Application field and optimal co-ordination of fixed interval timetable services in passenger interchanges

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Abstract

The adoption of fixed interval timetables generally simplifies the building of the timetables. It also allows a better use of infrastructures, vehicles and personnel. The main objectives of this study consist of the derivation of general laws and methodologies from the analysis of real situations concerning the passenger interchanges served by at least one fixed interval service. The parameter to be minimised is the waiting time for the passengers. The investigation covered a total of 25 interchanges in four European countries (Italy, Switzerland, Finland and Austria) including regional railway and metro services, tram and bus lines.

On the basis of this variegated set of situations an original widespread model was derived and tuned capable of: a) defining the limits of convenience for the adoption of fixed interval or “by appointment” timetables; b) outlining the programming of the optimum timetable in order to reduce the passenger waiting times for the correspondences; c) considering the consequences of the ordinary timetable perturbations by means of a probabilistic approach based on theoretical distribution of delays.

1 Objectives and hypothesis

This work is part of activities planned for the Ph. D in railway engineering at “La Sapienza” University in Rome.

The main objectives of this study is to derive general methodologies from the analysis of real situations with particular references to the interchanges where connections take place:
to co-ordinately define a frequency rhythm term in those services for which it is convenient to adopt a fixed interval rather than a “by appointment” timetables.

- to define a fixed optimum timetable, both for normal or particular situations, in comparison with the coincidence system between services.

For both objectives a reference parameter is to guarantee a margin of trust in coincidences.

The basic hypothesis adopted are:
- the possible connections were found for each interchange between different services, taking into consideration the conservative principle, according to which a traveller on a transport system may change to another system in more than one station, remains on the first train till the last useful station to change.
- the limitation of contemporaneous circulation on lines and stations were not considered.
- the running time and the stops were considered the minimum acceptable ones to have competitive transport services.

2 Analysis of the operation in a sample of interchanges

For each interchange of the chosen sample, the different typology of services in a rush hour (from 8.00 to 9.00) and the modalities of exchanges are considered.

The investigation interested a total of 25 interchanges in 4 European countries (Italy, Switzerland, Finland and Austria) including railway regional and metro services, tram and bus lines.

For any interchange has been examined the timetable system, which may vary because of the complexity of the interchange itself and includes its physical organisation and services.

<table>
<thead>
<tr>
<th>From \ To</th>
<th>FM1 (Orte)</th>
<th>FM1 (Fiumicino)</th>
<th>FM2 (Tivoli)</th>
<th>Metro B (Rebibbia)</th>
<th>Metro B (Laurentina)</th>
</tr>
</thead>
<tbody>
<tr>
<td>FM1 (Orte)</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>FM1 (Fium.)</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>FM2 (Tivoli)</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Metro B (Reb.)</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Metro B (Lau.)</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

0 = absence of connection; 1 = presence of connection

3 Identification of the interchange connections: graphs and matrices of spatial accessibility

The accessibility generally expresses the level of connection of a place with the surrounding environment.
The interchange accessibility measures the quality of its connection in the net. For each interchange the possible connections between the served origins/destinations have been identified, making it possible to get spatial accessibility matrices and corresponding graphs, which point out the possible changes within a station.

An example of an accessibility matrix (Table 1) and of its graphs of connection (Figure 1) is the one built up for the Roma Tiburtina station, which lays out at least two possible changes (connections) for each origin.

![Connection graph of Roma Tiburtina station.](image)

### 4 Interchange times calculation

Timing features have been calculated for each station according to the possible changes between the lines.

First of all it has been calculated the real waiting time $t_{ar}$ that the passenger waits in the interchange station between the arrival of a service and the departure of the corresponding service towards the final destination. This is calculated on the basis of the planned timetable. In relation to the number of departures towards a specific destination the waiting time varies with a reduced waiting time as the frequency increases.

The following analysis is developed on a schematised temporal basis with intervals of extension equal to a minute. Therefore the average interval $\Delta t_p$ between the $n_p$ departures within an hour has been calculated.

$$\Delta t_p = \frac{60}{n_p}$$

This interval has been used to calculate the hypothetical waiting time in the interchange, whenever the services were completely regular (fixed interval) with equal frequencies (equivalent fixed interval timetable), that is called mean
waiting time with fixed interval $t_{acc}$, calculated as an average between maximum and minimum waiting times:

$$t_{acc} = \frac{t_a \text{ max} - t_a \text{ min}}{2}$$

in which $t_{ama}$ is calculated as:

$$t_a \text{ max} = \Delta t_p + t_{int} - 1$$

$t_{amin}$ is equal to $t_{int}$, where $t_{int}$ represents the interchange time, which is the minimum time necessary for the passenger, once at the station, to access the following transport service.

Therefore the regular waiting time can be expressed in the following way:

$$t_{acc} = \frac{\Delta t_p + 2 \cdot t_{int} - 1}{2}$$

Subsequently for each connection the difference between the two waiting times $t_{acc}$-$t_{ar}$, has been calculated.

5 Convenience threshold of the fixed interval correspondences

The relationship between frequency and waiting time has been analysed by putting in relation (linear interpolation) the waiting times with the intervals of the departures in the Cartesian chart, in which the $\Delta t_p$ is on the ordinate and the difference between $t_{acc}$ and $t_{ar}$ is on the abscissa, chosen as a parameter of evaluation of the advantages for passengers to adopt the fixed interval system.

Evidently to high values of $\Delta t_p$ correspond low frequency systems in which it is preferable to organize the interchange by co-ordinating the different systems “by appointment”; vice-versa in one of low $\Delta t_p$ (high frequency systems) a system characterised by independent fixed interval timetables for the corresponding services.

On the basis of these considerations the prevalent course, resulting from the investigated relationships, is increasing with the difference between $t_{acc}$-$t_{ar}$: major intervals between trains ($\Delta t_p$) cause progressively major waiting times in a regular systems than in those by appointment.

In this hypothesis the convenience threshold between the two types of operation organizations can be defined by the condition $t_{acc}$-$t_{ar}$.

Figure 2 shows the overall diagram referring to the totality of the analysed stations.
It can be noted how the interception value for $t_{acc} - t_{w} = 0$, threshold of convenience between the optimal fixed interval and the co-ordinated one, is about 25 minutes.

Whenever the diagrams show a course discordant to the described (decreasing with $t_{acc} - t_{w}$) an anomalous functioning is pointed out, because it is possible to improve the co-ordinated timetable so that the waiting time can be at least equal to the optimum one with the fixed interval system.

In figure 3 it is shown the anomalous course of Valle Aurelia.

Figure 2: Interval between services vs. difference of waiting times: 25 interchanges.

Figure 3: Interval between services vs. difference of waiting times: Valle Aurelia interchange.
Obviously in the timetable band, which we have studied opening from 8.00 to 9.00 the co-ordination of the timetables relative to the railway line FM3 and line A of the underground represented by the order points, can be improved, and they must exchange quadrant in order to invest the tendency of the diagram.

In the specific case the timetables have fixed intervals of 15 minutes, thus the departure times are varied minute by minute from and to Viterbo and the difference between the waiting time with fixed interval timetable and the real one sustained by the passenger have been calculated.

In this way it is possible to individuate the optimum timetable for the passenger, that is the one minimising the mean waiting time.

The solution is the inversion of tendency shown in Figure 3.

6 Optimum interchange timetable: Roma Tiburtina case study

The second of the original objectives is to plan interchange operation in a way that the passengers spend a minimum waiting.

This objective has been reached by tuning an operative process (figure 4) experimentally applied to the Roma Tiburtina station.

![Figure 4: General scheme for the definition of the optimum interchange timetable.](image)

After the calculation of the present the waiting times of passengers for all the possible exchanges, the arrival of all the services have been progressively varied by one minute steps, up to a maximum of 10 minutes.

Finally the waiting times are compared till the minimum waiting time is found. The results for the trains arriving from Orte to Roma Tiburtina are shown in Figure 5.
For each origin it is possible to identify the arrival time, which minimises the waiting time for the different destinations.

Table 2: Mean waiting times in Roma Tiburtina.

<table>
<thead>
<tr>
<th>From \ To</th>
<th>Tivoli FM2</th>
<th>Orte FM1</th>
<th>Fiumicino FM1</th>
<th>Metro B</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tivoli FM2</td>
<td>-</td>
<td>7.5</td>
<td>10.0</td>
<td>6.0</td>
</tr>
<tr>
<td>Orte FM1</td>
<td>19.2</td>
<td>-</td>
<td>-</td>
<td>5.0</td>
</tr>
<tr>
<td>Fiumicino FM1</td>
<td>25.3</td>
<td>-</td>
<td>-</td>
<td>6.0</td>
</tr>
<tr>
<td>Metro B</td>
<td>19.3</td>
<td>10.6</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

Mean waiting time [min]: Optimised timetable

<table>
<thead>
<tr>
<th>From \ To</th>
<th>Tivoli FM2</th>
<th>Orte FM1</th>
<th>Fiumicino FM1</th>
<th>Metro B</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tivoli FM2</td>
<td>-</td>
<td>6.5</td>
<td>9.0</td>
<td>4.5</td>
</tr>
<tr>
<td>Orte FM1</td>
<td>16.2</td>
<td>-</td>
<td>-</td>
<td>4.4</td>
</tr>
<tr>
<td>Fiumicino FM1</td>
<td>15.3</td>
<td>-</td>
<td>-</td>
<td>4.5</td>
</tr>
<tr>
<td>Metro B</td>
<td>18.7</td>
<td>9.2</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

Mean value: 12.1

Mean value: 9.8

In Roma Tiburtina station the optimum timetable research allows to save a mean amount of 2.3 minutes (from 12.1 to 9.8 minutes of mean waiting time) (table 2).
The optimised timetable requires to delay respectively the arrivals from Orte of 3 minutes, from Fiumicino of 10 minutes and from Tivoli of 1 minute.

Table 3: Timetable variations minimising the mean waiting time for trains arriving from Orte to Roma Tiburtina - Roma Ostiense system.

<table>
<thead>
<tr>
<th>Timetable variation [min]</th>
<th>Roma Tiburtina</th>
<th>Roma Ostiense</th>
<th>Roma Tiburtina – Roma Ostiense system</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>11.0</td>
<td>7.9</td>
<td>9.5</td>
</tr>
<tr>
<td>+1</td>
<td>10.3</td>
<td>7.5</td>
<td>8.9</td>
</tr>
<tr>
<td>+2</td>
<td>14.7</td>
<td>7.5</td>
<td>11.1</td>
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<tr>
<td>+3</td>
<td>14.2</td>
<td>6.9</td>
<td>10.6</td>
</tr>
<tr>
<td>+4</td>
<td>13.5</td>
<td>6.9</td>
<td>10.2</td>
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<tr>
<td>+5</td>
<td>12.8</td>
<td>9.3</td>
<td>11.1</td>
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<td>+6</td>
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<td>+8</td>
<td>15.5</td>
<td>8.1</td>
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<tr>
<td>+9</td>
<td>15.2</td>
<td>7.5</td>
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<tr>
<td>+10</td>
<td>14.5</td>
<td>7.3</td>
<td>10.9</td>
</tr>
</tbody>
</table>

7 Optimal network timetable: Roma Ostiense – Roma Tiburtina system case study

After the application of the developed methodology to single interchanges, it has been investigated the possibility to apply it to portions of railway network including various stations and line sections, partially run by the same services. At this aim it has been selected the portion of network within the Rome railway node between the stations of Roma Ostiense and Roma Tiburtina, both interested by FM1 services from/to Orte and Fiumicino and Metro B Line.

The optimum network timetable will be looked for as a global one, by contemporary minimising the mean waiting time in both stations.

Some results of this combined process is shown in table 3: it can be noticed that the minimum waiting time in the system is obtained by delaying the arrivals from Orte of 1 minute (value coinciding with the minimising value for Roma Ostiense, but different from the minimising value for Roma Tiburtina).

Similar results may be obtained also for the arrivals from Fiumicino (to be delayed of 8 minutes in comparison with the present situation).

8 Methodological approach to determine the optimal co-ordinated timetable in perturbed conditions

The analysis carried out till now are considering traffic not perturbed by delays: trains arriving according to their planned scheduling.
The natural development of the study is a probabilistic approach capable to take into account also perturbations.

At this aim it is necessary to characterise the train arrivals with typical distributions.

An usual example of distribution is a linear decreasing one (triangularly shaped) with the maximum value (50% of the arrivals) in correspondence to the scheduled arrival time (delay = 0) and a maximum delay, that, for the concerned services, may be assumed of 10 minutes (Figure 6).

![Figure 6: Triangular distribution of arrivals.](image)

The generic probability $PR_i$ to observe generic delay value $R_i$ may be calculated as follows:

$$PR_i = \frac{PR_0 (R_{\text{max}} + 1 - R_i)}{R_{\text{max}} + 1}$$

where:

- $PR_0$ is the probability to have delay = 0 (arrival on schedule);
- $R_{\text{max}}$ is the maximum hypothetic delay (normally assumed lower than the interval between two trains).

The mean waiting time on perturbed conditions is calculated as the summation of the products of probability $PR_i$ which a particular value of delay takes place against the corresponding mean waiting time:

$$t_{\text{wmp}} = \sum_{i=1}^{R_{\text{max}}} [PR_i \cdot t_{\text{wm}}(R_i)]$$

On the basis of this modelling the waiting time in disturbed conditions for the trains arriving from Fiumicino and Orte results higher than the one calculated in unperturbed (ordinary) conditions.

In figure 7 an example of comparison of the mean waiting times in ordinary (deterministic model) and disturbed (probabilistic model) conditions is shown.
Figure 7: Medium waiting times in the ordinary and upset regime and for the arrivals from Orte.

9 Conclusions

On the basis of this variegated set of the analysed situations it was derived and validated an original widespread model capable to:

a) define the limits of convenience for the adoption of fixed interval or “by appointment” timetables;
b) outline the programming of the optimum timetable in order to reduce the passenger waiting times for the correspondences;
c) consider the consequences of the traffic perturbations by means of a probabilistic approach based on theoretical distributions of delays.

References