The use of mechanism design in the regulation of congestion

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Abstract

Since Pigou [3] it has been known that tax levels equal to marginal external costs give the most efficient social outcome. However, to be able to use these results the social planner has to know the value of the congestion costs in order to determine the size of the Pigou tax. Newer economic theory suggests that it may not be necessary to calculate the value of the external costs. The theory concerns the implementation of socially desired outcomes (e.g. the most efficient level of transport in a road network). In relation to internalisation of externalities, and also in relation to congestion, the Compensation Mechanism implements the outcome reached using Pigou taxes in an economy with externalities – that is, the economic efficient outcome. The mechanism implements this result by having each individual announce the amount (of money) he will pay in compensation for the delay he causes, and the amount he should receive in order to compensate him for the delays generated by other commuters.

The practical uses of mechanisms have been few, and no examples in transport settings have been seen. This paper demonstrates the important elements of the Compensation Mechanism and discusses some of the problems this new methodology rises.

1 Introduction

Congestion is one of the major problems in most major urban areas. In some cases congestion leads to a complete halt of the vehicles. Transport economist has favoured the use of the so-called marginal cost pricing as a solution to this problem. Ever since Pigou [3] focus has been on the benchmark case where a social planner levies a tax equal to marginal disutility incurred by other drivers
due to one additional vehicle using the road. This has been so in spite of the many practical difficulties this approach has had. Two major difficulties are the asymmetric information that makes the task of finding optimal Pigou taxes very difficult. Moreover it is recognised that whenever small changes to the system occurs then the taxes must actually be changed in order to retain optimality.

In this paper we look at some of these difficulties and propose the use of a new economic theory that solves these problems. This is done in Section 2. In Section 3 we translate this theory into a transport setting in which congestion occurs, and demonstrate how the methodology can come up with the same efficient solution as will be reached using Pigou taxes. We further demonstrate that loosening the assumption of perfect information does not alter the conclusion of the analysis in Section 3. This is done in Section 4. In the final section we give a short conclusion.

2 The implementation of socially desired outcomes

Since late 60'ties a new theoretical approach to the problem of ensuring the implementation of outcomes desired by a society has been developed. This theory is called Implementation theory, and the specific suggested solutions are Mechanisms or Mechanism Design. For a review of the development see Hurwich [1], and for a description of the main theoretical elements see e.g. Corchón [2]. However, we will in this paper not go into details on these theories and results, but instead discuss the implication of this new theory in a general sense.

The overall problem a society faces when it consists of a given number of individuals (call these commuters in a transport network for simplicity) having some (unknown) preferences, is to ensure that a desired outcome is implemented given these preferences. In our case of transport this could be ensuring efficient use of a given transport network. In order to obtain this the planner must know the preferences and provide incentives that ensures that only the drivers having the highest need (e.g. value of time) for use of the infrastructure actually use it. One way of doing this is levying a congestion tax equal to the marginal delay one additional commuter causes in the optimal (efficient) level of congestion. This tax is equal to the famous Pigou Tax [3].

Figure 1: The implementation triangle.
The overall problem can be described using the Implementation Triangle shown in Figure 1. Starting in the upper end of the triangle this is the desires and needs of the individuals. In the upper left the $E$ symbolises an economy consisting of the individuals and their preferences (e.g. value of time as described above). The upper right is the outcomes that we wish given a specific set of preferences; that is, we wish to ensure the implementation of a social choice correspondence $F$, which link preferences with outcomes.

The transport system defines a set of game rules $G$, which also consists of the regulation instruments introduced by a social planner (e.g. the Pigou tax mentioned above). Given these rules, the economy, $E$, and a common equilibrium concept, $EQ$, a set of possible strategies, $S$, is defined. The strategies in a transport system are choices of mode, departure time, route etc. An equilibrium is found when e.g. nobody can obtain more without somebody loosing some. Translated into a transport system with congestion this is when nobody can reduce his delay without someone incurring an increased delay. Finally, from the set of possible strategies defined by the equilibrium concept, the economy, and the game rules individuals choose strategies that maximise their preferences, or in a commuting setting ensuring them the fastest journey.

If the game rules are optimally specified the outcome will be equal to the desired outcomes. Unfortunately this not an easy task. As already mentioned this would mean that a planner has to assess the individual preferences and design appropriate regulatory rules according to this. In our transport setting this corresponds to measuring the value of time for each individual and calculating the optimal Pigou tax. It is well known that the assessment of individual values of time is very difficult. Almost any transport conference has a number of papers devoted to the value of time assessment. E.g. at the European Transport Conference 2001 at least 4 papers ([4], [5], [6], [7]) were devoted to this specific task. Furthermore, as the economy is subject to changes all the time through e.g. new commuters, change in jobs, locations etc. new taxes must be applied all the time. Hence the task of monitoring and adjusting is enormous.

The problem calls for another solution. The solution is quite simple in its abstract form. Simply apply game rules that are independent of the individual preferences and the economy as such. Hence, the rules do not have to be changed every time the economy changes and the planner does not have to assess the preferences in order to design the rules and set the appropriate taxes. The idea resembles that of heat regulation. In the old days the temperature in a room was regulated by opening the radiator when the temperature was too low, and turning it of again and opening the windows if the temperature got to high. This is the manually method a planner uses in the regulation of transport (traditional welfare economics). The introduction of thermostats made the problem much easier to handle. Simply define the temperature you want and the thermostat automatically takes care of the rest. The described mechanism design solution corresponds to the thermostat.

The solution is almost to simple to be true, but it is shown that such general rules do exist for specific equilibrium concepts (see Maskin [8], and Moore &
3 A simplified example solving the congestion problem

The general idea presented in the previous section may be very difficult to comprehend in a specific context as in the problem of relieving congestion. In this section we will describe a simple model of congestion (or externalities in general), and demonstrate how a mechanism can be designed so that the externalities are internalised in the decisions made by the individuals. The simple model can easily be generalised, and the solutions can be demonstrated to hold also in very general settings. This is done in Kveiborg [10]. However, for the understanding of the methodology this simple set-up is adequate, and the specific mechanism chosen is actually the same. The mechanism was originally proposed by Varian [12] for a general solution to the problem of externalities. Kveiborg [10], [11] generalise some of the results and analyse them in a transport related context.

3.1 Traditional welfare solutions to the problem of congestion

Consider a small economy of three commuters competing for the use of one road. The road is such that if any other driver is on the road there will be delays, which is perceived negatively by any of the commuters. The preferences of the three commuters are illustrated by the utility functions:

\[ u_1 = A_1 x_1^a - B_1 x_2 - C_1 x_3 + z_i \]  
\[ u_2 = -A_2 x_1 + B_2 x_2^\beta - C_2 x_3 + z_i \]  
\[ u_3 = -A_3 x_1 - B_3 x_2 + C_3 x_3^\gamma + z_i \]  

where \( x_i \) is transport demanded by commuter \( i \), and \( z_i \) is demand for other goods, which can be transferred from one individual to another. Transport on the other hand is personal and cannot be transferred or substituted from one commuter to another.

In an ordinary approach individuals maximise utility given some budget constraint. For commuter 1 this constraint would be

\[ p_1 x_1 + i_1 x_1 + q z_i \leq y_i \]  

where \( p_i \) is the price on transport by individual \( i \) (e.g. fuel and other variable expenses), \( i_1 \) is a tax levied on driving by individual \( i \) (e.g. a Pigou tax), \( q \) is the price on good \( z \), and finally \( y_i \) is income for individual \( i \).

In the choice made by each commuter the influence the choice has on the other commuters does not enter. Hence, there is an incentive to overuse the road, unless the tax is correctly specified. This is an issue we will return to in a little
while. On aggregate this lead to inefficient use, which the social planner wants to
solve. The way this can be done is to maximise overall utility, or maximise one
individual’s utility given that the other individual’s utilities do not decline. This
lead to the famous result that the congestion tax, \( r_i \), must be set equal to the
marginal disutility of the other commuters in order to ensure efficient usage of
the transport system. In our case this corresponds to setting \( r_i = A_2 + A_3 \), where \( A_2 \)
is the marginal disutility to commuter 2 of commuter 1’s choice of transport. In
this case this disutility is fixed irrespective of the level of transport, but this is
not crucial for the results. In other words the optimal (Pigou [3]) tax is equal to
the sum of the marginal disutilities of the other commuters.

Note that this corresponds to the static situation where preferences and
number of commuters are fixed from period to period. As mentioned above this
is not likely to be the case. Preferences change due to e.g. new job, children etc.
Hence, in our simple case this corresponds to a change in \( A_i \). The problems for
the planner is that she must assess \( A_i \) for each consecutive repetition of the
choices (e.g. once per day) in order to levy the optimal tax. But \( A_i \) is private
information not necessarily known by the social planner. One way of assessing
the \( A_i \)'s is to conduct a RP or SP survey. But in such a survey the individuals
have incentives to misrepresent their true values. If they state a value above \( A_i \)
they can induce less transport by the others and thus face less delay. But if they
all do that, the road will be underused.

3.2 Using mechanism design to solve the problem of congestion

An alternative way of solving the problem is to use the Compensation
Mechanism proposed by Varian [12]. Instead of levying a tax on commuting,
which is dependent on the actual preferences, we assign more general game
rules. The Compensation Mechanism consists of two stages.

In the first stage each individual must announce two sets of taxes (compensations). First, a compensation that he will pay to the others for the
delays he is causing them, call this \( r'_{ij} \). Secondly, a compensation that he should
receive from the others for the delay they are causing him, call this \( r''_{ij} \). Hence, in
our simple model each commuter has to announce 4 compensations.

In the second stage of the mechanism each individual maximise utility given
a slightly changed budget constraint. The additional terms in the budget are what
we termed “the additional game rules” in the previous section. The new budget
constraint commuter 1 faces is

\[
p_i x_i + (r_{21}^2 + r_{31}^2) x_i + q_x x_i \leq y_1 + r_{12}^2 x_2 + r_{13}^2 x_3 + (r_{21}^2 - r_{21}')^2 + (r_{31}^2 - r_{31}')^2,
\]  

and similarly for the other two commuters.

There are three additional terms in this constraint. The first is a term that
resembles the tax introduced in eqn (4), \( r_{21}^2 + r_{31}^2 \), the second is the
compensation that commuter 1 receives from the other commuters, \( r_{12}^2 x_2 + r_{13}^2 x_3 \).
This second term is thus a supplement to the ordinary income. The final change is a penalty term that must be paid if announcements differ, 
\[(t_{21}^{'1} - t_{32}^{'1})^2 + (t_{13}^{1} - t_{23}^{1})^2.\]

Note that individual 1 does not himself decide the tax he has to pay or the compensation that he receives. On the other hand he decides (part of) the tax the other commuters must pay. In this way he can induce the other commuters to travel less by rising the announced compensations. However, as he also demands other goods, \(z\), he would like to get the supplementary money from the other commuters, which mean that he cannot set the compensation too high. The optimal announcement is actually to announce the marginal disutility he suffers from the choices made by the others, which we demonstrate analytically below. The other thing to notice is that it is also optimal for the individuals to announce a tax equal to the one the affected commuters has announced \((t_{ij}^{1} = t_{ij}^{p})\) so that no penalty must be paid.

To solve the problem analytically we look at commuter 1. The problem is similar for the other two commuters. The problem is solved backwards. First step is to maximise utility (1) constrained by eqn (4) taking the announced compensations as given. This gives the following reaction function

\[x_i = \exp\left(\frac{\ln\left(\frac{p_i + t_{21}^{'1} + t_{31}^{'1}}{q_i\alpha_i}\right)}{\alpha_i - 1}\right),\]

and similarly for the other two commuters. Inserting the three reaction functions into the utility functions (1), (2), and (3) and maximising for the choice of \(t_{21}^{1}\) and \(t_{12}^{1}\) gives us

\[\frac{\partial u_i}{\partial t_{21}^{1}} = 0 = 2(t_{21}^{1} - t_{32}^{1}) \Rightarrow t_{21}^{1} = t_{32}^{1}\]

\[\frac{\partial u_i}{\partial t_{12}^{1}} = -\frac{B_{12}x_2}{(\beta - 1)(p_2 + t_{12}^{1} + t_{32}^{1})} = -\frac{\lambda t_{12}^{1}x_2}{(\beta - 1)(p_2 + t_{12}^{1} + t_{32}^{1})}\]

Eqns (7) and (8) gives us that \(t_{12}^{1} = t_{12}^{2} = B_1\), which is the marginal disutility for commuter 1 of transport by commuter 2. Solving this for all three commuters gives us that optimal announcements are exactly the marginal disutility of transport chosen by the other commuters.

We have found that using the Compensation Mechanism instead of having a social planner levying the optimal Pigou taxes, we obtain exactly the same efficient outcome. The main difference is that we did not have a social planner to assess individual preferences and define appropriate taxes, the individuals themselves accomplished this. Furthermore, if preferences for some reason change, then it will benefit the commuters to alter their announcements and
thereby change the outcome of the system. This is the automation that we discussed in Section 2.

4 Some properties of the proposed solution

The suggested solution described in the previous section does have a number of problems. In this section we will mention a few of them, and focus specifically on one particular problem related to the information requirements. But first let us look at some other problems that can be found.

It is immediately clear that as the system increases the number of announcements will tend to be prohibitively large. In a system of $N$ individuals, each must announce $2A'$ compensations. In a real world situation this cannot be accomplished, moreover how should any one individual be able to distinguish each of the $N-1$ other commuters and take each of their individual preferences for transport into account when making announcements. One thing worth mentioning in relation to this is that when $N$ goes to infinity the total compensations to be paid are equal. Hence, there may be a potential reduction in the number of announcements, which are required for the mechanism to work. However, this is an issue that has not been solved yet.

In order to find optimal announcements, the individuals were assumed to know the reaction functions of all the other individuals, which in a general setting is equal to possessing perfect information. Hence, instead of only having one individual (the social planner) assessing the preferences we now have that all individuals are supposed to do that. However, the proposed mechanism is an incentive scheme that induce the individual commuters to actually announce their true preferences, which as was argued above they did not have in the original situation.

In relation to the problem of imperfect information it is very likely that the commuters will make inefficient announcements in the first repetitions of the system. Whether this is a major problem can be analysed through the stability of the system. If the efficient equilibrium is stable, then the announcements will approach the optimal announcements over time. To analyse this problem a few additional assumptions must be made. First, which part of the system is suffering from imperfect information, and secondly, which adaptation procedure do the commuters use. Before we continue a small remark on the repetition should be made. If we consider a system of commuters there will gradually be some accumulation of knowledge contained in the different commuters. This knowledge about e.g. the size of the congestion problem can be utilised in the announcements of the taxes. The commuting situation is not very different for the commuters from one day to the next. Hence, we can consider the system of commuting as a repetition of (almost) the same case for every repetition.

Conditions for stability would involve two difference equations in a discrete setting. First it is recognised that an individual will try to ensure that penalties are not paid. Hence this lead to the condition
where \( k \) indicates the round of repetition. In equilibrium this condition ensures that no penalties are paid. The second observation is that uncertainty about the reaction function of the other agents prevails. Hence, uncertainty about the announcement of \( t_{12}^j \) is of interest. If optimal announcements are made then \( t_{12}^j(k) = t_{12}^j(k-1) \). However, when uncertainty about \( x_j \) prevails, there is also uncertainty about the value of the marginal disutility. This leads to the following condition

\[
t_{12}^j(k) = t_{12}^j(k-1) + \kappa \left( \frac{\partial u_i}{\partial x_j} - t_{12}^j(k-1) \right)
\]

where \( \kappa \) is an adjustment parameter. Analyse the stability of this system of difference equations is thus equivalent to analysing the convergence to the efficient solution of the transport system described in the previous section.

The system is stable if the eigenvalues of the following matrix are smaller than 1.

\[
\begin{bmatrix}
\frac{\partial t_{12}^1(k)}{\partial t_{12}^1(k-1)} & \frac{\partial t_{12}^2(k)}{\partial t_{12}^2(k-1)} \\
\frac{\partial t_{12}^1(k)}{\partial t_{12}^2(k-1)} & \frac{\partial t_{12}^2(k)}{\partial t_{12}^2(k-1)}
\end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\kappa & 1 \end{bmatrix}
\]

The eigenvalues are

\[
\lambda = \frac{1}{2} \pm \sqrt{1/4 - \kappa}
\]

It can then easily be shown that the system is stable if \( \kappa \in (0;1) \).

Hence, generally the announcements will converge to the efficient announcements. This is demonstrated in Figure 2 for a specific combination of marginal disutility (\( B_j=1 \)) and adjustment parameter (\( \kappa = 0.75 \)). The figure shows the evolution from inefficient initial announcements of \( t_{12}^1 = 0 \) and \( t_{12}^2 = 2 \) towards equilibrium and stability. The values are chosen a bit extreme in order to illustrate the point more clearly.
The adaptation procedure described above is quite simple and only information from the previous period is taken into account in the updatings. However, it may be more realistic to allow for usage of even more information in the updatings. This would ceteris paribus lead to an even faster adjustment.

In conclusion there is a tendency that over time equilibrium will be found. This also suggests that it may not be necessary for the commuters to assess the delays on a daily basis, but only make announcements if something has changed.

5 Conclusions

In this paper we have discussed a new theoretical method that can possibly be utilised to solve the problems of externalities. The discussion has been carried out firstly in a very general and abstract setting using the implementation triangle. However, in specific settings things can be formalised and simplified. We have done this using a very simple set-up with only three commuters having very specific utility functions. However, it should be clear that the methodology carries over to much more general settings as illustrated in [10]. We have demonstrated that using the Compensation Mechanism we can actually obtain the same efficient benchmark case as is found using traditional Pigou taxes.

There are several practical problems related to the use of Pigou taxes, some of those are solved using the proposed methodology. On the other hand this methodology rises new problems. A few of these have been touched in the paper, and arguments trying to overcome the stability problem have been provided.

It remains to be seen whether this methodology actually has a potential use in relation to congestion pricing and transport in general. However, the paper has demonstrated that it is not an artefact that the traditional welfare approach should always be used, there may be other solutions to be found in the economic theories.
References


