</td>
</tr>
</tbody>
</table>

By assuming a minimum station stop time for each train equal to 30 seconds, except for the terminus, the undisturbed running transit time referred to the whole line is about 20 minutes. In Fig. 2, for a set of different values for the length of block sections \( L \) the expected running transit time \( W \) is plotted as a function of traffic density.

![Figure 2: Expected running transit time as a function of train headway.](image)

To improve the readability of this diagram, the traffic density on the horizontal axis in Fig. 2 is represented as headway \( H \), that is time interval between two
subsequent trains running in the same direction, instead of the number of trains per second \( N \) introduced in the previous section. The relation between \( N \) and \( H \) is

\[
N \text{ [trains/s]} = \frac{1}{60 H \text{ [min]}}. \tag{13}
\]

Given a value for \( L \), when \( H \) decreases the expected transit time \( W \) increases because the probability that a train has to remain in a station due to a meet for a longer time than the minimum stop time becomes higher; moreover, it will be more likely that a train encounters a restrictive signal and must consequently slow down or even come to a stop in front of the home signal of the station where a meet is to occur. Vice versa, given a value for \( H \), when \( L \) increases the expected transit time \( W \) is due to increase because when a meet occurs a train is likely to wait for the train running in the opposite direction for a longer time; this fact can be deduced from equation (12), in which the term \( L/4s \) is prevailing on the term \( d_s^2/(2Ls_f^2) \) provided that realistic values for all the parameters are assumed.

In Fig. 3 \( W \) is plotted as a function of \( L \); a non-linearity in such a function is perceptible when the traffic density is relatively high.

![Figure 3: Expected transit time as a function of the distance between meet points.](image)

In Fig. 4 it is shown how the traffic capacity of the line considered as a case study decreases in function of the distance \( L \) between meet points, as it can be derived from equation (1). The traffic capacity \( P_1 \) is referred to downroad trains and is plotted for a set of four different values for the uproad traffic density \( N_2 \).
These plots can be used in the design of a transit system in order to achieve a given system performance when normal, undisturbed operation is considered.

4 Need for train regulation in single-track transit systems

Train delays due to a meet \( d_1(x) \) and \( d_2(x) \), expressed by equations (9) and (10) are plotted versus \( x \) in Fig. 5 together with their sum \( d^*(x) = d_1(x) + d_2(x) \). For \( x = x_m \) the function \( d^*(x) \) reaches its minimum value, that is \( d^*_s/s_r + S_{T_0} \); \( x_m \) is given by

\[
x_m = \frac{d_s' s}{2 s_r}
\]  

Given the distance between stations \( L \), the average train speed \( s \), and the departure time \( t_1 \) for train no. 1 (see Fig. 6), the departure time \( t_2 \) for train no. 2
can be fixed as to impose $x=x_m$ and consequently minimise the sum of train delays $d(x)$ by using the equation

$$t_2 - t_1 = \frac{L - 2x}{s}$$

(15)

Figure 6: Trajectories of trains meeting.

The regularity with which successive trains arrive at intermediate stations represents an important factor in determining the overall quality of the transport service. Within the design-level the train schedule can be planned as to minimise interference between trains; obviously, the level of service for a single-track transit systems is affected by train delays due to meets, unless the traffic density is very low, but under normal operating conditions such delays can be minimised as it is suggested by equations (14) and (15). Minimisation of these delays leads to maximisation of line traffic capacity. However, a train may deviate from its schedule; a train delayed will collect more passengers at subsequent stations and its delay will increase; without any regulating action the disturbance propagation will involve an increasing number of trains.

A technique to regulate train movements in a double-track metro system aiming at minimising disturbance propagation was developed by Mellitt & Chua [1]. This technique is based upon the principle of delaying some trains and increasing the speed of other trains; an increase in train average speed could be obtained by eliminating coasting mode which is used to save energy under normal undisturbed operating conditions. For single-track lines this technique should be adapted by introducing the minimisation of delays due to trains meeting. In the first part of this study, which is the subject of the present paper, the limits of single-track line operation are mainly investigated in a
macro sense. A particular version, specially suited to single-track LRT lines, of a general purpose railway operation simulator was developed. The simulation program used to this end had been developed to reproduce operation of conventional railway lines and it had been previously validated by comparing the results obtained by simulating the operation of several existing lines with the current train timetables or speed diagrams of the same lines. Several simulation tests were performed by the authors in order to validate the analytic model of single-track line operation presented in the Section 2 of this paper. A comparison between a set of experimental values for $d_2(x)$ resulted from a series of simulation tests and the plot of the same function as it is expressed by equation (10) is shown in Fig. 7.

Figure 7: Comparison between simulation results and the analytic model of train delays due to a meet.

In the next phase of this study the simulation program will be modified to evaluate the effectiveness of the regulation techniques above mentioned. The planned timetable contained in an input file which is accessed by the main procedure of the simulation program to reproduce train movements will be replaced by a dynamically updated schedule generated by a routine which will implement a regulation technique.

5 Conclusions

This paper has presented the first part of a study which aims at improving the quality of service in single-track transit systems by implementing appropriate regulation techniques of train running. Firstly, by developing a detailed analytic model of a train meet, an expression for the expected delay due to interference between trains along a single-track line has been derived. This
model has been compared with the results of several simulation tests referred to a case study. The model proposed in this paper can be used in assess traffic capacity of single-track railway lines. Then, effects of train delays on the traffic capacity of a single-track line have been investigated and train regulation criteria have been discussed. The implementation of a traffic control technique makes it possible to reach adequate traffic capacity within the constraints imposed by the availability of only one track.

References

