An explicit damage model for dynamic concrete behaviour. Numerical simulations and comparisons with experimental results on reinforced concrete plates under blast loading
A. Rouquand, C. Pontiroli, E. Canton
Centre D'Etudes de Gramat, 46500 Gramat, France

Abstract

Under severe mechanical loading, brittle materials, like concrete, can be described favourably using damage models. In order to determine the vulnerability of reinforced concrete structures submitted to accidental loading, the Centre d'Etudes de Gramat has developed an explicit damage model. This new concrete model uses two internal scalar variables to represent the material stiffness with opened or closed Inelastic tensile and inelastic compressive strains are introduced. Strain rate effects are also taking into account in order to separately increase the dynamic tensile and compressive material strength. Friction stresses are added to simulate stress strain hysteresis during unloading and reloading path. The Hillerborg regularisation concept is applied to reduced mesh size effects on failure process. The explicit model allows computation of the stress tensor directly and exactly, without any iterative process. Computation time are reduced drastically and convergence difficulties of the iterative procedure are suppressed.

The model has been implemented in an explicit finite element program. Numerical simulations has been done to simulate experimental results obtained on a reinforced concrete circular plates under shock wave loading. The concrete plates have a diameter of 1.3 meter and a thickness of 8 or 10 centimetres. The plates are supported on it external edge. Calibrated shock wave loading inducing static overpressure between 100 Kpa and 500 Kpa are applied successively to the reinforced concrete plates. Large and permanent deflections are obtained on the concrete structure. Strain gage measurements are recorded on the steel reinforcement. Comparisons between numerical and experimental results are presented and discussed.
1 Introduction

In order to determine the vulnerability of reinforced concrete structures under severe accidental loading, the Centre d’Etudes de Gramat (C.E.G.) simulates, on complex two or three dimensional structures, dynamic response and failure of reinforced concrete targets. Such numerical simulations take a long time to run and convergence problems often induce fatal numerical difficulties. Moreover, the complex behaviour of brittle material like concrete is not easy to model when qualitative and quantitative results are expected. The following essential physical mechanisms should be considered for reasonable numerical predictions in dynamic and highly non-linear problems:

- dissymmetry of the tension and compression strength
- crack closure effect.
- inelastic tensile and compressive strains.
- Strain rate effect on tension and compression strength.
- Friction phenomena.

An other important and practical point is that the constitutive equation has to be integrated in a finite element code in a very efficient numerical way in order to reduce computation time. A robust numerical procedure is necessary to obtain results during the fracture analysis of concrete structure. In order to take into account these physical mechanisms and these numerical aspects, it was necessary to develop a new model and to introduce it, in a finite element program. Validation of the new model on realistic experiments is necessary to be sure that numerical predictions give satisfactory results. A specific experimental facility has been developed for this purpose. Dynamic tests, with shock wave applied to large size concrete specimen, are able to give experimental results close to practical problems. The dynamic loading should induce large deflection and severe damage on the structure. Failure of the tested specimen has to be obtained and to be compared with numerical simulations for a complete validation procedure including a large loading range. Experimental dynamic measurements are obtained on the concrete specimen and on the steel reinforcements. Detailed comparisons on measured and computed values are proposed to evaluate the new explicit damage model.

2 Concrete damage model

2.1 General damage model characteristics

An explicit damage model mainly developed for reverse loading of concrete specimen is proposed. The new model [6] can be presented as an extension of the J. Mazars’ one scalar damage model (Mazars, [2], 1984). Two scalar damage variables are used here. These scalar variables are internal state variables that give the material stiffness with open and respectively with closed cracks. The scalar $D_t$ is introduced to represent the loss of material stiffness when cracks induced by extensions are open. The scalar $D_c$ refers to the loss of material stiffness when compressive loading is applied at the material point. Tensile damage variable $D_t$ is always greater than the compressive damage variable $D_c$. So crack closure effect induces an increase of the material stiffness and, in a other hand, a
crack reopening process reduce the material stiffness. Inelastic or irrecoverable strains are introduced in the constitutive equations. These strains are related to $D_t$ and $D_c$ damage variables. As damage grows, inelastic strain increases. They are physically associated to frictional effects in cracked zone which induce permanent strains even for zero stresses. Figure 1 shows the stress strain relation for uniaxial and cyclic tension or compression loads. Cyclic loading can be simulate with the present model. As for the J. Mazars’s model an explicit formulation is adopted in the stress strain relation. The formulation gives a stress tensor expression versus the strain increment tensor and then versus the total strain tensor. No iterative procedure are needed to get the exact final stress tensor. Details of the present model are given by Pontiroli (e.g. Pontiroli, [6]). Only general characteristics are shown here. The stress strain relation is given by:

$$\sigma - \sigma_{ft} = (1 - D_t) \alpha_t \left[ \lambda_0 \text{Trace} \left( \varepsilon_{ft} - \varepsilon_{ft} \right) I + 2 \mu_0 \left( \varepsilon_{ft} - \varepsilon_{ft} \right) \right]$$

$$+ (1 - D_c) \alpha_c \left[ \lambda_0 \text{Trace} \left( \varepsilon_{ft} - \varepsilon_{ft} \right) I + 2 \mu_0 \left( \varepsilon_{ft} - \varepsilon_{ft} \right) \right]$$

(1)

$\lambda_0$ and $\mu_0$ are the Lame coefficients. $\sigma_{ft}$ is the closure stress tensor, $\varepsilon_{ft}$ the associated closure strain tensor. $D_t$ and $D_c$ are the damage variables (scalar values). Evolution laws for $D_t$ and $D_c$ are controlled by an equivalent strain $\tilde{\varepsilon}$ which depends, like in the J. Mazars’s model [2], of positive principal strain components. $\alpha_c$ and $\alpha_t$ are scalar values between 0 and 1 which give the compressive and tensile part of a given loading. $\left( \alpha_c + \alpha_t = 1 \right)$.

![Figure 1: Cyclic behaviour of damage model under uniaxial loading](image-url)
2.2 Strain rate effects

Experimental results, obtained on dynamic tensile and compressive tests, show that concrete strength is strain rate dependent (Suaris and Shah, [3], 1985, Toulemonde, [5], 1995). This effect is significant specially on tensile tests when a strain rate equal to 1 s\(^{-1}\) can double the maximum concrete strength. In order to simulate these strain rate effects, damage evolution laws are strain rate dependent. The damage threshold that governs evolution law is written as follow:

\[ \varepsilon_0 = \alpha_t \varepsilon_0^t + \alpha_c \varepsilon_0^c \]  

(2)

\( \varepsilon_0^t \) and \( \varepsilon_0^c \) are tensile and compressive damage threshold. These threshold are strain rate dependent as follow:

\[ \varepsilon_0^t = \varepsilon_0^{ts} \left[ 1 + a_t (\dot{\varepsilon})^b_t \right] \]

\[ \varepsilon_0^c = \varepsilon_0^{cs} \left[ 1 + a_c (\dot{\varepsilon})^b_c \right] \]

(3)

\( \varepsilon_0^{ts} \) and \( \varepsilon_0^{cs} \) are static tensile threshold and respectively static compressive threshold. \( a_t, a_c, b_t, b_c \) are material coefficients which are obtained from dynamic tensile and compressive tests. \( \dot{\varepsilon} \) is a scalar measure of the strain rate calculated from octahedral strain (Liu and Owen, [4], 1986). This method introduces different strain rate sensitivity coefficients between tension and compression. The computed dynamic stresses increase proportionally to the dynamic damage threshold.

2.3 Friction effect

An additive friction stress, \( \sigma_{\text{fric}} \), is introduced in the formulation. This stress generates hysteresis in the stress strain relation during unloading and reloading path with constant and non zero damage variables. Dissipated energy is simulated in the elastic range for damaged material. This effect introduce a damping in the structure response which is strain rate independent.

The total stress tensor is given by:

\[ \sigma = \sigma + \sigma_{\text{fric}} \quad \text{if} \quad \dot{D}_t = 0 \quad \text{and} \quad \dot{D}_C = 0 \]

\[ \sigma = \sigma \quad \text{if} \quad \dot{D}_t \neq 0 \quad \text{or} \quad \dot{D}_C \neq 0 \]

(4)

2.4 Regularization method

In order to reduce mesh size effect on results, the regularization concept proposed by Hillerborg [1], is used. When crack localisation appears in the material, large strains take place along a finite element band. The energy dissipated in the localisation band is mesh size dependant. In order to get a constant dissipated energy for any mesh the damage evolution laws use a corrected equivalent tensile strain \( \tilde{\varepsilon}^* \) instead of the original equivalent tensile strain \( \varepsilon \). The corrected driver variable \( \tilde{\varepsilon}^* \) is defined by:
\[ \tilde{\varepsilon}^* = \tilde{\varepsilon} \quad \text{if} \quad \tilde{\varepsilon} \leq \varepsilon_p \quad (5) \]

\[ \tilde{\varepsilon}^* = \varepsilon_p + (\tilde{\varepsilon} - \varepsilon_p) \frac{L_e}{L_c} \quad \text{if} \quad \tilde{\varepsilon} \geq \varepsilon_p \quad (6) \]

$L_e$ is the finite element characteristic length (the square root of the element area). $L_c$ is a material internal length and $\varepsilon_p$ is the equivalent tensile strain obtained at the peak stress.

For a given loading, we can write:

\[ \varepsilon_p = \alpha_t \varepsilon_p^l + \alpha_c \varepsilon_p^c \quad (7) \]

$\varepsilon_p^l$ and $\varepsilon_p^c$ are strains at the peak tensile stress and at the peak compressive stress.

The regularization method modifies the post peak stress strain relation. It has an effect on tensile and compressive part of the material behaviour. Figure 2 gives an illustration of a numerical simulation with the Hillerborg concept. Using a fine mesh, damage localisation zones give a good idea of the crack pattern on a reinforced concrete beam under dynamic four point bending test [7].

Figure 2: Iso damage values of a reinforced concrete beam under bending test.
3 Experimental procedure

Experimental test have been conducted in order to evaluate the ability of the new model to predict the response of reinforced concrete plates under severe dynamic loading.

3.1 Reinforced concrete specimens

The circular concrete slabs which have been tested have a diameter of 1.3 meter and a thickness of 10 centimetres. These slabs are orthogonally reinforced in the top and bottom positions, at 1.5 cm of each surface. The diameter of the steel bars is \( d = 6 \text{ mm} \).

3.2 Shock tube facility

The slabs have been tested with the C.E.G blast simulator. In the shock tube, a plane shock wave is generated by bursting the diaphragm which separates the reservoir filled with compressed air from the expansion zone (figure 3). The concrete plates are placed at the end of the tube, perpendicularly to the shock wave direction (figure 3). The 2.4 m diameter shock tube allows to load samples with a maximum pressure of 0.66 Mpa.

3.3 Boundary conditions of concrete specimens

Two different boundary conditions have been studied (figure 3.(a) and 3.(b)):

- (a) correspond practically to a embedded slab due to the ring's rigidity
- (b) correspond practically to a simply supported slab

![Figure 3. Blast simulator lay-out](image1)

![Figure 4. Boundary conditions](image2)

3.4 Dynamic measurements

- Two pressure sensors in the slab plane allow to measure load history. These pressure profiles are introduced as data in numerical simulations.
- acceleration sensors on slabs, shock tube and support.
- slabs measurements:
  * local strain measurements are carried out by gauges bonded to reinforcement.
  * four displacement sensors to measure global displacements.
  * video cameras and 16 mm fast cameras (1000 images/second) record the slab during the shock and measure the central deflection.

4 Experimental and numerical comparisons

4.1 Identification procedure

The material properties of concrete and steel have been obtained with several characterisation tests on specimens:
- compressive tests (concrete's elastic modulus and compressive strength)
- split-cylinder (Brazilian test) and three points bending tests (concrete's tensile strength)
- tensile tests (steel's elastic modulus and yield stress)

<table>
<thead>
<tr>
<th>Concrete properties</th>
<th>Steel properties</th>
</tr>
</thead>
<tbody>
<tr>
<td>Young's modulus:</td>
<td>Young's modulus:</td>
</tr>
<tr>
<td>31 GPa</td>
<td>207 GPa</td>
</tr>
<tr>
<td>Poisson's ratio:</td>
<td>Poisson's ratio:</td>
</tr>
<tr>
<td>0.2</td>
<td>0.3</td>
</tr>
<tr>
<td>Compressive strength:</td>
<td>Yield stress:</td>
</tr>
<tr>
<td>32 MPa</td>
<td>660 MPa</td>
</tr>
<tr>
<td>Tensile strength:</td>
<td>Strain at failure:</td>
</tr>
<tr>
<td>3 MPa</td>
<td>2.5 %</td>
</tr>
<tr>
<td>(\sigma_{\text{f}})=crack closure stress:</td>
<td>Density:</td>
</tr>
<tr>
<td>-3 MPa</td>
<td>7850 Kg/m³</td>
</tr>
</tbody>
</table>

Density: 2323 Kg/m³

Strain rate parameters are identified from experimental results given by Suaris and Shah [3]: \(a_0=1.0, a_r=0.3, b_r=b_r=0.21\)

4.2 Finite element model

Computations are conducted using ABAQUS Explicit finite element code. Slabs with rings are modelled using 40 axisymmetric layered shell elements in which each element is composed of 3 concrete layers and 2 steel layers. The number of concrete integration points through the slab thickness is set to nine to follow the development of concrete's failure. Steel's behaviour is supposed to be elasto-plastic with perfect plasticity and a perfect link between concrete and steel bars is considered. Boundary conditions are applied on rings and unilateral contacts between reinforced concrete slab and steel rings are introduced (figure 4).
4.3 Experimental and numerical comparisons

Several days separate each shot. But in order to make easier the comparison between experimental and numerical results, successive shots have been reported on a reduced time scale, where a new shot starts, earlier, at the end of the response given by the previous one.

Experimental and numerical comparisons are presented for two reinforced concrete slab with same geometry and material properties but with different boundary conditions (figure 4 shows conditions (a) and (b)). Successive shots, with increasing maximum incident overpressure, allow to reach progressively the ultimate structure load and to follow damage evolution and the failure process. Under boundary conditions (a), figures 5 and 6 show central deflection versus time and central steel strain profiles for 4 successive shots at 0.17, 0.22, 0.3 and 0.4 Mpa. Moderate damage is obtained for these loads and numerical results seem to give a good prediction of slab’s behaviour. Permanent deflection and inelastic strain appear after each shot due to tensile concrete cracks. Yielding of steel reinforcement is not obtained with these loading.

![Experimental results](image1)

**Figure 5. Central deflection at successive shots**

![Numerical results](image2)

**Figure 6: Central reinforcement deformation at successive shots**
Under boundary conditions (b), figure 7 shows central deflection for 3 successive loads at 0.31, 0.38 and 0.48 Mpa.

![Graph](image1.png)

**Figure 7. Central deflection for 3 shots, boundary conditions (b)**

During the last test, at the slab centre, the cracks propagate rapidly through the thickness until collapse occurs when the steel bars yield (figure 8). Even after 3 tests on the same slab, numerical central deflection are in good agreement with experimental results (figure7). Failure of the reinforcement bars on the central part is predicted for the last shot. Other comparisons, such as the cracked area, the yield stress and the failure modes, are also analysed and found to be similar with the experiments.
5 Conclusions

A numerical and experimental procedure has been presented for predicting the non linear dynamic response of reinforced concrete slab under shock wave loading. The new concrete model uses a strain rate sensitive damage law which includes inelastic strains, crack closure and friction effects. This model can be used in conjunction with the Hillerborg regularization method to limit mesh size effect when localisation band appears. The explicit formulation of the two scalar concrete model gives very robust and fast computations to simulate failure of reinforced concrete structures. The constitutive model outlined in this paper gives a good estimate of the overall response of reinforced concrete plates submitted to several consecutive loads. Good agreement between numerical and experimental results are obtained for this structure under bending stresses. Finite element simulations gives satisfactory results for a large range of damage conditions going from small tensile cracks to the complete failure of the concrete slab.

6 References


