The schedule-based dynamic modelling for public transport networks: a new approach in path choice and assignment

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Abstract

This paper presents a general overview of the schedule-based approach in
dynamic path choice and assignment models for transit networks. The first part
describes different specification of path choice models in relation to different
user and service characteristics. The latter deals with a classification of
assignment models.

1 Introduction

Public transport networks are usually modelled using the frequency-based
approach, which considers services in terms of lines. In this case run scheduled
times are not considered explicitly, but the time dimension is considered through
the service frequency (inverse of the line headway), from which the name of the
approach derives. The most used frequency-based models refer to the concept of
hyperpath or optimal strategies (Nguyen and Pallottino [8], Spiess and Florian
[17]). The main hypothesis of the hyperpath model is that users choose the line
in an indifferent adaptive way, that is they board the first arriving run belonging
to their line choice set, defined as the set of lines that minimises the expected
total travel time. The underlying hypotheses that allow such results are that
vehicles arrive at stops in a random way and that users arrive at stops with constant rate. In recent years deterministic and stochastic equilibrium models have been presented in order to take into account congestion and perception errors of path attributes (De Cea and Fernandez [3], Wu et al. [20], Lam et al. [7]). The above described hypotheses are fully acceptable for services with high frequency, very low punctuality and low user information, but can generate considerable approximations when used in different contexts, in particular when ITS (Intelligent Transportation Systems) are present, or in the case of low-frequency services. For this reason in the last ten years a new approach, called schedule-based approach, has been developed; it refers to services in terms of runs, using the real vehicle arrival/departure time at stops to obtain attributes that can be explicitly considered in run choice modelling. As it will be better described in the following sections, this approach allows us to take into account the evolution in time both of supply and demand, as well as run loads and level of service attributes. Hence this approach is also called dynamic.

The core of the assignment modelling is the path choice that allows, in addition to a supply/demand interaction procedure, vehicle loads of transit services to be obtained. Transit path choice models can be differently specified according to user behavioural hypotheses that depend on specific user and service characteristics.

The service characteristics affecting user behaviour in path choice are frequency, regularity and information available to users.

Service frequency can be directly related to the frequency of line \( l \) in the reference period (i.e. the number of runs belonging to line \( l \) connecting the od pair in such a period) or, for overlapping lines, to the cumulative frequency (i.e. the sum of the frequencies of all "attractive" lines connecting the od pair). Usually service frequency is defined high if the average headway of vehicles is less than 12-15 minutes, while frequency is defined as low if the average headway exceeds 15 minutes.

Service regularity is a measure of to what extent the schedule is observed. If regularity is used to make assumptions on user behaviour in line-based systems, such as buses and trains, deviations from the schedule should be related to the average headway of runs belonging to the same line. Usually regular services are associated with low-frequency systems, typical of extraurban services. On the other hand, irregular services generally refer to high-frequency systems as in urban or metropolitan areas.

Pre-trip and/or en-route real-time information on services can be available to users in different places (for example at stops or at home) and concerns at least waiting times of arriving vehicles at the chosen access stop, while more advanced information systems could give information on travel times and on-board occupancy, too. Static information on run schedule is traditionally available with timetable. Intelligent Transportation Systems (ITS) have significantly expanded the range of information available to the traveller through Advanced Traveller Information Systems (ATIS). ITS also improves the performance of transit services, in terms of regularity, through the use of Advanced Public Transportation Systems (APTS).
As regards user characteristics, the main difference to consider concerns whether they are frequent or occasional users. Frequent users travel frequently and know routes and scheduled timetable as well as the real system functioning based on previous experience. Occasional users use sometimes transit services, so they only know some line routes (the most important) and their scheduled timetable.

One of the first schedule-based path choice models was the one introduced for high-frequency transit services by Tong (described in Wong and Tong [18,19]), even if the schedule-based approach was initially developed for low-frequency transit systems, allowing the explicit consideration of the early/late schedule penalties, which play a key role in path choice for extraurban services. Nuzzolo and Russo [11], Carraresi et al. [1], Nguyen et al. [9], Florian [4] specified deterministic path choice models, while Nuzzolo and Russo [11], Cascetta et al. [2], Nielsen and Jovicic [10], Nuzzolo et al. [13] defined stochastic path choice models.

Recently the schedule-based approach was proposed for urban transit systems in order to consider more coherent user behavioural hypotheses in relation to service characteristics. Hickman and Wilson [5], Hickman and Bernstein [6], Nuzzolo and Russo [12], Wong and Tong [18,19] presented within-day dynamic path choice models that allow time variation in supplied services to be considered. They have been specified for regular and irregular high frequency services, with or without information systems to users at stops. The Nuzzolo and Russo [12] path choice models were extended in a doubly dynamic stochastic path choice model (Nuzzolo et al. [14,16]) that explicitly considers the within-day and day-to-day variations of services and user learning on attributes. The extension to different user classes, considering explicitly frequent and occasional users, was presented in Nuzzolo et al. [15].

In the following sections the scheduled-based approach is dealt with. Section 2 and 3 describe the path choice and assignment models developed in the literature, while section 4 ends the paper with some considerations about the application fields of the traditional frequency-based and the new schedule-based approaches.

2 Schedule-based path choice models for transit services

User behaviour hypotheses and path choice models are specified in the framework of the (random) utility theory, in which users are assumed rational and behave with the aim of maximising their perceived utility.

User decisions can be classified according to two types of choice behaviour:

- pre-trip choice behaviour, which underlies user choices before departure. It includes the comparison of possible alternative paths and the choice of one of them on the basis of expected values of attributes. Pre-trip choices are analogous to those assumed for path choice in road networks;

- en-route choice behaviour, which underlies user choices during the trip. This behaviour describes how users respond to unknown or unpredictable events.
The type of en-route choice behaviour can be defined as *indifferent* (if users board the first arriving vehicle belonging to the set of alternatives) or *intelligent* (if users, when a vehicle belonging to the path choice set arrives, compare the disutility of the arriving vehicle with the disutility of the next arriving vehicles belonging to the path choice set).

As the main factor that greatly affects user behaviour is service frequency, the following sections will describe path choice models for both high and low frequency services.

### 2.1 Path choice models for high-frequency services

In relation to the high frequency of services we can assume that the origin departure time coincides with the desired origin departure time, so user arrival at the stop is not related to run departure scheduled times. Furthermore, it is assumed that users do not have full information before starting their trip and they follow a mixed pre-trip/en-route choice behaviour. En-route choices occur at stops and are relative to the decision to board a particular run or to wait for another run of the choice set. The choice of boarding stops is considered before starting the trip, since it is not influenced by unknown events.

In the sphere of the schedule-based modelling approach for *high-frequency services*, as this kind of services are typical of urban areas, generally several boarding stops can be reached for the same origin and departure time and, even if considering the same user arrival time at stop, many runs can be available. Thus path choice implies choice of access stop and choice of run (or sequence of runs) leading users to their destination.

![Figure 1: Example of path choice set](image)

Figure 1 reports an example of path choice set: considering the origin departure time $t_{0n}$, three different access stops are available, from which different runs of different lines can be used to reach the destination (e.g., from stop 2, run 3 of line $f$ or the combination of run 2 of line $g$ and run 7 of line $h$ can be considered).
In the schedule-based approach a path between origin \( o \) and destination \( d \), departing from \( o \) at a given time \( \tau_{Di} \), is defined by the space-time sequence which includes: origin \( o \) with origin departure time \( \tau_{Di} \), access to access stop \( s \) with relative arrival time \( \tau_{Dis} \), run (or sequence of runs) with run departure time from access stop and run arrival time to egress stop \( s' \), egress from stop \( s' \) to destination \( d \) with relative arrival time at destination.

The probability \( p_{od}[r,s]\mid \tau_{Di} \) of choosing a path including run \( r \) at boarding stop \( s \), given the \( od \) pair and the origin departure time \( \tau_{Di} \), can be written as

\[
p_{od}[r,s]\mid \tau_{Di} = \frac{p_{od}[r]\mid \tau_{Dis} \cdot p_{od}[s]\mid \tau_{Di}}{p_{od}[\tau_{Di}]} (1)
\]

which is the product of the probability of choosing run \( r \) at stop \( s \), given the arrival time at stop \( \tau_{Bi} \) by the probability of choosing stop \( s \), given the origin departure time \( \tau_{Di} \). In the following the index \( od \), when not reported, is understood.

The probability \( p[r\mid s, \tau_{Dis}] \) of choosing the arriving run \( r \in K^s \) at stop \( s \) can be expressed as:

\[
p[r\mid s, \tau_{Dis}] = \text{prob}(U_r > U_r) = \text{prob}(V_r + \varepsilon_r > V_r' + \varepsilon_r), \quad r \neq r', r, r' \in K^s (2)
\]

where the perceived utility \( U_r \) of the generic run \( r \), belonging to the choice set \( K^s \), can be expressed as

\[
U_r = V_r + \varepsilon_r = \sum \beta_j X_{jr} + \varepsilon_r (3)
\]

in which \( \beta_j \) are the weights of attributes \( X_{jr} \), which make up the systematic utility \( V_r \), and \( \varepsilon_r \) is a random residual. Attributes \( X_{jr} \) usually considered are: waiting time, on-board time, transfer time, number of transfers, “route” on-board comfort (function of on-board crowding on the following links), “stop” boarding comfort (function of on-board crowding at the stop), monetary cost.

The run choice model can be deterministic (Hickman and Wilson [5]) or stochastic (Hickman and Bernstein [6], Nuzzolo and Russo [12], Wong and Tong [18,19], Nuzzolo et al. [14,15,16]) if random residuals \( \varepsilon_r \) are assumed null or otherwise, and different random utility models (logit, probit, etc.) can be specified according to the distribution of random residuals \( \varepsilon_r \).

The random residuals \( \varepsilon_r \) usually take into account aggregation errors (e.g., zoning, network model), missing attributes (e.g., scenic quality, habit), dispersion of user behaviour (e.g., value of time) and user perception errors (e.g., travel time). Moreover, some models include in the random terms some aspects they do not consider explicitly, like service irregularity.

Users assess in different ways attributes of eqn (3) if they are frequent users or occasional ones, and if a user information system, especially on waiting time of arriving runs, is available at stops. Users typically forecast attributes at the time in which they decide on the basis of available information and past experience. Anyway, if a user information system is working, some attributes can be assessed directly through real-time information at stops (e.g., the waiting time of next arriving runs provided to users). Moreover, in the case of high-frequency services, sometimes there is a substantial difference between real
arrival/departure times of vehicles at stops and scheduled times (irregular services). This aspect, jointly with the type of user (frequent or occasional), leads to considerable different run choice mechanisms. As above described, in the framework of run choice, three types of behaviour are possible: pre-trip, intelligent en-route, indifferent en-route.

For irregular services with user information at stops about waiting times, considering frequent users, the run choice at stop is typically en-route and can be simulated through a sequential mechanism that considers an intelligent en-route choice behaviour. When a run \( r \) of the path choice set \( K \) arrives at stop \( s \), the user chooses to board \( r \) if the perceived utility \( U_r \) is greater than the utility \( U_{r_i} \) of all other runs \( r_i \in K \) yet to arrive.

In this case, \( U_r \) includes the waiting time of the next run \( r' \), given by the difference between the arrival time of run \( r' \) and the arrival time of run \( r \) (supplied by the user information system), while in \( U_r \) the waiting time of run \( r \) is replaced by the time already spent at the stop (equal to the difference between arrival time of run \( r \) and the user arrival time \( \tau_{Di} \) at stop \( s \)) simulating a possible “impatience effect”. Other attributes of eqn (3) are considered as previously described.

Of course, if the user does not choose run \( r \), the choice is reconsidered when the next run arrives and so on (sequential run choice mechanism with intelligent en-route behaviour). Occasional users behave in the same way, but they consider a reduced run choice set (i.e., they only consider the most important runs connecting the od pair).

For irregular services without information, frequent users consider a run choice set that minimise their average perceived cost, within which they choose comparing the first arriving run with the others belonging to the run choice set on the basis of the further expected waiting times, in addition to the other level of service attributes (intelligent en-route behaviour). If no information is provided and services are irregular, occasional users do not have enough experience of system functioning and board the first arriving run belonging to their run choice set (indifferent en-route behaviour). For frequent users, if vehicle arrivals are random as Poisson process, the indifferent en-route behaviour can also be assumed.

Finally, one of the most important classification parameter in path choice modelling is the dynamic characteristic of the model. In the sphere of schedule-based approach, path choice models consider at least the within-day dynamic, which allows the system evolution (level of service attributes) within the reference period to be taken into account. Schedule-based path choice models can also consider the day-to-day dynamic (i.e., the evolution of system characteristics from one day to another) if they include a learning process on attributes of eqn (3). Path choice models that consider both the within-day and day-to-day dynamics are usually referred in literature as doubly dynamic path choice models (Nuzzolo et al. [16]).

For what concerns the stop choice, it is usually assumed to be fully pre-trip, as no real-time information is typically available at origin. The probability \( p[s] \tau_{Di} \)
of choosing the boarding stop \( s \), within a choice set of boarding stops, \( S_{od} \), can be expressed as

\[
p[s|\tau_{D}] = \text{prob}(U_s > U_{s'}) = \text{prob}(V_s + \varepsilon_s > V_{s'} + \varepsilon_{s'}) \quad s \neq s', s,s' \in S_{od} \quad (4)
\]

where \( U_s \) is the perceived utility, sum of the systematic utility \( V_s \) and of a random residual \( \varepsilon_s \). The systematic stop utility is a function of stop-specific attributes (e.g., access or egress times, presence of shops, etc.) and "inclusive utility" expressing the average utility associated with all runs available at stop \( s \). As described for the run choice, deterministic or stochastic stop choice models can be specified according to the assumptions on random residuals \( \varepsilon_s \).

2.2 Path choice models for low-frequency services

From a practical point of view, low-frequency services, as regional bus and intercity railways, are usually characterised by regular service functioning and we hypothesise users have full information before starting their trip. In this case there is no difference in user behaviour for frequent and occasional users, and the presence of user information is unnecessary to support user choices, assuming that they at least know routes and timetable. Furthermore, as low-frequency services are typical of extraurban areas, it is common that only one access terminal as well as a single egress terminal are available, so stop choice can be easily simulated. The run choice is assumed fully pre-trip and, in addition to the other service attributes, we need to consider the disutilities that occur because of the difference (that can be considerable) between desired user departure time and vehicle scheduled departure time or between desired user arrival time at destination and run scheduled arrival. In the literature, this difference is called early schedule penalty or late schedule penalty (Nuzzolo et al. [15]).

The probability \( p[r|\tau_{D}] \) of choosing run \( r \), given the \( od \) pair and desired departure time \( \tau_{Di} \) (or desired arrival time \( \tau_{Ai} \)) can be expressed as

\[
p[r|\tau_{D}] = \text{prob}(U_r > U_{r'}) = \text{prob}(V_r + \varepsilon_r > V_{r'} + \varepsilon_{r'}) \quad (5)
\]

where the systematic utility \( V_r \) is a linear combination, through \( \beta \) parameters, of attributes \( X_r \). A set of attributes that can be considered (Nuzzolo et al. [13]) are: access and egress times and costs, on-board times, transfer times, number of transfers, monetary cost, comfort, early/late schedule penalty.

Most of schedule-based path choice models for low-frequency services take into account user target times (desired departure times at origins or destination arrival times at destinations), considering as path choice set alternatives the two "nearest" paths in terms of minimum early and late times with respect to the user target time (Nuzzolo and Russo [11], Cascetta et al. [2], Nielsen and Jovicic [10], Nuzzolo et al. [13]). Florian [4] considers maximum earliness and lateness values to define a time slice around user target time (see figure 2), within which the path choice set is defined, while Carraresi et al. [1] and Nguyen et al. [9] consider only the late alternative with respect to the origin target time or the early alternative in the case of desired arrival time at destination. If the path choice model is deterministic, the minimum disutility alternative is considered,
while if it is stochastic a certain probability to all alternatives of the choice set, calculated through eqn (5), is associated.

Deterministic (Carraresi et al. [1], Nguyen et al. [9], Florian [4]) and stochastic (Nuzzolo and Russo [11], Cascetta et al. [2], Nielsen and Jovicic [10], Nuzzolo et al. [13]) path choice models can be specified according to the hypothesis of null $\varepsilon$ or otherwise. Assuming $\varepsilon$ different from zero, different random utility models can be specified. In particular, in the case of multi-class multi-service transit systems (e.g., railways), the existing correlations between alternatives have to be taken into account, and nested-logit or probit models should be used (Nuzzolo and Russo [11], Nuzzolo et al. [13]).

Moreover, considering a learning process on attributes of $V_r$, day-to-day dynamic schedule-based path choice models for low-frequency services can be specified to take into account the day-to-day evolution of on-board loads and level of service attributes.

3 Schedule-based assignment models for transit services

Assignment models allow the on-board load of transit vehicles to be obtained. Using a schedule-based approach, in which all runs of the transit services are explicitly considered, it is possible to obtain very detailed results in terms of on-board loads and level of service attributes for each vehicle.

The classification of assignment models usually refers to the classical one adopted in road network modelling, in which different assignment models can be specified according to the type of behavioural path choice model (deterministic or stochastic), the type link performance functions (flow-dependent or otherwise, which lead to uncongested or congested networks), the assignment approach (network loading, user equilibrium, dynamic process, etc.), and the dynamic evolution (within-day and/or day-to-day) they take into account.
Table 1 reports the classification parameters of assignment models for transit networks. In particular, for uncontested transit networks, both AoN (All or Nothing) and SNL (Stochastic Network Loading) assignment models can be considered to calculate on-board loads, as well as DUE (Deterministic User Equilibrium) and SUE (Stochastic User Equilibrium) assignment models can be specified for congested transit networks.

### Table 1: Classification of schedule-based transit assignment models

<table>
<thead>
<tr>
<th>Transit network</th>
<th>Assignment approach</th>
<th>Deterministic</th>
<th>Stochastic</th>
</tr>
</thead>
<tbody>
<tr>
<td>uncontested [c=cost]</td>
<td>Network Loading</td>
<td>AoN</td>
<td>SNL</td>
</tr>
<tr>
<td>congested [c=c(t)]</td>
<td>Dynamic</td>
<td>DUE</td>
<td>SUE</td>
</tr>
<tr>
<td></td>
<td></td>
<td>DDP or SDP</td>
<td>DDP or SDP</td>
</tr>
</tbody>
</table>

AoN=All or Nothing; SNL=Stochastic Network Loading; DUE=Deterministic User Equilibrium; SUE=Stochastic User Equilibrium; DDP=Deterministic dynamic process; SDP=Stochastic dynamic process.

For what concern the dynamic evolution, as in the schedule-based approach the timetable is explicitly considered, the within-day dynamic is a native characteristic of the schedule-based approach, in the sense that all schedule-based assignment models are at least within-day dynamic. Moreover, considering a learning process on attributes, the day-to-day dynamic can be also considered, leading to doubly dynamic (within-day and day-to-day dynamic) assignment models. The dynamic evolution can be simulated by dynamic process models, both stochastic and deterministic.

All above described assignment models can be specified in the case of regular service functioning or otherwise, considering the randomness of on-board loads in relation to the randomness (irregularity) of transit services.

### 4 Conclusions

In the last ten years the improvements in computer science and the increasing computation capacity have allowed the schedule-based approach to be developed and applied. The schedule-based approach was initially developed for low-frequency systems and has recently been proposed for urban transit systems in order to consider more coherent user behavioural hypotheses in relation to user and service characteristics.

Frequency-based models are suitable to be used when a high degree of detail is not necessary and aggregate results can be obtained with few input data. For example in the case of the strategic planning of a new transit network, we need to specify only the line paths and stops in addition to service frequencies, and the average line loads, obtained as output, are sufficient for the aims of the project. In this case, the definition of the timetable, required by the schedule-based approach, and the output in terms of load for each run is not consistent with the level of detail of the other components of the simulation models, and is not necessary. By contrast, in the sphere of operative planning (e.g., with the aim of
supporting the definition of a new timetable or to assess the introduction of ITS systems) we need more precise and detailed results that can be obtained only through a schedule-based approach.

Further developments are in progress and mainly regard the use of the schedule-based approach in applications, as the estimation of transit O/D matrices from traffic counts and the transit network design.

References


