Modeling failure waves in brittle materials
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Abstract
In an attempt to elucidate the failure mechanism responsible for the so-called failure waves in glass, numerical simulations of plate impact experiments, with a microcracking multiple-plane model, have been performed. These simulations show that the failure wave phenomenon can be modeled by propagating surfaces of discontinuity from the specimen surface to its interior. Lateral stress increase, reduction of spall strength, and progressive attenuation of axial stress behind the failure front are properly predicted by the multiple-plane model. Numerical simulations of high strain rate pressure shear experiments indicate the model predicts reasonably well the shear resistance of the material at strain rates as high as 1x10^6 /sec. The agreement is believed to be the result of the model capability in simulating damage-induced anisotropy. By examining the kinetics of the failure process in plate experiments, we show that the progressive glass spallation in the vicinity of the failure front and the rate of increase in lateral stress are more consistent with a representation of inelasticity based on shear-activated faults (narrow shear lines) and microcracking, rather than pure microcracking.

1 Introduction
The use of microscopic surface flaws to explain the low measured strength values of glass compared to its theoretical strength is a well established concept. By contrast, the role of these flaws in the overall material degradation under dynamic loading is not well understood. The generation and propagation of a damage front in dynamically loaded glass samples has been reported by Kanel et al.¹, and Brar et al.² It was experimentally shown that behind this damage front, glass loses shear and tensile strength.

Three theories have been proposed to explain this so-called failure wave in glass. Kanel et al.¹ argued that a system of microcracks intersecting in space is formed during shock compression leading to the forma-
tion of blocks with flat surfaces. These authors also reported that the block boundaries presented a weak but not vanishing tensile strength. Another explanation of the observed failure wave phenomenon, based on theories of inhomogeneous plastic flow in amorphous materials (Spaepen, Argon, Steif et al. and experimental observations in indentation testing (Hagan, Lawn et al., Kurkjian et al.) was postulated by Espinosa and Brar. Their interpretation consists of the initiation of shear-activated microfaults, planes of localized plastic deformation, at the impact surface which propagates to the interior of the sample. Within this mechanism, microcracks nucleated at the intersection of microfaults can explain the observed reduced but finite tensile strength behind the failure front. A third hypothesis invoking phase transformation to crystalline phases within the bulk of glass samples was advanced by Raiser and Clifton. Their motivation for postulating phase transformation was based on the fact that their experimental results showed that surface roughness appears to play no role in the formation of failure waves.

Recently, we have conducted rod recovery experiments in which extensive fragmentation is observed in the recovered glass samples (see Espinosa et al.). It should be noted that these observations are performed after unloading, and consequently the history of the fragmentation process is not known. Microscopy studies performed on fracture surfaces confirm the formation of mostly planar faults intersecting in space. Furthermore, the fracture surfaces presented features (twist hackles, Wallner lines, and mist hackles) commonly encountered in glass fracture under tension. Post-test X-ray experiments also revealed that the material retains its amorphous structure.

In view of these observations and the existence of several hypotheses about the physics of the problem, it was decided to numerically investigate the failure event by means of a moving damage front originated at the impact surface at the start of the compression pulse. The material degradation is simulated with the multiple-plane microcracking model discussed in Espinosa. We are aware of only one other simulation of failure waves in glass in which the shear stress and material strength are relaxed to zero behind the failure front, see Kanel et al. The adopted relaxation process was empirically rather than mechanistically justified.

In this investigation a micromechanical study of glass failure due to dynamic microfracture is performed.

2 Microcracking Multiple-Plane Model

The inelastic response of brittle materials is modeled through a microcracking multiple-plane model based on a dilute approximation (Taylor model). Details of the mathematical formulation can be found in Espinosa. The basic concept is that microcracking and/or slip can occur on a discrete number of orientations (Fig. 1). In our computational model, the slip plane properties (friction, initial microcrack size, micro-
rack density, etc.) and their evolution are independently computed on each plane. The macroscopic response of the material is based on an additive decomposition of the strain tensor into an elastic part and an inelastic contribution arising from the presence of microcracks within the solid. In contrast to scalar representations of damage, the present formulation is broad enough to allow the examination of damage induced anisotropy and damage localization in the interpretation of impact experiments.

Fig. 1: Schematic of microcracking multiple-plane model.

The general structure of the constitutive equations corresponds to that of a solid with a damage-induced anisotropic stress-strain relation with elastic degradation. In particular, the effective behavior of the solid is predicted to be rate dependent due to crack kinetics effects. From a computational standpoint, this ensures numerical reliability and mesh independence. This is in contrast to quasi-static formulations of damage for which the governing equations become ill-posed in the softening regime. If the material is subjected to a predominantly tensile stress state, microcracks along orientations perpendicular to the direction of maximum tensile stresses will grow. In this case, significant dilation is expected due to mode I crack opening. If a predominantly compressive state of stress with shear is imposed, then crack opening is inhibited but inelasticity is manifested by the growth of penny-shaped cracks in modes II and III (shear modes).

3 Numerical simulations

A dynamic finite element analysis is performed to simulate the experiments reported in Espinosa et al.\textsuperscript{11}, and in Raiser and Clifton\textsuperscript{10}. In all the calculations reported here, the failure wave phenomenon is modeled as a propagating damage boundary. This is accomplished by activating the microcracking model on elements located within a damage region defined by a failure front that initiates at the sample surfaces and propagates to the inside.
A strip of plane strain four-node quadrilateral elements is used in the simulation of normal and pressure shear experiments. Periodic boundary conditions are imposed to simulate the corresponding deformation fields. The set of model parameters used in the calculations are $N = 1 \times 10^{11} / \text{m}^3$ on all planes, $K_{IC} = 0.5 \text{ MPa} \sqrt{\text{m}}$, $\mu = 0.15$, $a_0 = 1.\mu\text{m}$, $n^+ = m^+ = 0.3$, and $n^- = m^- = 0.1$. When different parameters are used, the corresponding plots indicate the variation with respect to these values.

Model parameters like fracture toughness, $K_{IC}$, are taken from values reported in the literature. Other parameters like crack density are selected such that they are in agreement with the observations performed on recovered samples, see Espinosa et al.\textsuperscript{11} When such information is not experimentally available, the density is selected consistent with the impact surface roughness. For highly polished surfaces, a density of $1 \times 10^{11}$ is utilized, while, for intentionally roughened surfaces (pressure-shear configuration) a density of $N = 1 \times 10^{13}$ is used.

Fig. 2: Transverse stress histories from experiment 7-1719. Comparison between experimental records and simulations with three different initial crack sizes. Lateral and axial stresses are plotted for the case $a_0 = 100 \mu\text{m}$.

Having introduced the model parameters used in the simulations, we next examine the effect of propagating a damage front in the experimental configurations reported in Espinosa et al.\textsuperscript{11}, and Raiser and Clifton\textsuperscript{10}. We start with the lateral gauge configuration, experiment 7-1719. In Fig. 2, the experimentally recorded transverse stresses, through lateral manganin gauges, are compared to the computed stresses for three different values of initial crack size. It can be observed that the stress increase resulting from glass cracking exhibits different rates as a function of $a_0$. For $a_0 = 1 \mu\text{m}$ the rate of lateral stress increase, at the location of the front gauge, is smaller than the one recorded experimentally. Furthermore, the lateral stress increase upon arrival of the failure wave, at the back gauge, presents...
a delay with respect to the experimental record. This delay is the result of crack kinetics effects rather than failure front speed. In all calculations the failure front has been propagated at 2000 m/sec. When the initial crack size is increased to 100 μm better agreement is obtained. These observations appear to indicate that the propagating failure front consists of the sweeping of propagating defects rather than the nucleation of new ones. In the present simulations, \( a_0 \) is the initial crack size nucleated behind the failure front and should not be interpreted as an initial defect in the intact material. The numerically predicted axial stress, \( \sigma_x \), is also plotted in Fig. 2 for the case \( a_0 = 100 \mu m \). A progressive reduction in axial stress is predicted.

![Graph showing stress history](image)

**Fig. 3**: Longitudinal stress history from experiment 7-1754. Progressive stress decay upon arrival of failure wave to gauge location is observed.

The axial stress behind the failure wave also has distinct features as observed in normal impact experiments with in-material stress measurements close to the impact surface, Espinosa et. al. It has been seen that upon arrival of the failure front to an in-material gauge located at 3 mm from the impact surface, a progressive reduction in longitudinal stress occurs. This feature can be correlated to the kinetics of the failure process behind its front. In Fig. 3 the computed axial stress trace corresponding to experiment 7-1754, the double shock experiment, is compared with the experimental measurement. A small discrepancy in the stress magnitude of the second pulse is seen. However, good agreement is observed in the progressive stress decay produced by the propagating damage front after arrival of the failure wave to the gauge location. The parameters used in this calculation are those previously reported. The failure front was propagated at 2000 m/sec.

Normal impact experiments investigating the existence of failure waves have also been reported by Raiser and Clifton. In their experiments the interaction of an unloading wave, from the back surface of a target glass
plate, with the advancing failure front was monitored by means of free surface interferometric measurements. Depending on the impactor thickness, spall planes in front and behind the failure front were produced. In Fig. 4, the computed and experimentally recorded free surface velocities for the case in which tensile loading is produced in front of the failure wave are shown. A difference in the rising part of the free surface velocity results because glass densification is not incorporated in the present model. Such densification results in a nonlinear material behavior which is manifested by the attenuation of the wave front and the generation of a tail following stress release. Since our main interest is the identification of the failure wave mechanism, we will not account for glass densification in our modeling. From the computed velocities, it is clear that interaction of the unloading wave with the advancing failure front generates waves that results in a reacceleration of the target free surface. In the simulations, two different values of n+ have been used to assess the effect of crack growth rate in the free surface velocity. The smaller crack tip velocity, smaller n+, seems to provide the best fit of the experimental record. The rate of free surface velocity increase is well captured by this simulation. When a higher crack speed is used, the arrival of the second unloading wave, at approximately 1.3 μsec, does not become evident in the trace, and the overall reacceleration is excessive.

From these calculations, one can conclude that the free surface reacceleration is produced by crack growth behind the failure front when the material is subjected to tensile dynamic loading. A relevant feature to note is that the increase in free surface particle velocity is progressive; i.e., the material behind the failure front has a reduced but finite spall strength. The case in which the two unloading waves, one from the target back surface and the other from the impactor back surface, meet behind the failure front is examined in Fig. 5. For details of the generated wave fronts in this experiment see Raiser and Clifton. The experimental traces show a fast increase in velocity followed by a plateau with a free surface velocity of .9 mm/μsec. In the case of experiment glass2, a small reacceleration, at approximately 1.1 μsec, instead of a velocity reduction, is reported in Raiser and Clifton. This feature indicates the material located at a distance from the failure front exhibits an almost complete loss in spall strength. In experiment glass1, a small reduction in free surface velocity is observed at a time corresponding to the arrival of the unloading wave. When the computed velocity is compared to the experimental record, once again, there is an initial difference in particle velocity due to the fact that our model does not address glass densification. An almost complete lack of spall strength is predicted in our model when n+ = 0.3 is used in the equation for crack evolution. By contrast, when a smaller crack tip velocity is used in the simulations, the material behind the failure front presents a reduced but finite spall strength. It should be noted that the nonlinear behavior induced by glass densification will in general
Fig. 4: Free surface particle velocity prediction for experiment called 93-04 in Raiser and Clifton\textsuperscript{11}. Increase in particle velocity, after initial unloading, is due to progressive spallation of glass.

Fig. 5: Free surface particle velocity predictions for experiment called glass2 in Raiser and Clifton\textsuperscript{11}. The velocity history for experiment glass1 is also shown in the same figure. An almost complete loss of spall strength is predicted when $n^+ = 0.3$ is used.

produce a pulse tail that could reduce the decrease in particle velocity observed in the case in which $n^+ = 0.1$. Hence, it appears that modeling of glass densification will provide a more accurate interpretation of these experimental records. Another feature revealed by the numerical simulations is a slow decay in free surface particle velocity after the initial rising part. This decay seems to be linked to the accumulation of damage behind the failure front. Although this feature is not present in the experimental traces, recent experiments performed by Grady\textsuperscript{13} appear to confirm our numerical prediction.

Additional insight about glass failure can be gained by modeling the high strain rate pressure-shear experiments reported in Espinosa et al.\textsuperscript{11} The
Fig. 6a: Normal velocity history from high strain rate pressure-shear experiment, test 91-11 in Espinosa et al.\textsuperscript{12} Comparison of numerical simulation and experimental record.

Fig. 6b: Transverse velocity history from high strain rate pressure-shear experiment, test 91-11 in Espinosa et al.\textsuperscript{12} Comparison of numerical simulation and experimental record.

normal velocity-time profile corresponding to this test, is shown in Fig. 6a. The experimental record exhibits an initial pulse with a short duration, approximately 100 nsec, due to the existence of a small gap at the Glass-WC/6Co interface in the flyer assembly. The velocity rise upon closure of the gap is associated with the reverberation of waves within the specimen. Ultimately, a homogeneous state is achieved when the normal particle velocity rises to the value predicted by the elastic impact of the WC/6C anvils. The multiple-plane microcracking model with the set of parameters previously defined is utilized. In the numerical simulation, normal motion is applied along the y-y direction, and shear motion is applied along the x-x direction (see Fig. 1). We can observe that the numerical simulation captures the reduction in particle velocity introduced by the
specimen-flyer gap as well as the axial stress build up within the glass sample. The transverse particle velocity, shown in Fig. 6b, initially rises to a level of 10 m/sec followed by a progressive increase up to a maximum of 28 m/sec. Examination of the computed transverse velocity reveals that the microcracking model predicts reasonably well the shear flow of the material as well as the progressive increase in transverse particle velocity with the increase in axial stresses. After gap closure, at approximately 150 nanoseconds, the numerical simulations slightly overpredicts the rate of transverse velocity increase. This discrepancy can be the result of interface slippage, due to the variations in normal traction introduced by the gap, which is not simulated in the calculations. However, it should be emphasized that good agreement in the dynamic shear resistance of glass, after the initial transient, is obtained.

4 Concluding remarks

When the experiments reported in the literature and their modeling are analyzed in their totality, the shear-activated microfault mechanism appears more likely than the pure microfracture mechanism. We have noted in the discussion of lateral gauge experiments, that the failure wave seems to consist of the sweeping of propagating defects, presumably planar, rather than the nucleation and growth of new defects. Moreover, when the previously mentioned progressive spallation behind the failure front is taken into consideration, it appears reasonable to postulate the nucleation, at the intersection of microfaults, and growth of microcracks with accumulated inhomogeneous flow in the material. If the failure wave is interpreted as the propagation of a system of cracks from the impact surface to the interior of the specimen, an inconsistency with the observed progressive spallation immediately behind the failure front arises.

In summary, this work has examined micromechanical explanations of the so-called failure waves in glass which cannot be investigated by experiments alone. Our study was based in a micromechanical description of inelasticity behind the failure wave rather than its mathematical description in terms of a transformation shock (Clifton14.) Numerical simulations have shown that a propagating microcracking front can capture the main features observed in a variety of impact configurations. Lateral stress increase, reduction of spall strength, and progressive attenuation of axial stress behind the failure wave front are properly reproduced when a multiple-plane microcracking model is utilized. The simulations provide additional insight into the kinetics of the failure process by modeling of wave interaction and damage evolution.

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6 References