

Resource planning in risky environments

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Abstract

The amount of any individual risk acceptance criterion is directly related to the correspondent amount of expected revenue. At the same time, a decision maker often searches for optimal strategies operating under contexts affected both by random events and limited information. This paper is focused on the so-called portfolio risk problem, concerning the risk management of financial resources. The problem leads to an interesting framework that should be applied to other fields, such as risk evaluation and system analysis of transport networks in emergency conditions, as well as risk management forms to improve engineer's decisions during large projects development.

Keywords: portfolio risk problem, risk management, emergency conditions.

1 Introduction

It is quite easy to imagine the situation that each individual faces when he desires to increase his assets by investing in financial products. It is equally easy to imagine that several difficulties actually arise in this situation. Firstly, our investor, or decision maker, has to act in a non deterministic context and his decisions will lead to results that can be only partially governed by the investor itself. Therefore, it is necessary to make use of statistic instruments, in order to have information about the future from what has happened in the past, and of the probability theory, in order to be able to make previsions characterized by a certain degree of confidence, and it is at the same time admitted that the results of any decision are judged, a priori, according to the expected events. While, in retrospect, the actions taken will be assessed according to the facts that actually took place and the comparison between what was expected and what has happened could be used to better target similar subsequent choices.

Another difficulty that arises in front of the investor is that it does not seem to be a universally valid measure to determine what, in the broad sense, could be



said to be the result of an investment decision. At a first superficial approach, this result could be measured with the net gain, i.e. with the difference between what has been earned and what has been initially invested. However, this represents an excessive simplification for several concurrent reasons.

Firstly, the moment in which decisions to invest are taken is usually is usually referred to a different time from the moment the same investments are to be matured. Several times, actually, the amount of money invested and the incomes are staggered on completely different time frames, therefore the general trend can be evaluated only in terms of cash flow. There are several standardized methods to calculate the cash flow value; however, each method to determine the cash flow is a model and, therefore, is subjected to approximations and meanwhile it is an object of criticism. The main difficulty that arises in quantifying an investment strategy or in comparing one or two alternative financial plans consists on the definition itself of “value”: the same quantity of money assumes a different meaning for different people, until it changes from one situation to the other for the same individual who invests the money.

In order to better understand this last concept, a well-known example is reported in the following, which has been stated for the first time by Bernoulli in 1973 and is known as the “paradox of St. Petersburg”. This paradox is the following. We suppose of launching a coin until we get heads for the first time; and we suppose that N is the number of launches before we get heads (i.e. we get heads the first time at the launch $N+1$); in this case the player will win 2^{N+1} euros. We suppose now that the game has a total prize money of K euros, how much you will accept to spend (or invest, or risk) in order to play?

It is quite simple to calculate the gain in such a game. Actually the probability of gaining 2^{N+1} euros is equal to $2^{-(N+1)}$, which is the probability that at the first N launches of the coin we always get tails, while at the launch $(N+1)$ we get heads. Therefore the expected gain G is equal to:

$$G = \sum_{N=0}^{\infty} 2^{N+1} \cdot 2^{-(N+1)} = \sum_{N=0}^{\infty} 1 = +\infty \quad (1)$$

and the net gain will therefore be equal to: $+\infty - K = +\infty$.

As a result, considering the effectiveness of these simple calculations, there is not any reason to define any maximum limit to the amount with which we enter in the game while on the contrary there is an infinite number of good reasons to participate, in its literal sense. However, if you are going to ask a person, even an expert of finance, to really participate to the game, it would be quite difficult to find someone willing to risk a certain amount, albeit modest, to sit down to play.

What has been described is known as a paradox, but it is not at all in the strict sense of the term and it is rather a simple example to show how the value of money is not perceived directly. What governs the decisions of investors is not actually the absolute value of the gain they expect, nor the relative increase of their assets, but what could be defined by the name of utility, and which in some way is a subjective measure of the degree of propensity, or aversion, to the risk of the individual. The hypothesis put forward by Bernoulli [3] was that the individual utility associated with a certain amount of money is function of

the patrimonial consistency owned by the individual, therefore you could assume that the contribution of a given sum of money was, to the purpose of the utility, inversely proportional to the degree x of overall patrimonial consistency. In other words, calling x the patrimonial state and considering an infinitesimal increase dx of money, we will have a variation of utility dU given by:

$$dU = k \frac{dx}{x} \quad (2)$$

from which we obtain by integration:

$$U(x) = k \log(x) + C \quad (3)$$

where k and C are constant terms of, respectively, proportionality and integration.

The hypothesis that the expression above of the utility $U(x)$ is a correct interpretation of the aversion to the risk became known, in later times, as the principle of diminishing gains. It is obvious that this formula is actually a utility model, moreover it should be remembered that investors would have different attitudes of propensity to risks and that probably their utility function (if it exists) would assume different functional forms. The paradox of St. Petersburg has however not been recalled in order to propose a peculiar functional form of the utility, but only in order to put in evidence that any investor will always have his own propensity to risk and an equally subjective perception of the value of the money, therefore possibly giving rise to different types of utility functions. This subjectivist approach of the individual's behavior introduces high complexity in evaluating and comparing different alternative investment strategies.

1.1 Random utility models

The modern theory of random utility takes its origin in 1947 with the publication of the fundamental work by Von Neumann and Morgenstern entitled "Theory of games and economic behaviors". Without going into details, it is considered sufficient to mention here that this treaty presented the utility theory from the point of view of an axiomatic approach. The major axioms were related to completeness (two quantities can always be compared in terms of utility), transitivity (if A has a higher utility than B and B has a higher utility than C, than A has a higher utility than C), continuity (given three objects A, B, C with respectively increasing utilities, there is always a probability $p \in [0,1]$ therefore it is indifferent to get B or to play at a lottery where it is possible to win A with probability p or C with probability $1-p$; in this case B is called "equivalent certainty" of the lottery). There are also other axioms, but it is not worth to recall them herewith for synthesis sake. Like any other axiomatic theory, the utility theory of Van Neumann and Morgenstern has never been immune from criticism. In any case, it represents the first attempt to describe within a rigorous framework the economic behavior of rational decision-makers.

Immediately below a fairly simple introductory description of the basic concepts of utility theory is performed. We make use of an example and we

assume that an individual has two possibilities A and B of investment. Both investments show different results under different scenarios. Again for example, we could have the following possibilities for A:

- result 10 with $p = 1/3$; result 20 with $p = 1/3$; result 30 with $p = 1/3$;

while for investment B the possibilities are:

- result 5 with $p = 1/4$; result 20 with $p = 1/2$; result 30 with $p = 1/4$.

If the perceived value of the investment is the real value of the investment itself, i.e. the utility function is the identity, taking into account the probabilities associated to the three different scenarios, the investment A has an expected revenue of 20, while the investment B has an expected revenue of 21.25 therefore it seems preferable to A. If instead we adopt the model suggested by Bernoulli and therefore we use the logarithmic utility function $U(x) = \log(x)$ then the situation changes and the investment A shows an expected value of the logarithmic utility of 2.90, while the analogous expected value for the investment B is equal to 2.82: in this second case therefore A is better than B.

Because the utility function is quite variable from individual to individual, different investors will perceive different utilities and therefore will invest in different manners. What is important is to try to understand what are the characteristics and the attributes of an investor which will condition the shape of a utility function. In the simple example exposed before it is clear that if we move from an identity utility function to a logarithmic utility function, we move from a subject indifferent to the risk to a subject who is averse to risk, or prudent.

In order to better understand the concept of aversion to risk, i.e. of prudence, it should be remembered what previously called "equivalent certainty". We are supposed to have an investment with two known results, which can be referred as x and y and which can be referred to two different scenarios, each of which have a given probability of respectively p and $(1-p)$; therefore an investor indifferent to the risk will remain exactly indifferent to the investment by accepting the secure payment corresponding to $(px + (1-p)y)$. This attitude characterizes an investor indifferent to risk (or risk neutral) if the indifference keeps the same in front of any triple x, y, p , i.e. the utility function for a risk-neutral investor is:

$$U(px + (1-p)y) = pU(x) + (1-p)U(y) \quad (4)$$

We can easily demonstrate that equation (4) is satisfied if and only if U is a linear function and therefore the linearity of the utility function implies the indifference to the risk.

On the other hand, an investor averse to risk would always prefer a secure result rather than a lottery, i.e. his utility function satisfies the following condition:

$$U(px + (1-p)y) \geq pU(x) + (1-p)U(y) \quad (5)$$

Equation (5) coincides with the expression of concavity of a function. For example: the logarithmic utility function is concave therefore it corresponds to a

subject who opposes to risk. To conclude, it should be stated that an investor which prefers however to risk in order to get a result, independently from the costs supported, will have a convex risk function:

$$U(px + (1-p)y) \leq pU(x) + (1-p)U(y) \quad (6)$$

It is also worth underlining that it is fully reasonable to assume that, for any investor, the utility function is a non decreasing function and therefore, if it is continuous and non differentiable, its derivative is always non negative (although in general it is strictly positive).

2 Measure of portfolio risks

Given a finite set of possible patrimonial placements, or packages, we assume that a portfolio is defined through the percentage x_j of each of the j placements of the patrimony that an investor may decide to reach with his investment. Therefore it immediately follows that $x_j \geq 0$ and that $\sum_j x_j = 1$.

Assuming that a set Ω of possible scenarios with its respective associated probabilities π_ω and assuming that, for each scenario ω and for each collocation j , the revenue, or the result, of the investment $R_{j\omega}$ is known, then, for each patrimonial arrangement in exam, it is possible to calculate the expected value for the revenue as:

$$r_j = E[R_{j\omega}] = \sum_\omega R_{j\omega} \pi_\omega \quad (7)$$

which actually constitutes the expected result, given some possible scenarios, of a unitary investment (e.g. 1 euro) in the j^{th} package and for which it is possible to immediately obtain, given that the average is a unitary operator, the expected revenue for the entire portfolio as:

$$r = \sum_j x_j r_j \quad (8)$$

Regarding the risk, it is usually associated to the variance (or to the standard deviation) of the portfolio revenue. In order to calculate this dispersion measure it is necessary to have available some information on the correlation (or the covariance) between different collocations. The variance of a given portfolio is actually given by:

$$\sigma^2 = \text{var}\left(\sum_j x_j R_j\right) = \sum_j x_j^2 \text{var}(R_j) + \sum_j \sum_{k \neq j} x_j x_k \text{cov}(R_j, R_k) \quad (9)$$

where:

$$\text{var}(R_j) = E\left[(R_{j\omega} - r_j)^2\right] \quad (10)$$

and:

$$\text{cov}(R_j, R_k) = E\left[(R_{j\omega} - r_j)(R_{k\omega} - r_k)\right] \quad (11)$$

We call Q the variance-covariance matrix, whose generic element is:

$$Q = \begin{cases} \text{var}(R_i) & i = j \\ \text{cov}(R_i, R_j) & i \neq j \end{cases} \quad (12)$$

and we call x the array whose j^{th} component is x_j ; therefore the variance of the portfolio can be written:

$$\sigma^2 = E\left[\left(\sum_j x_j R_j - r\right)^2\right] = x^T Q x \quad (13)$$

As will be shown in the next section, a risk-averse investor will give preference to a diversified investment plan rather than opt for a position dominated by a single acquisition, in order to reduce the overall risk of the investment.

The essential tool for managing risk containment, therefore, becomes the correlation that exists between the results of different patrimonial collocations, while the variance of the portfolio is proposed as a good measure of the degree of dispersion around the mean value of the result. It follows that a prudent person will prefer a portfolio characterized by a very low variance, however appropriate to its aversion to take risks. It must in any case be noted that the variance is a good measure of the degree of dispersion around the mean, but at the same time it weighs equally variances of negative sign as those of positive sign. However, it is quite normal that an investor, even a prudent individual, would always be rather pleased to get a result corresponding to a positive variation with respect to the expected value. This implies that in current practice in the financial field people often use other terms of measurement, such as to put greater emphasis on the risk of loss of capital and not only the risk of deviating from a given expected value of the revenue of the investment.

An important measure of the risk of losing real capital in a given investment is the so-called semi-variance, also known as the *downside risk*:

$$E\left[\left(\sum_j x_j \max\{0; (r - R_{j0})\}\right)^2\right] \quad (14)$$

For how the downside risk is defined, it is evident that the results above the expected value are not taken into account, whereas only the losses, i.e. the results below the expected value, contribute to composing the total degree of risk. In most cases the results can be similar to the random variables distributed according to a normal and then minimizing the variance corresponds to minimize the downside risk. In many practical applications, in particular when the results are presented with a non-symmetrical probability density function, the downside risk reflects better than any other magnitude propensity investor to minimize the risk.

3 Diversify in order to risk less

Previously it was shown that a prudent person, i.e. a risk-averse investor, usually prefers to use a strategy of diversification of his investments in order to contain any measure of risk.



The distribution of the invested capital over several different packages is a recurrent strategy for investors who want to guarantee a certain coverage for the possibility of experiencing poor results of their choices of placement of the portfolio. The key to understanding the relationship between diversification and risk reduction lies in the correlation between different investments and is defined by the relationship:

$$\rho_{jk} = \frac{\text{Cov}(R_j, R_k)}{\text{Var}(R_j)\text{Var}(R_k)} \quad (15)$$

We take now under consideration, by way of example, a limit case: let us say that there are only two possible placements, A and B, and that a portfolio has been realized for a portfolio consisting of a $\lambda\%$ of A and of a $(\lambda-1)\%$ of B; if r_A and r_B are the respective expected results of the two placements in question, the expected revenue of this portfolio is simply given by:

$$r = \lambda r_A + (1 - \lambda) r_B \quad (16)$$

If we indicate now with σ_A^2 and with σ_B^2 the variances of the two investments, while σ_{AB} is their covariance, it is equally easy to develop the calculation of the variance of the portfolio that will be expressed by:

$$\sigma^2 = \lambda^2 \sigma_A^2 + (1 - \lambda)^2 \sigma_B^2 + 2\lambda(1 - \lambda)\sigma_{AB} = [\lambda(1 - \lambda)] \begin{bmatrix} \sigma_A^2 & \sigma_{AB} \\ \sigma_{AB} & \sigma_B^2 \end{bmatrix} \begin{bmatrix} \lambda \\ (1 - \lambda) \end{bmatrix} \quad (17)$$

Assuming that the two locations are perfectly correlated, i.e. $\rho_{AB} = 1$, then, being $\rho_{AB}\sigma_A\sigma_B = \sigma_{AB}$, it results:

$$\sigma^2 = (\lambda\sigma_A + (1 - \lambda)\sigma_B)^2 \quad (18)$$

therefore the standard deviation of the portfolio can be obtained as a linear combination of the standard deviations of the two packages that form it. In this example, the risk of each package is transferred, without any kind of modification, to the composition given to the portfolio, therefore the diversification among perfectly correlated patrimony collocations is useless, at least from the risk reduction point of view. The example given represents the mathematical formulation of a quite simple rule: in order to lower the level of risk it is necessary to build a portfolio so that each time a group of placements goes to poor performances, there is another group which instead gets results that exceed the desired revenues. In the case of perfect correlation, all patrimonial collocations behave in the same way and therefore it is impossible to compensate the risks. Conversely, if two collocations are in perfect anti correlation, i.e. $\rho_{AB} = -1$, then the investment risk becomes:

$$\sigma^2 = (\lambda\sigma_A - (1 - \lambda)\sigma_B)^2 \quad (19)$$

and if we set:

$$\lambda = \frac{\sigma_B}{\sigma_A + \sigma_B} \quad (20)$$

we can realize a portfolio without any possibility of risk.



If instead the correlation is neither 1, nor -1 , as in the majority of situations, we will find an intermediate case. In general, given the characteristics of the two investments, it is possible to build a portfolio with a minimal risk choosing a value for λ in the interval $[0,1]$, therefore the expression of the risk $(\lambda^2\sigma_A^2 + (1-\lambda)^2\sigma_B^2 + 2\lambda(1-\lambda)\sigma_{AB})$ is minimized. If we now calculate the first derivative of this expression with respect to λ , we obtain:

$$\lambda(\sigma_A^2 + \sigma_B^2 - 2\sigma_{AB}) - \sigma_B^2 + \sigma_{AB} \quad (21)$$

therefore if this equation were valid, i.e. $2\sigma_{AB} = (\sigma_A^2 + \sigma_B^2)$, the coefficient λ would vanish and no risk minimization would be possible.

From a more accurate examination it emerges that, however, in this case it would be necessary that both $\sigma_A = \sigma_B$ and $\rho_{AB} = 1$, i.e. that we have two collocations perfectly and positively correlated and with a risk percentage totally equal. In all the other cases the risk becomes minimum assuming a value $\bar{\lambda} \in [0,1]$ given by the ratio

$$\bar{\lambda} = \frac{\sigma_B^2 - \sigma_A\sigma_B\rho_{AB}}{\sigma_A^2 + \sigma_B^2 - 2\sigma_A\sigma_B\rho_{AB}} \quad (22)$$

To summarize, the minimum portfolio risk is reached when the following values for $\bar{\lambda}$ are assumed:

$$\bar{\lambda} = \begin{cases} 0 & \text{for } (\sigma_A/\sigma_B) < \rho_{AB} \\ 1 & \text{for } (\sigma_A/\sigma_B) = \rho_{AB} \\ \frac{\sigma_B^2 - \sigma_A\sigma_B\rho_{AB}}{\sigma_A^2 + \sigma_B^2 - 2\sigma_A\sigma_B\rho_{AB}} & \text{otherwise} \end{cases} \quad (23)$$

Each aware investor would decide his patrimonial collocation strategy according to the portfolio risk minimization criterion in order to ensure the maximum revenue.

A rational investor would generally choose an efficient portfolio, i.e. a portfolio representative of a Pareto optimum in a multi target problem of risks minimization and of expected gains maximization.

One of the possible strategies to adopt in order to have efficient portfolios is to establish the risk level, which could be placed between the minimum risk and the risk level to get the maximum revenue. In this case, because the risk of level is fixed a priori, we arrive at defining the portfolio composition of the maximum expected revenue.

An alternative decision could be when the investor decides a predetermined gain, intermediate between that obtained with minimum risk and the highest possible expected. In this second case, because the gain is established a priori, we compose the portfolio with the minimum corresponding risk.

These concepts have been generalized and represented by Markowitz [8], who formulated the Markowitz portfolio risk model: given a set of n patrimonial collocations, each with an expected revenue $r \in R^n$ and characterized of a matrix

$Q \in R^{n \times n}$ of variance – covariance, the Markowitz model allows to determine the composition of the portfolio of minimum risk respect to a given revenue r_0 :

$$\min x^T Q x \quad (24)$$

subject to:

$$r^T x = r_0 \quad (25)$$

$$1^T x = 1 \quad (26)$$

$$x \geq 0 \quad (27)$$

The cost function (24) of the Markowitz model represents the minimization of the risk associated to the portfolio consisting of the purchase of a share x_1 of position 1, x_2 of position 2, ... x_n of position n . The fixed revenue r_0 is guaranteed by the constraint (25), while the constraint (26) establishes that the various quotas x are expressed in percent values and therefore by summing all of them we get the unit; at the same time, the x cannot be negative because only acquisition are possible, as stated in the constraint (27).

The Markowitz model of portfolio risk leads to a quadratic programming model, which is convex because the variance-covariance matrix is semi defined positive. For this reason, several solution algorithms are at disposal, also when for high numbers of n the size of the problem is quite large.

Besides the work by Markowitz [8], several variants of the model have been developed. For example, if we admit the possibility of “uncovered” for some collocations, i.e. if the eventuality of $x_j < 0$ is admitted, if we call x_j^+ the decision of acquiring the j^{th} position, and x_j^- the decision of operating at the same position as j , the Markowitz model is extended by adding the following constraints:

$$x = x^+ - x^- \quad (28)$$

$$1^T x^- \leq \eta 1^T x^+ \quad (29)$$

$$x^+, x^- \geq 0 \quad (30)$$

where: the expression (28) establishes that x is the difference between the positive and negative positions of each collocation (in this case, obviously, constraint (27) becomes useless). The inequality (29) states that it is not good to work in such a way that too many uncovered positions are, and this is performed through the parameter η which fixes the value assigned to the maximum percentage of uncovered positions admissible in the portfolio. The (30) matches the non negativity constraints.

Another variant of a certain interest is that proposed for the case in which the investor is already at the beginning owner of a certain number of patrimonial collocations in a quantity equal to $x_{initial}$ and has to decide which quota to sell and which quota to buy of each of the above mentioned collocations. The original model could be adapted to the case under study adding a balancing equation and a non negativity constraint:

$$x = x_{initial} + y_{buy} - u_{sell} \quad (31)$$

where y_{buy} and y_{sell} are the quantities respectively bought and sold, which actually require to add a further non negativity constraint ($y_{buy}, y_{sell} \geq 0$).

It is necessary to take into account that often the purchase and sell operations are accompanied by management costs. If we call f_{buy} and f_{sell} the unitary cost for respectively purchasing and sell a unitary quantity of patrimonial collocation, then equation (26), i.e. $1^T x = 1$, of the Markowitz model, is replaced by the following:

$$(1 - f_{sell})y_{sell} = (1 + f_{buy})y_{buy} \quad (32)$$

which establishes the total amount of money acquired from the sale of y_{sell} units (i.e. the sale amount without expenses) is equal to the amount necessary for acquiring positions, burdened with the relative expenses. All these modifications do not modify the convexity of the problem, therefore the model is still solvable with the available calculus techniques for quadratic programming.

However, if in this last example the transaction costs would consist of a constant rate, i.e. of a part of fixed cost and independent from the amount of purchase or sell, then the convexity would be lost and the model would become that of mixed non linear integer programming, which implies several solution difficulties also for cases of limited dimensions.

A further variant of certain interest is derivable from the basic model through the introduction of stochastic constraints which impose some limits to the probability of hitting a substantial loss. If we indicate with α a given revenue, although modest, and with β a given confidence level, then a stochastic constraint assumes the following form:

$$\text{Prob}(R \leq \alpha) \leq \beta \quad (33)$$

In (33) R assumes the meaning of random variable representative of the portfolio revenue if its distribution probability function is known in some way; therefore the (33) is equal to a non linear constraint condition. In the specific case that revenues of patrimonial collocations are distributed as a normal with average r and variance-covariance matrix Σ , then we can demonstrate that the constraint (33) can be transformed into the following deterministic expression:

$$r^T x + \Phi^{-1}(\beta) \left\| \Sigma^{1/2} x \right\|_2 \geq \alpha \quad (34)$$

in which $\Phi(\cdot)$ indicates the distributive function of the standard normal random variable. A model which takes into account the maximization of then expected results and which is subjected to a control of the risks of the type seen now could be formulated as follows:

$$\max r^T x \quad (35)$$

subject to:

$$r^T x + \Phi^{-1}(\beta) \left\| \Sigma^{1/2} x \right\|_2 \geq \alpha \quad (34 \text{ rep.})$$

$$1^T x = 1 \quad (26 \text{ rep.})$$

$$x \geq 0 \quad (27 \text{ rep.})$$

This problem, although non linear, is convex in the case in which the assigned confidence level β has such a value that results $\Phi^{-1}(\beta) \leq 0$, i.e. $\beta \leq 0.5$; this actually implies that the model is equal to a linear optimization model on a convex domain and therefore it is solvable with any of the several available algorithmic procedures. Otherwise, if $\beta > 0.5$, the model becomes non convex and there is the actual possibility of getting trapped in some local optimum instead of the global one, therefore the search of the global optimum could become difficult and challenging on the computational field.

Changing context, an investor could have a fairly precise knowledge of his portfolio while instead he could be quite uncertain about the composition of the variance-covariance matrix. Therefore, given a portfolio, it would be impossible to calculate the variance and therefore determine the risk. However, the investor could be interested in determining significant limitations on the risk to be assumed by adopting a quite imprecise information for the variance-covariance matrix. Bringing all of this to the extreme, we could assume that the variance-covariance matrix Σ is completely unknown, but however we know a generic interval $[L_{ij}, S_{ij}]$ within which the generic element Σ_{ij} falls. In this specific situation the investor could desire to know the maximum possible risk that his portfolio x could encounter and therefore he should solve the following problem:

$$\max_{\Sigma} x^T \Sigma x \quad (35)$$

subject to:

$$L_{ij} \leq \Sigma_{ij} \leq S_{ij} \quad (36)$$

$$\Sigma = \Sigma^T \quad (37)$$

$$\Sigma \geq 0 \quad (38)$$

The cost function (35) states the risk maximization with respect to all the possible variance-covariance matrixes. The constraint condition (36) states the belonging of each element to the respective known interval. The matrix must be symmetrical and semidefinite positive, as stated respectively by the constraints (37) and (38). The model now described is known as semi definite programming problem, or SDP, and is configured as a particular case of a special class of convex problems of non linear programming, which could be solved in a quite efficient manner, at least at the current knowledge status, until their dimensions remain limited enough.

4 Concluding remarks

In the technical literature now exists a well defined complex of studies which deal with the so-called *portfolio risk* problem, i.e. the strategic management of patrimonial investments. In all of this, the risk evaluation of a given investment, respect to the prevision of a corresponding expected revenue, assumes central importance. The relative models attempt to reproduce the behavior of the investor who makes choices in a random context. In this context the investor has



to evaluate in some way risks and expected revenues having scarce and however uncertain information, in order to develop an optimal strategy for the investment. This paper has shown an overview of the main aspects of the problem of strategic investment management, believing that its general aspects sometimes occur, therefore useful parallels can be found also in other sectors of the research that require evaluations of situations determined by risk events in relation to limited sources of information on events themselves.

The analysis of project risks is traditionally neglected in the field of public engineering works, according to a wider interpretation of the Arrow and Lind Theorem [2]. According to this theorem, the State has at its disposal a comparative advantage on privates because of its capability of diversifying its risks and spreading them over a wide number of contributors. In practice, the diversification of risks on a wide portfolio of projects is never perfect.

Similar situations can be found in the management of public transport systems, where the portfolio risk theory could be useful in setting up emergency plans, or in managing the risk of accidents which may occur during the construction of major engineering works.

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