The optimal cost allocation for urban transport management

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Abstract

Several researchers have discussed the grounds for higher priority of investment in urban mass transit systems within the context of the Downs-Thomson’s paradox. The optimal cost allocation of the introduction of a new transit system between urban transports users were examined as the second best pricing problem constrained by the binary mode choice/assignment model and the restriction of the CO\textsubscript{2} discharge in the urban transport network in Gifu city. The results are that the effects of introducing a guide-way bus system are limited in the city because its urban transport system is inclined to very heavy automobile use. However, charging for automobile use is possible. Furthermore, if the public transport agency that operates the guide-way bus system were to be subsidized by profits from automobile charge substitutable transport modes could be made available economically for automobile users. This combination policy is the most effective way to transfer urban transport demand from automobiles to the public transport system.

Keywords: optimal cost allocation, multi-mode network equilibrium, MPEC, urban transport problem, Downs-Thomson's paradox.

1 Introduction

The progress of motorization has resulted in urban transport systems that heavily depend on automobile use. Excessive automobile demand has caused a number of social problems that cause external diseconomies. Typical examples of this are the increase in traffic congestion and the discharge of greenhouse gases. In the long term, it causes the spreading of urban areas so that energy consumption becomes inefficient and de-urbanization occurs in CBD.
For urban transport systems to be sustainable under these recent circumstances, new public transport systems have been introduced to transfer automobile traffic demand. The social cost of public transport is lower than that of automobiles. Introducing more mass transit systems will decrease the total social cost of urban transport systems. These effects will spread extensively over urban areas due to the network effect, for example, the interaction between traffic demands on road sections. Several researchers have been discussed the grounds for higher priority of investment of urban mass transit systems in urban areas within the context of the Downs-Thomson’s paradox [1, 2, 3, 4].

However, most public transport agencies such as bus and train companies are unsuccessful in urban areas. Public transport agencies cannot bear the huge investment costs necessary to match private automobile investment. Further investment to reduce social external costs due to automobile uses cannot be borne by public transit agencies single-handedly and neither is it efficient. However there is the possibility of sharing the burden of introduction costs with those other than public transport users who would benefit from it. This would make the introduction of new public transit sustainable and increase the efficiency of the urban transport system, by being consistent with the principle of beneficiary burden.

Miyagi et al. [5] and Miyagi and Suzuki [6] analyzed the optimal cost allocation of a new transit system between urban transport users as the second best pricing problem to compete with automobiles. Suzuki and Miyagi [7] extended this model, restricting the total amount of discharge gases from the urban transport system. As a result, additional costs are charged in proportion to the amount of discharge gases for each individual transport user. This minimizes the loss in total social surplus that results from introducing a new transit system to control discharge gases.

Extended second best pricing models constructed through established researches are being applied to simplified urban transport networks. This study is examining the optimal cost allocation through urban transport pricing systems. Through applying this model, urban transport systems are simulated corresponding to several pricing schemes, subsidy systems and criteria for discharge gases.

2 The second best pricing model for public transport system that competes with automobiles

Banmol and Bradford [8] show that the Ramsey pricing rule [9], which is the second best method of excision, can be applied to regulating public transit fares. The Ramsey price is deduced by maximizing social welfare, which is constrained by the non-negative profit conditions of public transport agencies. The Ramsey pricing rule is important in that it integrates the traditional full cost principle, which guarantees reasonable profits for transit agencies, with the marginal cost principle, which maximizes social welfare related to transport the market.

Our model is an extended Ramsey pricing model that assumes the following.
(i) The government sets all fares that maximize the total social surplus, which includes public transport systems and automobiles.

(ii) The government subsidizes the public transport agency with the fixed rate of the construction cost of the new transit system.

(iii) The government necessitates the reduction of discharge gases produced by urban transport systems within their criteria and to be sustainable the public transport agency.

(iv) The public transport agency complies with the fare regulation set by the government.

(v) All transport users have the freedom to decide their travel modes and routes to minimize their travel costs.

(vi) Fare setting affects road congestion through the interaction between transport users’ travel choices and the performance of transport facilities.

The framework of the second best pricing model must be integrated with a multi-modal network equilibrium model to realize the above assumptions within the pricing problem. The binary mode choice/assignment model proposed by Florian and Spiess [10] is adapted and used in this study. The structure of this problem is known as Stackelberg’s leader-follower problem [11] or MPEC: Mathematical Program with Equilibrium Constraints, [12].

The model is represented as Equations (1)-(5) corresponding to the simplest network that consists of one OD pair, one automobile route and one public transport route. Equation (1) shows that the government sets transport fares to maximize total social surplus considering urban transport systems. The total social surplus is denoted by the sum of consumer surplus and producer surplus in Equation (6). Inequality (2) expresses non-negative profit conditions for the public transport agency. The limitation of discharge gases produced by urban transport systems is indicated by Inequality (3). The equilibrium conditions and flow conservation conditions for the binary mode choice/assignment model are formulated as VI, Variational Inequality (4) and (5), where $w(q)$ is a function of modal split that represents the difference in generalized cost between automobile use and public transport use. In this study, a logit-type modal split function is assumed, and $w(q)$ is its inverse function.

\[
\text{Max.}\; \Pi(p) \quad (1)
\]

\[
s.t. \sum_{i=1}^{2} p_i q_i^*(p) - \sum_{i=1}^{2} VC_i(q_i^*(p)) - \sum_{i=1}^{2} FC_i + gN \geq 0 \quad (2)
\]

\[
\sum_{i=1}^{2} \gamma_i d_i q_i^*(p) \leq G \quad (3)
\]

where $q^*$ are the solutions to the following VI.

\[
C(h^*(p))^T(h(p) - h^*(p)) - w(q_i^*(p))(q_i(p) - q_i^*(p)) \geq 0 \quad (4)
\]

\[
\Omega = \{x : x = \Delta h, q = \Lambda h_1, h \geq 0\} \quad (5)
\]
\[ \Pi(p,q(p)) = \theta \bar{q} \ln \sum_{i=1}^{2} \exp \left( \frac{y - C_i(q_i(p), p_i)}{\theta} \right) + \sum (p_i q_i(p) - VC_i(q_i(p))) \] (6)

\[ \frac{q_i}{\bar{q}} = \frac{1}{1 + \exp\left( (C_1 - C_2) / \theta \right)} \] (7)

\( \bar{q} \): Total travel demand between the OD pair (trip)
\( y \): Average household income (yen/day)
\( q_i \): Travel demand using mode i (i = 1 automobile, = 2 public transport) (trip)
\( N \): Number of household
\( p_i, t_i \): Travel fare (yen) and time (min) using mode i
\( g \): Lump sum tax (yen/day)
\( VC_i, FC_i \): Variable cost and fixed cost operating mode i (yen/day)
\( gN \): Subsidy from government (yen/day)
\( \gamma_i \): Discharge gas arising from unit travel distance using mode i (m^3/km)
\( d_i \): Average travel distance using mode i (km)
\( \theta \): Dispersal parameter
\( G \): Limitation of discharge gas arising urban transport systems (m^3)
\( x, h, C \): Travel demand of links, paths (trips) and generalized travel cost of path (yen)
\( \Delta, \Lambda \): Link-path, OD-path incident matrix

3 Application to real scale urban transport networks

3.1 Study area and network setting

This chapter examines multi-mode fare setting that targets realistic urban transport systems in Gifu city. Gifu has a population of about 400,000 people, and over one million trips are generated per day. This city is categorized as a typical mid size Japanese city that depends heavily on automobile use; excluding on foot and by bicycle, this occupies 90% of the demand. Although the public transport system composed of buses and streetcars covers the whole urban area, demand for it is slight, especially for streetcars. There is traffic congestion on the ring-road and bridges over Nagara River, which lowers the service of not only private automobiles but also buses.

The urban transport system situation is calculated as the binary mode choice/assignment model as mentioned above. The system is approximated by a road network and a public transport network that consists of 2,046 links, 496 nodes and 70 centroids. The travel cost functions of the automobile links are
defined as a flow-dependent function, and the public transit links are defined as a flow-independent function. The travel costs of automobile uses on the links are indicated by

\[ c_a = p_a + \omega t_a + \beta \left( \frac{x_a}{Q_a} \right)^{\beta_2}, \]  

which are the generalized cost functions through integrating fares and travel times using value of time saving: \( \omega \). The travel times on the links are represented as a standard link performance functions related to traffic demands on the links. The travel costs of public transport systems are defined as

\[ c_a = p_a + \omega t_a. \]  

The variable costs of public transport agencies are assumed as constants because transport services do not alter their schedules to correspond to short term variation in the volume of demand.

CO\(_2\) discharge is chosen as a representative discharge gas. The coefficient of CO\(_2\) discharge from automobiles and buses on the link is defined as Equation (10), which is estimated by the Tokyo Metropolitan Research Institute for Environmental Protection [13].

\[ EF_a = \alpha_0 + \alpha_1 V_a + \alpha_2 V_a^2 + \alpha_3 / V_a \]  

\( EF_a \) : Coefficient of discharge gas produced by automobiles on link a (g/km)

\( V_a \) : Average travel speed of automobiles on link a (km/h)

\( \alpha_0, \alpha_1, \alpha_2, \alpha_3 \) : Parameter \( (= 169,-1.38,0.0116,2398) \)

It is assumed that the public transport system is improved by introducing a guide-way bus system. We have investigate the differences between cost allocations between four cases, which are combinations of introducing a guide-way bus system or not and imposing fare for automobile use or not. The guide-way bus system is where buses run in the guide-way separated from automobile roads in congested areas such as CBD. These buses run on normal roads in areas without congestion, such as suburban areas. Therefore, these buses can run faster than normal buses especially on the guide-way route. As a result the travel times of users are reduced, particularly during peak hours. We assume a guide-way route would be constructed between the Memorial Center and Gifu station, which is most congested from the northern area around Nagara River to Gifu City Center.

The bus fares are uniform throughout the urban area and in proportion to distance in suburban areas; this is equivalent to the real situation. The area covered by a uniform bus fare is assumed to be within the ring road. However, for the sake of simplicity automobile fares are set uniformly regardless of distance. The uniform bus fare and automobile fare system are set using the extended second best pricing model.
The limitation of the total CO₂ discharge is calculated from the total in standard equilibrium, which is equivalent to the real urban transport situation where guide-way bus systems have not been introduced, where the uniform bus fare is set at 200 yen and where any automobile fares have not been corrected. A reduction in the total amount of CO₂ by 10% from the standard equilibrium amount is attempted by introducing the guide-way bus system. Moreover, the variable cost of the public transport agency is set as earnings and expenses in the balance of the standard equilibrium for the same reason.

3.2 Calculation method

We adopt nonlinear sensitivity analysis [14,15] for MPEC as the calculation method for our model. The method is explained as follows. First an initial equilibrium is defined corresponding to a certain decision variable. Secondly, in this equilibrium, the gradient of the equilibrium equation is calculated with respect to perturbation of the decision variable. Then the descending direction of the objective function of the main optimization problem is searched with consideration to the gradient of equilibrium equation information, which restricts the main optimization problem. Thirdly, the equilibrium point, in which the value of objective function can be smaller than in the initial equilibrium point, is chosen according to the descending direction. This process is repeated until a smaller objective function value can no longer be found.

Consequently, to find the optimal fare setting we have to calculate the multi-modal network equilibrium several times corresponding to fares in each repeated process. We use emme/2 produced by INRO to calculate this real-scale network equilibrium. Fig. 1 is the flow chart of the algorithm. Equilibrium traffic volumes on the links that correspond to initial fares are calculated in Steps 1 to 3. Then the objective function value according to the travel volume and cost in this equilibrium is computed, whose objective function is a combined objective function with the constraints of [P] using the penalty function method. The problem can be reformulated as a minimization problem if the extended second best pricing problem [P] is simplified as (11), and the optimization problem in this computational system is represented as (12) and (13).

\[
\max_{p,q,h} \Pi(p,q,h) \quad s.t. \ f(p,q,h) \leq 0 \quad (11)
\]

\[
\min_{p,q,h} E(p,q,h) \quad (12)
\]

\[
E(p,q,h) = -\Pi + \alpha f
\]

\[
\text{if} \quad f > 0 \quad \text{then} \quad \alpha = \alpha_0 \quad \text{else} \quad \alpha = 0 \quad (13)
\]

\( \mathbf{f} \): constrained function,

\( \mathbf{E} \): objective function,

\( \mathbf{a} \): penalty parameter,

\( \mathbf{\alpha}_0 \): extremely large value
The equilibrium is calculated considering perturbed fares from the present one, then the gradient is computed from difference between the present equilibrium and the perturbed equilibrium in Steps 5 to 8. The most steeply descending direction and step size are decided based on the gradient and then fares are then renewed in Steps 9 to 10. After that, the algorithm returns to Step 2 to calculate the new equilibrium and its objective function value corresponding to renewal fares. The optimal fares and optimal equilibrium variables such as travel demands on the links, the modal splits, etc., are ascertained by repeatedly calculating until the objective function value converges whose criteria are evaluated in Step 4. This algorithm is programmed as the macro of emme/2.

3.3 Studies on charging automobile users for the introduction costs of guide-way bus system

With the limitation of the 10% decrease in the CO₂ discharge the optimal fare setting problem can be solved only when automobile users are charged without considering introducing a guide-way bus system. This implies that the reduction of CO₂ discharge cannot be achieved without charge for automobile uses because the urban transport system heavily depend on automobile uses and luck of the potential capability of public transport services in Gifu city.
Figure 2: Variation in the traffic volume of automobiles.

Figure 3: Variation in transport demand for the bus system.
Within the two cases of charging automobile uses, by introducing guide-way bus system the optimal bus fare is 40 yen and the automobile fare is 100 yen and without it 17 yen and 118 yen in the same manner. Comparing the two cases, fares are discriminated strongly without introduction of guide-way bus systems to secure the incentive to transfer from automobile use to public transport system use.

The total social surplus is larger with the introduction of the guide-way bus system than without. Furthermore, the total surplus is also larger than at present which is without introduction of guide-way bus systems and the limitation of discharge gases. Therefore, the optimal cost sharing between urban transport users does not only lower social cost to reduce discharge CO₂ but also improves the efficiency of the urban transport system.

Figs. 2 and 3 show the variation in the volume of automobile and bus demand on links between the most successful case of introduction of guide-way bus systems and charging automobile uses, and the present case. The blue links represent the decrease of demand, red ones represent the increase of demand and the widths represent variation of demand. Automobile users change their mode choices due to charging automobile uses and improved bus route services between Gifu station and the northern suburban areas around Nagara River. The decreasing traffic volume of automobiles is spread to the road network of the northern urban areas through the interactions of traffic flows on the links. However, reduction in road congestion causes a slight increase in automobile demand that runs in the easterly and westerly directions.

4 Conclusions

This study applies to the second best pricing model restricted by the multi-modal network equilibrium to the real scale urban transport network systems. It examines the sharing of introduction costs of a guide-way bus system aiming to reduce discharge gases with automobile users. The results of applying this system to Gifu city are as follows. The effects of introducing a guide-way bus system are limited in this city because its urban transport system is inclined to very heavy automobile use. However, charging for automobile use is possible. For this reason, imposing fares for automobile use and introducing a guide-way bus system should be enforced simultaneously. Furthermore, if the public transport agency that operates the guide-way bus systems were subsidized with profits from automobile charge substitutable transport modes could be made available economically for automobile users. This combination policy is the most effective way to transfer urban transport demand from automobiles to public transport. The structure of the urban transport system can be described on the network through our model, therefore we may proceed to explain the spatial relation between cost and benefit in urban transport systems and investigate more realistic transport policies such as alternative guide-way route planning, restructuring the bus network system, and several ways of enforcing automobile charges.
References