A multi-agent model for the Vehicle Routing Problem with Time Windows

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Abstract

The purpose of this paper is to present a coalition based multi-agent model for the Vehicle Routing Problem with Time Windows, which is one of the most important ones in distribution and transportation. The VRPTW objective is to design least cost routes from one depot to a set of customers. A fleet of homogeneous vehicles performs the routes to serve all the customers, while respecting capacity and time window constraints.

The proposed model is detailed, illustrated through a numerical example, experimented on the base of Solomon s benchmark and compared to the optimal solution.
Keywords: vehicle routing problem, multi-agent system, coalition formation, negotiation protocol.

1 Introduction

The VRPTW is an important problem occurring in many distribution systems. Some of the most useful applications of the VRPTW include bank deliveries, postal deliveries, school bus routing and JIT manufacturing. The VRPTW is defined as the problem of minimising time and costs in case a fleet of vehicles has to distribute goods from a depot to a set of geographically scattered customers while respecting spatio -temporal constraints.

Because of the high complexity level of the VRPTW and its wide applicability to real life situations, a number of exact and approximative methods have been proposed in the literature. Among the most recent and efficient exact methods, we can mention that of Larsen *et al.* [1] or Cook and Rich [2].

The most successful heuristic methods in solving the VRPTW are embedding tabu searches Taillard *et al.* [3] and ant colony system Gambardella *et al.* [4].

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To our knowledge, distributed methods are also applied to the general VRP such as the Contract Net Protocol by Fisher *et al.* [5], or in the context of task allocation namely the coalition formation method of Shehory and Kraus [6].

We noticed that the distributed resolution of the VRPTW has not been sufficiently and comprehensively surveyed and compared, especially concerning the coalition formation methods. The aim of our study is to try to fill this gap by proposing a multi-agent model based on coalition formation, contributing to the global optimisation of the VRPTW. Henceforth, this model is referred by the acronym Coal-VRP.

The remainder of this paper is organised as follows. In Section2, the problem we intend to solve (the VRPTW) is mathematically formulated. In section 3, the coalition formation is reviewed as a method for dealing with situations of cooperation and competition in open environments. In section 4, the proposed model Coal-VRP is exhaustively described. The results of the proposed model for Solomon's benchmark test problems are given and compared with optimal solutions in section 5. Some conclusions are drawn in section 6.

2 Problem formulation

Here, the main objective of VRPTW is to determine least total travelled distance routes while respecting spatio-temporal constraints. The VRPTW can be formulated mathematically as follows:

Let:

G (A, N): a graph

N: a node set {0, *n*+1} of *n* customers denoted by *C*, and the nodes 0 and *n*+1which represent the depot.

V: a vehicle set of *m* vehicles

i, *j, k, n*: a given different customers

v: a given vehicle

Q: a given vehicle capacity

q_i: a customer *i* demand

c_{ij}: a cost associated with each arc (*i*, *j*)

e_i, l_i: the lower bound (earliest start time) and the upper bound (latest start time) of the customer *i* time window, respectively.

a^v_i: the time vehicle *v* starts service at customer *i*

s_i: the customer *i* service time.

t_{ij}: the travel time between *i* and *j*.

$$x_{ij}^{v} = \begin{cases} 1 & \textit{if the arc } (i,j) \textit{ is covered by the vehicle } v \\ 0 & \textit{otherwise} \end{cases}$$

$$\text{Minimize} \sum_{v\in V} \sum_{i\in N} \sum_{j\in N} c_{ij} x_{ij}^{v} \tag{1}$$

Subject to:

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$$\sum_{v \in V} \sum_{j \in N} x_{ij}^{v} = 1 \qquad \forall i \in C \tag{2}$$

$$\sum_{i \in C} q_i \sum_{j \in N} x_{ij}^{v} \leq Q \qquad \forall v \in V \tag{3}$$

$$\sum_{j \in N} x_{0j}^{v} = 1 \qquad \forall v \in V \tag{4}$$

$$\sum_{i \in N} x_{ik}^{v} - \sum_{j \in N} x_{kj}^{v} = 0 \qquad \forall k \in C, \forall v \in V \tag{5}$$

$$\sum_{i \in N} x_{i(n+1)}^{v} = 1 \qquad \forall v \in V \tag{6}$$

$$x_{ij}^{v}(a_i^{v} + s_i + t_{ij}) \leq a_j^{v} \qquad \forall (i, j) \in A, \forall v \in V \tag{7}$$

$$e_i \leq a_i^{v} \leq l_i \qquad \forall i \in C, \forall v \in V \tag{8}$$

$$x_{ij}^{v} \in \{0,1\} \qquad \forall (i, j) \in A, \forall v \in V \tag{9}$$

In formulating the VRPTW, the objectif function, eqn. (1) minimises the total travelled distance incurred by vehicle fleet, under the following constraints:

- Each route must be covered by one vehicle.
- Each customer must be visited only once by exactly one vehicle. Hence, split deliveries are not allowed, eqn. (2).
- The total demand delivered in one route should never exceed the vehicle capacity, eqn. (3).
- Each route must start and end at the depot, eqns. (4,5,6).
- Each customer has a required service rime.
- We consider hard time windows i.e. a vehicle is permitted to arrive before the earliest time window, and wait without cost, but it is not permitted to arrive after the latest time window, eqns. (7,8).
- Integrity constraints, eqn. (9).

3 Coalition formation

The coalition formation is an important method of cooperation in multi-agent systems. It consists in dividing agents being able to communicate and negotiate into groups. These groups are called coalitions.

A coalition is defined Vauvert and Seghrouchni [7] as a short-term organisation based on specific and contextual commitments. This allows agents to coexist while profiting from their respective competencies.

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4 Solving process

The proposed model, Coal-VRP, consists in determining a final set of routes given a set of orders received from different customers .

It is based on multi-agent approach which allows problem solving in terms of cooperation, competition and negotiation. Two agent types are implied: Interface agent and Customer agent.

4.1 Assumptions and definitions

- A Customer agent corresponds to a given customer.
- The interest of a Customer agent is to be delivered in least cost route while respecting capacity and time windows constraints.
- A coalition is an ordered list of Customer agents. It corresponds to a route having to start at the depot, deliver all the associated customers according to their appearance order, and finally return to the depot.
- The Customer agent i is a parent of the Customer agent j if j is accessible from i while respecting spatial constraints (distance between i and j) and temporal ones (i and j time windows).
- The Customer agent i is a child of the Customer agent j if j is a parent of i.
- A relationship graph is a directed and acyclic graph where every arc, connecting two different nodes i and j, indicates that the Customer agent i is parent or child of the Customer agent j.
- The Customer agent i is an ancestor of the Customer agent j if i is a parent of j or a parent of an ancestor of j.
- The Customer agent i is a descendant of the Customer agent j if j is an ancestor of i
- A compromise ratio is a value associated to a given coalition *coal* It is equal to the ratio between the cost of *coal* and its size. It represents the preference criterion of a coalition to another one.
- The state of the Customer agent j with regard to the Customer agent i is ‘wait’ if they are able to belong to the same coalition.
- The state of the Customer agent j with regard to the Customer agent i is ‘enemy’ if they do not belong to the same coalition.
- The state of the Customer agent j with regard to the Customer agent i is ‘friend’ if they belong to the same coalition.

4.2 Dynamic of Coal-VRP

Here, we provide a description of the dynamic of the set up multi-agent system Coal-VRP: the main steps of the solving process, and the most important messages exchanged between the different agents and incorporated into their behaviour.

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4.2.1 Distributed formation of the relationship graph

This step provides both of the parent list and children list of each Customer agent *i* while respecting its time window bounds and a spatial neighbourhood heuristic. Hence, *i* diffuses to its acquaintances a message called formlParChild <lStatknow$_i$> enclosing its static knowledge

4.2.2 Distributed formation of coalition lists, descendant lists and ancestor lists

In this step, each Customer agent *i* constitutes a coalition list with respect to the capacity constraint, and updates its ancestor and descendant lists, on the base of the relationship graph obtained in the previous step. It sends on the one hand a message formlCoal $< lCoal_i\ ,\ e_i\ ,\ l_i >$ and a message updatelDesc $< i >$ to its parents, and on the other hand it sends a message updateIAnc $< i >$ to its Children.

4.2.3 Determination of the most preferable coalition

In this final step, the preferred coalitions are retained, and the coalition configuration is gradually achieved.

Indeed, we define a negotiation protocol that guarantees an agreement within all the Customer agents. It is based on six types of messages exchanging proposals of coalitions between two different Customer agents *i* and *j*:

- Message 1: requestFriendship $<$j, $indivSol_J>$: *j* proposes to *i* its current individual solution $indivSol_j$, which coalition belongs to its coalition list $lCoal_i$ formed in the second step.
- Message 2: confirmFriendship$<j, indivSol_j>$: this message expresses that *j* accepts the coalition proposed by *i* in the message requestFriendship.
- Message 3: join $<indivSol_j>$: *j* informs *i* that it is definitively joined to its individual solution $indivSol_j$.
- Message 4: refuseFriendship $<j$, $indivSol_j>$: given *j* satisfied in $indivSol_j$, it informs *i* that it does not accept to form a coalition with it.
- Message 5: requestExcuse $<$j$>$: *j* asks *i* whether it can accept its excuse and form a coalition with it again under certain conditions.
- Message 6: abandon $<indivSol_j>$: *j* informs *i* that it abandons the coalition $indivsol_j$.

5 Numerical results

This section reports computational results of Coal-VRP experiments compared with the optimal solution [8]. It has been tested on a set of 56-problems benchmarks Solomon *et al.* [9] organised in six different problem classes (Cl, C2, R1, R2, RC1 and RC2). Each class contains eight to twelve 100-node problems and all problems in any one class have the same customer locations, and the same vehicle capacities; only the time windows differ.

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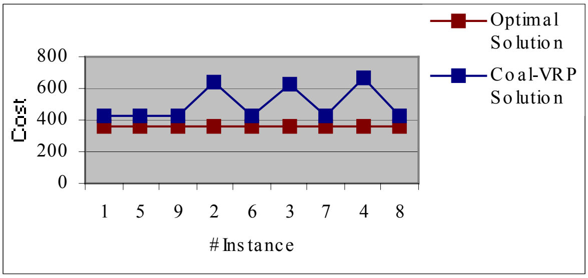

Figure 1: The class Cl of 50 customers.

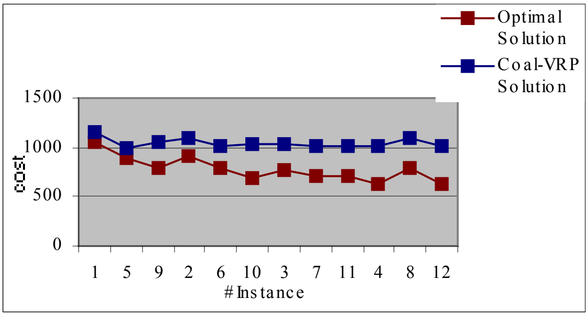

Figure 2: The class R1 of 50 customers.

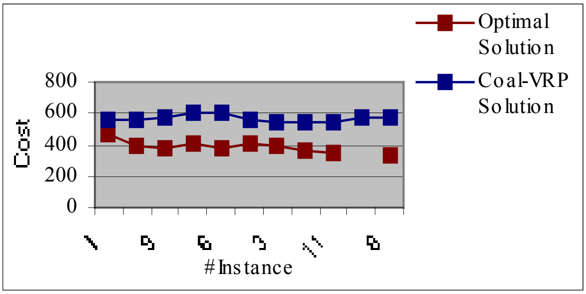

Figure 3: The class R2 of 25 customers.

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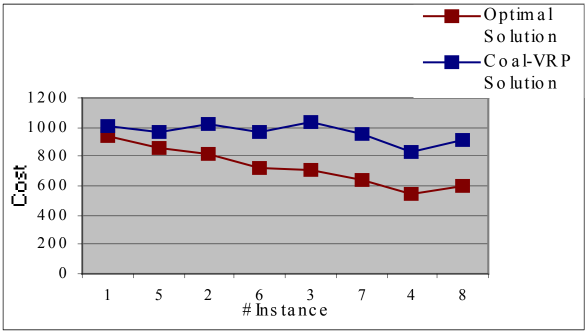

Figure 4: The class RC1 of 50 customers.

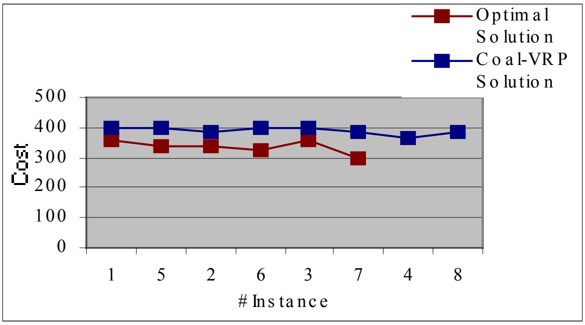

Figure 5: The class RC2 of 25 customers.

6 Conclusion and future prospects

In this paper, we developed a multi-agent model Coal-VRP to solve the VRPTW via coalition formation. This model involves uniquely Customer agents, which cooperate, coordinate and negotiate within a solving process.
This paper provides empirical and comparative studies showing that Coal-VRP results are promising and that it can be very competitive to tackle VRPTW.
Currently, we are trying to improve Coal-VRP through dynamic generation of coalitions in the second step of the solving process, then merging the second and third steps.

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