Interface effects for SPH impact computations
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Abstract

SPH (Smooth Particle Hydrodynamics) algorithms can introduce errors at free boundaries, material interfaces, attachment to finite element grids, and sliding interfaces on finite element grids. This paper identifies and evaluates these error sources for a variety of impact problems.

1 Introduction

SPH methods are now being used for a wide variety of impact problems. The basic SPH technique was first introduced by Lucy [1] and Gingold and Monaghan [2] in 1977. More recently, the effect of strength was added by Libersky and Petschek [3], axisymmetric algorithms were developed by Johnson, Petersen, and Stryk [4], and Petschek and Libersky [5], and a Normalized Smoothing Function (NSF) algorithm was developed by Johnson and Beissel [6]. SPH nodes have also been linked to finite elements by Johnson, Petersen, and Stryk [4, 7], and Attaway, Heinstein, and Swegle [8]. This paper identifies and evaluates errors at free boundaries, material interfaces, attachment to finite element grids, and sliding interfaces on finite element grids.

2 Background

Figure 1 represents some features of the SPH technique. Node i is designated as the center node and the neighbor nodes are designated as nodes j. The distance between nodes i and j is $r_{ij}$, the diameters of the nodes are $d_i$ and $d_j$, and
the masses of the nodes are $M_i$ and $M_j$. The masses remain constant throughout the computation, and are obtained from $M = \rho_0 V_0$ where $\rho_0$ and $V_0$ represent the initial density of the material and the initial volume represented by the node.

Figure 1: SPH characteristics.

For axisymmetric geometry, the three normal strain rates ($\dot{\varepsilon}_x, \dot{\varepsilon}_z, \dot{\varepsilon}_\theta$), the shear strain rate, $\dot{\gamma}_{xz}$, the rotational rate, $\Omega_{xz}$, and the volumetric strain rate, $\dot{\varepsilon}_v$, for center node $i$, are as follows [4, 6]:

\begin{align}
\dot{\varepsilon}_x &= -\sum_j \beta_x W_{ij} V_j (\dot{u}_j - \dot{u}_i) \ell_x / 2\pi x_j \\
\dot{\varepsilon}_z &= -\sum_j \beta_z W_{ij} V_j (\dot{v}_j - \dot{v}_i) \ell_z / 2\pi x_j \\
\dot{\varepsilon}_\theta &= -\sum_j \beta_\theta W_{ij} V_j r_{ij} \dot{\theta}_j / 4\pi x_j^2 \\
\dot{\gamma}_{xz} &= -\sum_j W_{ij} V_j [\beta_z (\dot{u}_j - \dot{u}_i) \ell_z + \beta_x (\dot{v}_j - \dot{v}_i) \ell_x] / 2\pi x_j \\
\Omega_{xz} &= \sum_j W_{ij} V_j [\beta_z (\dot{u}_j - \dot{u}_i) \ell_z - \beta_x (\dot{v}_j - \dot{v}_i) \ell_x] / 4\pi x_j \\
\dot{\varepsilon}_v &= \dot{\varepsilon}_x + \dot{\varepsilon}_z + \dot{\varepsilon}_\theta
\end{align}
where $W_{ij}' = \frac{\partial W_{ij}}{\partial t}$ is the derivative of the smoothing function, $V_j$ is the current volume of the node $j$, $u_i$ and $u_j$ are the $x$ velocities of nodes $i$ and $j$, $v_i$ and $v_j$ are the $z$ velocities, $\ell_x$ and $\ell_z$ are the direction cosines from node $i$ to node $j$, and $x_j$ is the $x$ coordinate of node $j$.

The three $\beta$ factors are used to normalize the smoothing functions such that they will provide the exact strain rates in the three principal directions for states of constant strain rates [6]. They are obtained from equations (1) - (3) by setting $u_j - u_i = \varepsilon_{xx} r_{ij} \ell_x$ for a constant strain rate in the $x$ direction, $v_j - v_i = \varepsilon_{zz} r_{ij} \ell_z$ for a constant strain rate in the $z$ direction, and $\dot{u}_j = \varepsilon_{\theta} x_j$ for a constant hoop strain rate. The resulting normalizing factors are

$$\beta_x = \frac{-1}{\sum W_{ij}' V_j r_{ij} \ell_x^2 / 2\pi x_j}$$

$$\beta_z = \frac{-1}{\sum W_{ij}' V_j r_{ij} \ell_z^2 / 2\pi x_j}$$

$$\beta_\theta = \frac{-1}{\sum W_{ij}' V_j r_{ij} / 4\pi x_j}$$

The nodal force in the $x$ direction on node $j$, due to the stress of node $i$, is

$$P_{ji}^x = P_{ji}^{x \text{ (plane)}} + P_{ji}^{x \text{ (hoop)}}$$

The force due to the in-plane stresses is

$$P_{ji}^{x \text{ (plane)}} = W_{ij}' V_j V_i \left[ \beta_x \left( \sigma_{xx} - Q_{ij} \right) \ell_x + \beta_z \tau_{xz} \ell_z \right] / 2\pi x_j$$

where $\sigma_{xx} = s_{xx} - \left( P_x + Q_{ij} \right)$ is the net normal stress in the $x$ direction, composed of the deviator stress, pressure, and nodal artificial viscosity, and $\tau_{xz}$ is the shear stress. This expression is slightly different from that presented previously [4]. The initial formulation for the forces included the term, $M_i M_j / \rho_j^2$, and this has been replaced by $M_i M_j / \rho_i \rho_j = V_i V_j$, where $\rho_i$ and $\rho_j$ are the current densities of the nodes.

There is also an artificial viscosity, $Q_{ij}$, which is dependent on the relative velocities of nodes $i$ and $j$ [9]. It is intended to stabilize the grid and keep adjacent nodes from becoming too close to one another. This will be
designated as a bond viscosity, because it acts on the bond between nodes i and j. It can also introduce a significant amount of artificial strength [6, 10].

The force due to the hoop stress is

$$P_{ij}^{\text{hoop}} = \beta_{ij} W_{ij} V_{ij} \frac{r_{ij} \sigma_{ij}^\theta}{4 \pi x_j^2} \tag{12}$$

and the force in the z direction is

$$P_{ij}^z = W_{ij} V_{ij} \left[ \beta_z \left( \sigma_{ij}^z - Q_{ij} \right) \ell_z + \beta_x \tau_{ij}^x \ell_x \right] / 2 \pi x_j \tag{13}$$

Equations (11) - (13) provide the forces only on the neighbor nodes j. The forces on center node i, due to the stresses in node i, are equal and opposite to the in-plane forces in the x and z directions.

$$P_{ii}^x = - \sum_j P_{ij}^x \quad \text{(plane)} \tag{14}$$

$$P_{ii}^z = - \sum_j P_{ij}^z \tag{15}$$

3 Examples

All of the examples in this paper use the quadratic smoothing function and a normalized smoothing distance of $\alpha = 1.0$ (2.0 nodal diameters) [6]. Unless noted otherwise, they also use the NSF algorithm [6] and the bond viscosity [9, 10].

Figure 2 shows equivalent strain rates in a radially-stretching ring of material in axisymmetric geometry. The stretching velocity is $u = 3x / 2$ and the corresponding strain rates in the x and $\theta$ directions are $\dot{\varepsilon}_x = 3 / 2$ and $\dot{\varepsilon}_\theta = 3/2$. The resulting equivalent strain rate is $\bar{\varepsilon} = 1.0$ [6]. The left side of Figure 2 shows how the SPH equivalent strain rates can be significantly low on the boundaries ($0.35 \leq \bar{\varepsilon} \leq 0.68$), and also somewhat low in the interior ($\bar{\varepsilon} = 0.87$), when the smoothing functions are not normalized ($\beta_x = \beta_z = \beta_\theta = 1.0$). When the NSF algorithm is applied, the results are much more accurate, as shown on the right side of Figure 2.

The upper two cylinder impact computations in Figure 3 show the effect of the NSF algorithm when a nodal artificial viscosity is used. The outline of the deformed cylinder and the plastic strain contours on the left side of the cylinder come from a finely gridded standard finite element computation. The darkened
Figure 2: Equivalent strain rates in stretching cross-sections.

Figure 3: Cylinder impact computations.
SPH nodes are for plastic strain regions of 0.25–0.75 and 1.25–1.75, the same regions as shaded for the finite element computation. The results with the NSF algorithm show excellent agreement with the finite element solution. When the bond viscosity is used instead of the nodal viscosity, additional artificial strength is introduced. Unfortunately, not all problems can be run with nodal viscosity only, and some form of a bond viscosity is often required [9, 10].

Another important observation is that the finite element computation and the two SPH computations with the NSF algorithm have a higher strained region (0.75–1.25) on the outer surface near the impacted end of the cylinder. This higher strained region does not exist in the two computations without the NSF algorithm because of the boundary errors illustrated in Figure 2. Clearly the NSF algorithm improves the accuracy in both the interior regions and at the free boundaries.

Errors can also be introduced for the three interface conditions shown in Figure 4. The upper portion of Figure 4 shows two different SPH materials in contact. If the materials are not bonded together, then the errors are unacceptably large because the standard SPH algorithm develops both shear and tension at the interface. The center and lower portions of Figure 4 show SPH nodes attached to, and sliding on, a standard finite element grid. These cases are discussed in more detail by Johnson [7], and are addressed in subsequent examples.

The upper left portion of Figure 5 shows a high strength steel penetrator (L = 88.9 mm length and D = 12.9 mm diameter) perforating a 6061-T651 aluminum plate (T = 26.3 mm thickness). The initial striking velocity is \( V_s = 500 \) m/s and the computed residual exit velocity at 200 \( \mu \)s is \( V_r = 405 \) m/s, which is in good agreement with experimental results [11]. This problem is ideally suited for the linkage of SPH nodes and standard elements. It allows for severe distortions in the softer SPH material, a structural response in the harder standard element material, and a well defined sliding interface between the two materials.

The lower right portion of Figure 5 shows the same problem, except that both the penetrator and target are represented by SPH nodes. In this case the penetrator is significantly deformed and it does not perforate the target plate \( (V_r = 0) \). This occurs because the standard SPH algorithm treats materials as if they are bonded together, and this results in high shear stresses at the interface. Clearly, if SPH approaches are to be applied to problems such as this, then some special sliding interface algorithm will need to be developed.
Figure 4: Description of various interface conditions.
Figure 6 shows two wave propagation computations for water impacting a rigid surface at $V_s = 300$ m/s. The results are shown at the centerline of a plane strain grid that has sufficient width to not allow any signals from the side boundaries. This essentially provides a state of one dimensional strain. The impacting end of the water is represented by quad elements and the other end is represented by SPH nodes such that the wave passes through the interface. The computation on the left side of Figure 6 has the SPH nodes attached to the finite element grid in a manner similar to that shown in the center portion of Figure 4. The computation on the right side utilizes a sliding interface similar to that shown in the lower portion of Figure 4.

In both cases the wave has the correct velocity and it oscillates about the correct pressure. The oscillation at the leading edge of the wave could be damped by using higher artificial viscosity coefficients, rather than the baseline coefficients [10]. The reflected wave for the sliding interface occurs because the effective mass at the interface (standard node plus SPH node) is greater than the other nodal masses. Also, in both cases, the SPH node at the interface has a lower pressure. This is due to the NSF algorithm that computes higher $\beta$ factors at the SPH boundary nodes, as discussed previously. These $\beta$ factors increase the forces on the boundary nodes, and the result is that the pressure is reduced to provide the proper forces to support the wave. This problem could probably be corrected by allowing the SPH interface nodes to use nodal data from the standard finite element grid [7], or by utilizing SPH ghost nodes on the standard element side of the interface. Both of these approaches could be very complex, however, and the current capability is probably adequate for many applications, (such as that shown on the left side of Figure 5).

## 4 Summary and Conclusions

This paper has provided an assessment of the accuracy of various interface conditions for SPH computations. Free boundaries are handled in an accurate manner by using the NSF algorithm. When SPH nodes are attached to a standard finite element grid, or are allowed to slide along the standard grid, the example wave propagation and penetration computations provide a level of accuracy that is probably acceptable for most applications. The most significant errors occur for conditions where two adjacent SPH materials are allowed to slide along one another in an unbonded condition. This case requires the development of a special interface algorithm.
**SPH nodes and standard elements**

**Steel penetrator**
- $L = 88.9\,\text{mm}$
- $D = 12.9\,\text{mm}$
- $V_s = 500\,\text{m/s}$
- $V_r = 405\,\text{m/s}$

**Aluminum target**
- $T = 26.3\,\text{mm}$

**Figure 5:** Impact of a steel penetrator onto an aluminum plate.

**Figure 6:** Wave propagation through attached and sliding interfaces.
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References