Trip distribution forecasting in fuzzy multi-commodity transportation networks

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Abstract

In this paper, the author models trip distribution in fuzzy multi-commodity transportation networks, and indicates its dimension consideration. First, current trip distribution is learnt by an approximation function for each commodity. Then, a quadratic model is used to calibrate these functions. The implementation of the proposed model is demonstrated through the use of a numerical example.

Keywords: fuzzy, multi-commodity, approximation function, trip distribution.

1 Introduction

Trip distribution constitutes the second stage in the transportation planning process. Trip distribution models are used to determine the number of trips between each pair of zones when the number of trips generated/attracted by particular zones is known. Continual utilization of this model will enable future capital needs to be updated and identified. Generally, such information is essential to a variety of operational tasks; failure analysis to capacity planning and traffic engineering.

A survey of these models may be divided into the following types. In gravity-based models, regression techniques are applied to calibrate the parameters. The models are divided into linear [1–3] and nonlinear [4, 5] regression models. The pioneers of these models are Willumsen [6] and Van Zuylen [7, 8]. Statistical models include the constrained generalized least squares model [9] and constrained maximum-Likelihood models [10–13]. Another group has been developed based on sort computing techniques such as fuzzy logic and neural network [14, 15]. They compared their model based on multilayer perception neural network with maximum-likelihood doubly-constrained models. Then, Kalic and Teodorović [16] attempted to develop a technique for modeling trip

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distribution. The model developed represents the application of Genetic Algorithm (GA) for trip distribution forecasting. All the above-mentioned models forecast the trip distribution for single-commodity transportation networks.

Therefore, the basic research task in this article is to develop trip distribution forecasting from single-commodity to multi-commodity. This paper is organized into three sections. The second section contains the proposed method followed by a new method and dimension consideration of the problems in the case of large scale. A numerical example is given. Conclusions are presented in the third section.

2 The proposed method

Let us first define parameters of multi-commodity networks:

T_{ij}^k = Number of interchanged trips of commodity *k* between *i* and *j*.

P_i^k = Number of trips produced by commodity *k* at zone *i*.

A_j^k = Number of trips attracted by commodity *k* at zone *j*.

t_{ij}^k = Travel time of commodity *k* between *i* and *j*.

L_{ij}^k = Optional adjustment factor for interchanges between zones *i* and *j* for commodity *k*.

$i =$ an origin zone, $j =$ a destination zone, $\hat{T}_{ij}^k$ = estimation of T_{ij}^k

Now, the proposed method is described as follows:

STEP 1 (normalize the parameters): using the linear scale transformation, the various scales are transformed into a comparable scale. Therefore, the parameters can be normalized:

$$P_i^k(s) = \frac{P_i^k}{\max\limits_i P_i^k},\ A_j^k(s) = \frac{A_j^k}{\max\limits_j A_j^k},\ L_{ij}^k(s) = \frac{L_{ij}^k}{\max\limits_{i,j} L_{ij}^k},\ t_{ij}^k(s) = \frac{\min\limits_{i,j} t_{ij}^k}{t_{ij}^k} \qquad (1)$$

If these parameters have trapezoid numbers, they can be converted to crisp numbers (2) by the Fuller method [17]:

$$\widetilde{P}_i^k = (P_{i1}^k, P_{i2}^k, P_{i3}^k, P_{i4}^k) \qquad P_i^k = \frac{1}{3}[P_{i1}^k + P_{i3}^k + \frac{1}{2}(P_{i2}^k + P_{i4}^k)] \qquad (2)$$

STEP 2 (Calculate approximation functions): our objective is to minimize the sum of squares errors for all pairs in T_{ij}^k and $\hat{T}_{ij}^k$:

$$\hat{T}_{ij}^{k} = f_k(P_i^k(s), A_j^k(s), L_{ij}^k(s), t_{ij}^k(s)) \tag{3}$$

$$SSE = \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{m}[T_{ij}^{k} - f_k(P_i^k(s), A_j^k(s), L_{ij}^k(s), t_{ij}^k(s))]^2 \tag{4}$$

Based on regularization, it is proven by the RBF neural network that this eqn is minimized by the approximation function (5): [15, 18]

$$\hat{T}_{ij}^{k} = \sum_{h=1}^{N} w_h^k G(\underline{X}_{ij}^k(s), \underline{X}_{[h]}^k(s)) \tag{5}$$

$$G(\underline{X}_{ij}^k(s), \underline{X}_{[h]}^k(s)) = e^{-(\left\|\underline{x}_{ij}^k(s) - \underline{x}_{[h]}^k(s)\right\|)^2} \tag{6}$$

$$\underline{X}_{ij}^k(s) = [P_i^k(s), A_j^k(s), L_{ij}^k(s), t_{ij}^k(s)] \tag{7}$$

Also, eqn (5) can be expressed in matrix format:

$$\underline{T}^k = G^k.\underline{W}^k \tag{8}$$

Therefore, the above approximation functions can be calculated by the weight vector (9):

$$\underline{W}^k = (G^{k'} G^k)^{-1} G^{k'}.\underline{T}^k \tag{9}$$

STEP 3 (Calibrate the approximation functions): for calibrating the above approximation function, the proposed model (P) is formulated, which is one of the main contributions of this paper.

A. Objective Functions 1: based on conservation of flow [19], the sum of trips produced for *k*-th commodity in zone *i* should be equal to P_i^k :

$$\sum_{j=1}^{m} \hat{T}_{ij}^{k} \cong P_i^k \qquad i = 1,...,n \tag{10}$$

However, the sum of trips produced by commodity *k* in zone *i* may not be equal to P_i^k . Therefore, $\hat{T}_{ij}^{k}$ can be calibrated by coefficients x_i^k, x_j^k :

$$\sum_{j=1}^{m} \hat{T}_{ij}^{k}.x_i^k.x_j^k \cong P_i^k \tag{11}$$

B. Objective Functions 2: similarly, the sum of trips attracted by commodity *k* to zone *should* be equal to A_j^k :

$$\sum_{i=1}^{n} \hat{T}_{ij}^{k} .x_i^k .x_j^k \cong A_j^k \tag{12}$$

C. Objective Functions 3: also, the calculated trip-time frequency distribution for commodity *k* should be equal to the trip-time frequency distribution of real trip distribution [20], i.e.,

$$\sum_{t} \hat{T}_{ij}^{k} .x_i^k .x_j^k - \sum_{t} T_{ij}^{k} \cong 0 \tag{13}$$

D. Constraints: let u_{ij} be the upper limit of the sum of all commodities in arc *(i, j)* of the multi-commodity network. Then:

$$\sum_{k} \hat{T}_{ij}^{k} .x_i^k .x_j^k \leq u_{ij} \tag{14}$$

The above objective functions and constraints construct the following mathematical programming. It should be mentioned that in Zimmerman [20], Hwang [22], Masud [23] and Murty [24] a fuzzy objective function has been converted to constraint linear programming. Now, we have utilized this idea to propose the following new model:

Find:

$$x_i^k , x_j^k \tag{15}$$

st:

$$f_i^k = P_i^k - \sum_{j=1}^{m} \hat{T}_{ij}^{k} .x_i^k .x_j^k \cong 0$$

$$f_j^k = A_j^k - \sum_{i=1}^{n} \hat{T}_{ij}^{k} .x_i^k .x_j^k \cong 0$$

$$f_t^k = \sum_{t} \hat{T}_{ij}^{k} .x_i^k .x_j^k - \sum_{t} T_{ij}^{k} \cong 0$$

$$\sum_{k} \hat{T}_{ij}^{k} .x_i^k .x_j^k \leq u_{ij}$$

$$x_i^k , x_j^k \geq 0$$

Obviously, the fuzzy model (15) calibrates all of the approximation functions simultaneously. Now, according to Zimmermann's approach [20], the optimal solution of the above fuzzy model can be obtained by solving eqn (16):

$$\text{Minimize}[\text{Min}[\mu_i(\hat{T}_{ij}^{k}), \mu_j(\hat{T}_{ij}^{k}), \mu_t(\hat{T}_{ij}^{k})]] \tag{16}$$

If the membership functions are defined as follows:

$$\mu_j(\hat{T}_{ij}^k) = \begin{cases} 1 + \dfrac{f_j^k}{r_j} & -r_j \le f_i^k \le 0 \\ 1 - \dfrac{f_j^k}{r_j} & 0 \le f_i^k \le r_i \\ 0 & \text{otherwise} \end{cases} \tag{17}$$

where the tolerances q_i, r_j and v_t are given initially. By substituting:

$$\text{Min}\,[\mu_j(\hat{T}_{ij}^k), \mu_i(\hat{T}_{ij}^k), \mu_t(\hat{T}_{ij}^k)] = \lambda \tag{18}$$

We get:

$$(\lambda - 1)q_i \le f_i^k \le (1-\lambda)q_i \;, \; (\lambda - 1)r_j \le f_j^k \le (1-\lambda)r_j \;, \tag{19}$$

$$(\lambda - 1)v_t \le f_t^k \le (1-\lambda)v_t$$

Therefore, the proposed model (P) is obtained from eqns (15), (16) and (19):

Min λ (Model P)

st:

$$(\lambda - 1)r_j \le A_j^k - \sum_{i=1}^{n} \hat{T}_{ij}^k(I) \le (1-\lambda)r_j$$

$$(\lambda - 1)q_i \le P_i^k - \sum_{j=1}^{m} \hat{T}_{ij}^k(I) \le (1-\lambda)q_i$$

$$(\lambda - 1)v_t \le \sum_t \hat{T}_{ij}^k(I) - \sum_t T_{ij}^k \le (1-\lambda)v_t$$

$$\sum_k \hat{T}_{ij}^k(I) \le u_{ij}$$

$$\hat{T}_{ij}^k(I) = \hat{T}_{ij}^k . x_i^k . x_j^k$$

$$x_i^k, x_j^k, \lambda, \hat{T}_{il}^k(I) \ge 0$$

Indeed, this model (P) calibrates the approximation functions (5). Fortunately, since most of the constraints in model (P) are of quadratic form, this model can be handled by Lingo software. It is worth mentioning that the quadratic model is convex [24].

STEP 4 (Forecast trip distribution): suppose that $P_i^{k'}$, $A_j^{k'}$, $L_{ij}^{k'}$, $t_{ij}^{k'}$ are the parameters of a multi-commodity network in the future. Trip distribution in the future ($T_{ij}^{k'}$) is forecasted by eqn (20):

$$T_{ij}^{k'} = f_k(P_i^{k'}(s), A_j^{k'}(s), L_{ij}^{k'}(s), t_{ij}^{k'}(s)) \rightarrow$$
$$T_{ij}^{k'} = \sum_h W_h^k G(\underline{x}_{ij}^{k'}(s), \underline{X}_{[h]}^{k'}(s)).x_i^{k'}.x_j^{k'} \qquad (20)$$

Finally, the proposed model (F) calibrates $T_{ij}^{k'}$ and trip distribution in the future is obtained by solving this model:

Min λ (Model F)

st:

$$(\lambda - 1)r_j' \le A_j^{k'} - \sum_{i=1}^{n} \hat{T}_{ij}^{k'}(I) \le (1-\lambda)r_j'$$
$$(\lambda - 1)q_i' \le P_i^{k'} - \sum_{j=1}^{m} \hat{T}_{ij}^{k'}(I) \le (1-\lambda)q_i'$$
$$\sum_k \hat{T}_{ij}^{k'}(I) \le u_{ij}'$$
$$\hat{T}_{ij}^{k'}(I) = \hat{T}_{ij}^{k'}.x_i^{k'}.x_j^{k'}$$
$$x_i^{k'}, x_j^{k'}, \lambda, \hat{T}_{ij}^{k'}(I) \ge 0$$

2.1 Dimension consideration

If we consider the multi-commodity transportation network with n nodes, r arcs and m commodities:

a. Maximum size of the model P will be $(n$+r). m constraints and $n.m$ variables which could be handled in the case of large scale.
b. Size of the model F will be $n.m$ constraints and variables.

It is worth mentioning that the main contribution of this research is to develop two new optimization problems (P) and (F). The optimal solution of model (F) can be obtained by Lingo software. Now, the proposed model is illustrated by a numerical example.

2.2 Numerical example

Consider the following two-commodity network. Trips produced, trips attracted, the travel time and trip distribution at present between zones for each commodity are given in Table 1. Find trip distribution based on given data in Table 2.

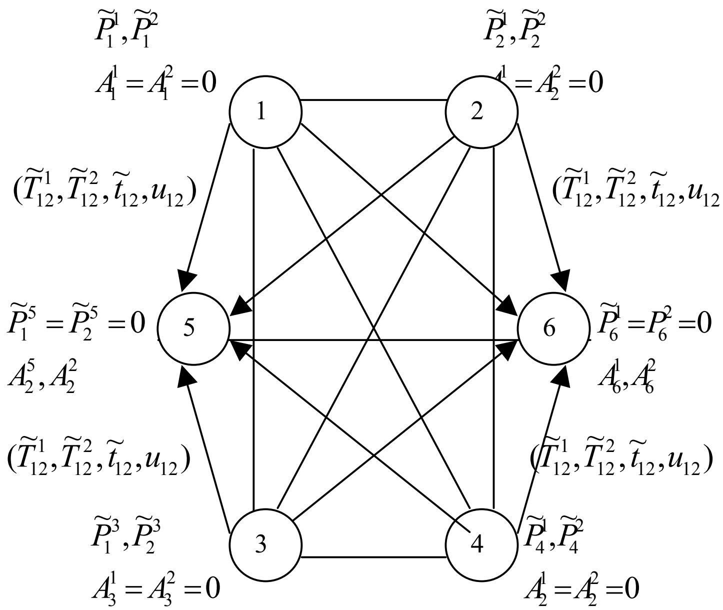

Figure 1: Two-commodity transportation network.

Table 1: Parameters at present (given data).

$\tilde{T}_{15}^{1} = (230, 240, 245, 250)$	$\tilde{P}_{1}^{1} = (390, 400, 405, 410)$
$\tilde{T}_{15}^{2} = (410, 420, 425, 430)$	$\tilde{P}_{1}^{2} = (690, 700, 705, 710)$
$\tilde{T}_{16}^{1} = (150, 160, 165, 170)$	$\tilde{P}_{2}^{1} = (490, 500, 505, 510)$
$\tilde{T}_{16}^{2} = (270, 280, 285, 290)$	$\tilde{P}_{2}^{2} = (390, 400, 405, 410)$
$\tilde{T}_{25}^{1} = (290, 300, 305, 310)$	$\tilde{P}_{3}^{1} = (490, 500, 505, 510)$
$\tilde{T}_{25}^{2} = (150, 160, 165, 170)$	$\tilde{P}_{3}^{2} = (90, 100, 105, 110)$
$\tilde{T}_{23}^{2} = (190, 200, 205, 210)$	$\tilde{P}_{4}^{1} = (90, 100, 105, 110)$
$\tilde{T}_{26}^{1} = (290, 300, 305, 310)$	$\tilde{P}_{4}^{2} = (190, 200, 205, 210)$
$\tilde{T}_{26}^{2} = (230, 240, 245, 250)$	$\tilde{A}_{5}^{1} = (690, 700, 705, 710)$
$\tilde{T}_{35}^{1} = (190, 200, 20, 210)$	$\tilde{A}_{5}^{2} = (490, 500, 505, 510)$
$\tilde{T}_{35}^{2} = (30, 40, 45, 50)$	$\tilde{A}_{6}^{1} = (790, 800, 805, 810)$
$\tilde{T}_{36}^{1} = (290, 300, 305, 310)$	$\tilde{A}_{6}^{2} = (890, 900, 905, 910)$
$\tilde{T}_{45}^{1} = (50, 60, 65, 70)$	$\tilde{T}_{36}^{2} = (50, 60, 65, 70)$
$\tilde{T}_{46}^{1} = (30, 40, 45, 50)$	$\tilde{T}_{45}^{2} = (110, 120, 125, 130)$
$\tilde{t}_{13} = \tilde{t}_{24} = (5, 6, 6.5, 7)$	$\tilde{T}_{46}^{2} = (70, 80, 85, 90)$
$\tilde{t}_{12} = \tilde{t}_{15} = \tilde{t}_{34} = \tilde{t}_{26} = \tilde{t}_{45} = \tilde{t}_{36} = (2, 3, 3.5, 4)$	$\tilde{t}_{14} = \tilde{t}_{23} = \tilde{t}_{16} = \tilde{t}_{25} = \tilde{t}_{35} = \tilde{t}_{56} = (4.5, 5.5, 6, 6.5)$
$\tilde{t}_{46} = (4, 5, 5.5, 6)$	

Table 2: Parameter in future (given data).

$\tilde{P}_4^{1'} = (140, 150, 155, 160)$	$\tilde{P}_1^{1'} = (590, 600, 605, 610)$
$\tilde{P}_4^{2'} = (240, 250, 255, 260)$	$\tilde{P}_1^{2'} = (890, 900, 905, 910)$
$\tilde{A}_5^{1'} = (890, 900, 905, 910)$	$\tilde{P}_2^{1'} = (590, 600, 605, 610)$
$\tilde{A}_5^{2'} = (690, 700, 705, 710)$	$\tilde{P}_2^{2'} = (490, 500, 505, 510)$
$\tilde{A}_6^{1'} = (940, 950, 955, 960)$	$\tilde{P}_3^{1'} = (490, 500, 505, 510)$
$\tilde{A}_6^{2'} = (1090, 1100, 1105, 1110)$	$\tilde{P}_3^{2'} = (140, 150, 155, 160)$
$\tilde{t}_{56}' = (4, 5, 5.5, 6)$	$\tilde{t}_{13}' = \tilde{t}_{24}' = (4.5, 5.5, 6, 6.5)$
$\tilde{t}_{14}' = \tilde{t}_{23}' = \tilde{t}_{16}' = \tilde{t}_{25}' = \tilde{t}_{35}' = \tilde{t}_{46}' = (3, 4, 4.5, 5)$	$\tilde{t}_{12}' = \tilde{t}_{15}' = \tilde{t}_{34}' = \tilde{t}_{26}' = \tilde{t}_{45}' = \tilde{t}_{36}' = (2.4, 2.5, 2.55, 2.6)$

Min λ (Model F)

st:

$$-5(1-\lambda) \le 600 - (\hat{T}_{15}^{1'}(I) + \hat{T}_{16}^{1'}(I)) \le 5(1-\lambda)$$
$$-5(1-\lambda) \le 900 - (\hat{T}_{15}^{2'}(I) + \hat{T}_{16}^{2'}(I)) \le 5(1-\lambda)$$
$$-5(1-\lambda) \le 600 - (\hat{T}_{25}^{1'}(I) + \hat{T}_{26}^{1'}(I)) \le 5(1-\lambda)$$
$$-5(1-\lambda) \le 500 - (\hat{T}_{25}^{2'}(I) + \hat{T}_{26}^{2'}(I)) \le 5(1-\lambda)$$
$$-5(1-\lambda) \le 500 - (\hat{T}_{35}^{1'}(I) + \hat{T}_{36}^{1'}(I)) \le 5(1-\lambda)$$
$$-5(1-\lambda) \le 150 - (\hat{T}_{35}^{2'}(I) + \hat{T}_{36}^{2'}(I)) \le 5(1-\lambda)$$
$$-5(1-\lambda) \le 150 - (\hat{T}_{45}^{1'}(I) + \hat{T}_{46}^{1'}(I)) \le 5(1-\lambda)$$
$$-5(1-\lambda) \le 250 - (\hat{T}_{45}^{2'}(I) + \hat{T}_{46}^{2'}(I)) \le 5(1-\lambda)$$
$$-5(1-\lambda) \le 900 - (\hat{T}_{15}^{1'}(I) + \hat{T}_{45}^{1'}(I) + \hat{T}_{25}^{1'}(I) + \hat{T}_{35}^{1'}(I)) \le 5(1-\lambda)$$
$$-5(1-\lambda) \le 700 - (\hat{T}_{15}^{2'}(I) + \hat{T}_{45}^{2'}(I) + \hat{T}_{25}^{2'}(I) + \hat{T}_{35}^{2'}(I)) \le 5(1-\lambda)$$
$$-5(1-\lambda) \le 950 - (\hat{T}_{26}^{1'}(I) + \hat{T}_{36}^{1'}(I) + \hat{T}_{16}^{1'}(I) + \hat{T}_{46}^{1'}(I)) \le 5(1-\lambda)$$
$$-5(1-\lambda) \le 1100 - (\hat{T}_{26}^{2'}(I) + \hat{T}_{36}^{2'}(I) + \hat{T}_{16}^{2'}(I) + \hat{T}_{46}^{2}(I)) \le 5(1-\lambda)$$

$$\hat{T}_{15}^{1'}(I) = 318.709 x_1^{1'}.x_5^{1'} \qquad \hat{T}_{15}^{1'}(I) = 318.709 x_1^{1'}.x_5^{1'}$$
$$\hat{T}_{16}^{1'}(I) = 263.204 x_1^{1'}.x_6^{1'} \qquad \hat{T}_{16}^{1'}(I) = 263.204 x_1^{1'}.x_6^{1'}$$
$$\hat{T}_{25}^{1'}(I) = 281.775 x_2^{1'}.x_5^{1'} \qquad \hat{T}_{25}^{1'}(I) = 281.775 x_2^{1'}.x_5^{1'}$$
$$\hat{T}_{26}^{1'}(I) = 300.404 x_2^{1'}.x_6^{1'} \qquad \hat{T}_{26}^{1'}(I) = 300.404 x_2^{1'}.x_6^{1'}$$
$$\hat{T}_{35}^{1'}(I) = 235.419 x_3^{1'}.x_5^{1'} \qquad \hat{T}_{35}^{1'}(I) = 235.419 x_3^{1'}.x_5^{1'}$$
$$\hat{T}_{36}^{1'}(I) = 256.766 x_3^{1'}.x_6^{1'} \qquad \hat{T}_{36}^{1'}(I) = 256.766 x_3^{1'}.x_6^{1'}$$
$$\hat{T}_{45}^{1'}(I) = 69.095 x_4^{1'}.x_5^{1'} \qquad \hat{T}_{45}^{1'}(I) = 69.095 x_4^{1'}.x_5^{1'}$$
$$\hat{T}_{46}^{1'}(I) = 53.881 x_4^{1'}.x_6^{1'} \qquad \hat{T}_{46}^{1'}(I) = 53.881 x_4^{1'}.x_6^{1'}$$

$$\hat{T}_{15}^{2'}(I), \hat{T}_{16}^{2'}(I), \hat{T}_{25}^{2'}(I), \hat{T}_{26}^{2'}(I), \hat{T}_{35}^{2'}(I), \hat{T}_{36}^{2'}(I), \hat{T}_{45}^{2'}(I), \hat{T}_{46}^{2'}(I) \ge 0$$

The optimal solution of model (F) is given in Table 3 (solved by Lingo software).

Table 3: The optimal solution.

New Model				Survey model		Trip distribution
Forecasting		Present		Present		
Commodity 2	Commodity 1	Commodity 2	Commodity 1	Commodity 2	Commodity 1	
404.3265	310.5853	420	240	420	240	T15
490.6735	284.4147	280	160	280	160	T16
145.1731	277.2482	160	200	160	200	T25
359.8269	327.7518	240	300	240	300	T26
44.47258	224.0430	40	200	40	200	T35
110.5274	270.9570	60	300	60	300	T36
111.0278	83.12359	120	60	120	60	T45
133.9722	71.87641	80	40	80	40	T46

By referring to Table 3, we could see the forecasted trip distribution between the zones for some commodities gets higher (e.g. T16 for commodity 2 which is 280 at present rises to 490.67 in the future). If we consider the Table 3 in more detail, we could find similar cases. This trip distribution forecasting identifies that the volume of traffic will exceed the capacity of paths. Thus, capacity planning and traffic engineering are required.

3 Conclusions

The author has presented a model to learn the present state of real trip distribution by RBF neural network. In addition, a novel model has been introduced to forecast the trip distribution in multi-commodity transportation networks in both crisp and fuzzy environments. This new approach is constructed based on a fuzzy multi-objective optimization model, which can be solved easily using Lingo software. As an advantage, the model incorporates all calibrations simultaneously to save the execution time. On the other hand, the special structure of the proposed model is very close to quadratic programming where its convexity is confirmed.

A numerical example has been then presented to demonstrate the accuracy of solutions gained for the present state of trip distribution from the proposed model. The efficiency of this method has been indicated using dimension consideration for large-scale transportation networks.

Modeling data distribution on intelligent multi-commodity transportation networks is the subject of further research.

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