Strength effects in long-rod penetration
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Abstract

The paper reviews our recent work concerning the influence of target and penetrator strength on the penetration process of long-rod penetrators. This work has been done with the Eulerian processor of the 2-D program PICSES 2DELK. Using these simulations, we were able to critically examine various features of penetration model of Alekseevskii and Tate, which has been considered the most successful analytical model for the past 25 years. Also, the issue of geometric scaling is discussed (for the axisymmetric case of penetration into semi-infinite targets). We present some data to enhance our assumption which relates the nonscaling phenomenon with the semi-brittle nature of the failure mechanisms of the rod material. Finally, a set of simulations is presented in which the role of the strain to failure is demonstrated. This strain to failure should be linked with physical properties which characterize adiabatic shear processes.

1 Introduction

The penetration of long-rod projectiles into semi-infinite metallic targets is one of the basic issues in terminal ballistics. For almost 30 years the 1-D analytical model of Alekseevskii [1] and Tate [2] (henceforth termed the AT model) has played a major role in analyzing test results with penetrators having length-to-diameter (L/D) greater than 10. Recently, some major predictions of this model have been shown to contradict experimental findings which prompted an extensive 2-D simulation work to better understand the complex interaction between long-rods and metallic targets. The purpose of the present paper is to review our recent work which was aimed to investigate these discrepancies, between the model and experiments, using the Eulerian processor of the PISCES-2DELK code [3]. It is convenient to present the results of these simulations in terms of the normalized penetration (P/L, where
P is penetration depth and \( L \) the projectile length. Briefly, the 1-D analytical model of References [1] and [2] builds on the simple hydrodynamic model of penetration, which was developed in the mid 40's, to account for shaped-charge jets penetrations into thick steel targets. The AT model adds two strength parameters for the target \( (R_t) \) and the penetrator \( (Y_p) \), starting with a modified Bernoulli equation:

\[
\frac{1}{2} \rho_p (V-U)^2 + Y_p = \frac{1}{2} \rho_t U^2 + R_t
\]

where \( U \) and \( V \) are penetration and tail velocities, respectively. Two more equations are needed to describe the erosion rate and deceleration of the rigid part of the penetrator:

\[
\frac{dL}{dt} = -(V-U)
\]

\[
\rho_p L \frac{dV}{dt} = -Y_p
\]

in order to form a set of equations which is integrated numerically to obtain the various features of the penetration. Note that the simple hydrodynamic model is the special case of the AT model for \( R_t = Y_p = 0 \). Also, if one neglects only the strength of the penetrator \( (Y_p=0) \) one obtains the successful model, of the mid 50's, for penetration of jets into armored steels.

We shall also deal with other issues which are related to strength effects, such as the issue of geometrical scaling in terminal ballistics and the difference between the penetration performance of depleted uranium (DU) and tungsten-alloy (WA) penetrators. These issues are related to the maximum strain to failure of the rod material.

### 2 The influence of L/D

The most important prediction of the AT model is that for each penetrator-target combination a single curve can be constructed for the variation of normalized penetration \( (P/L) \) versus impact velocity, for all long-rods. However, the experimental work of Hohler and Stilp [4] resulted in large differences between the normalized penetrations of long-rods with \( L/D=10, 20 \) and \( 30 \). The rods were made of either tungsten alloy or steel while the targets were armored steels. Clearly, this major discrepancy between model and experiment had to be accounted for. Our first simulations (described in Reference [5]) used Johnson and Cook [6] material models to describe the properties of both tungsten-alloy and armored steel with a setup which insured that the target is indeed "semi-infinite" in both lateral and thickness dimensions. We used seven cells on the penetrator radius which seemed to be enough from the point of view of convergence. Figure 1 summarizes our results in terms of normalized penetration \( (P/L) \) versus penetrator's aspect ratio.
(L/D) for two tungsten alloy penetrators (density of 17.1 g/cc) having yield strengths of 1.25 GPa and 0 GPa. The impact velocity in these simulations was 1.4 km/s. We shall see later that the same trends also occur for other impact velocities.

![Figure 1: Simulation Results for the Normalized Penetration (P/L) versus Length-to-Diameter ratio (L/D) at 1.4 km/s (from [5])](image)

As is clearly seen, the regular penetrator (Y=1.25 GPa) penetrates less efficiently as its aspect ratio increases, even up to L/D=40. Comparing these results with the experimental studies (both in [4] and ours [5]) shows a very good agreement between simulation and experiment. Thus, both experiments and 2-D simulations "agree" that a single normalized penetration curve cannot be constructed, in contrast with the prediction of the AT model. Moreover, our simulations with a zero-strength penetrator resulted in a crossover phenomenon, as shown in Figure 1, according to which at very high aspect ratios (L/D>30) the penetration capability of zero strength rod is superior to that of the regular rod. This (counter-intuitive) result can be explained by realizing the fact that increasing penetrator strength improves its penetration depth by a small amount. On the other hand, the deceleration of the rear part of the penetrator is a strong function of its strength since it is due to elastic waves which move back and forth along its rigid part. These waves, with an amplitude of Y (yield strength of rod material), carry an increment of velocity decrease of ΔV=Y/ρC, where ρ and C are the density and sound speed of the rod. Thus, with each stress reverberation, the rear portion of the rod loses 2•Y/ρC of its velocity (which can amount to several tens of meters per
second). If the rod is long enough its back portion can decelerate by several hundreds of meters per second, resulting in a much lower penetration. For the zero-strength rod we observe an asymptotic behavior of $P/L$ with the aspect ratio, as expected. The initial decrease is due to the entrance phase which is more significant for the shorter rods. During the first stages of penetration (entrance phase) the target is less resistant and the relative penetration is higher as compared with the later parts of the penetration process.

3 Optimal penetrator strength

It is clear from the above discussion that the strength of the penetrator plays a double role as far as penetration depth is concerned. On the one hand, the stress which it exerts on the target is increased (augmenting penetration), but the rear portion decelerates more strongly, diminishing its penetration capability. This opposing trend can lead to an optimum in the penetration vs. strength curve for a given penetrator-target configuration. In Reference [7] we investigated this phenomenon for tungsten alloy penetrators with strengths in the range of 0-3.5 GPa impacting steel targets at velocities in the range of 1.4 to 2.2 km/s. Figure 2 shows our simulation results for $L/D=10$, 20 and 30 penetrators at 1.4 km/s, from which clear maxima at $Y=0.8$ GPa can be deduced. It is worth noting that here again we find the $L/D$ dependence of $P/L$ for each penetrator strength in the range investigated. Such optima do not result from the AT model which predicts an ever increasing depth of penetration with penetrator strength. Similar optima were found for other impact velocities and the dependence of the optimal strength on impact velocity is much stronger than its dependence on the $L/D$ ratio (see [7]).

![Figure 2: Simulation Results for L/D=10 to 30 Penetrators at V₀ = 1.4 km/s](image.png)
4 A critical examination of the AT model

In order to critically evaluate the AT model we performed an extensive study [8,9] with numerous simulations of the penetration process for different long-rods into various targets. Our aim was to check the validity of the basic assumptions of this model, namely, the existence of a steady-state during penetration and the validity of the assignment of strength parameters to the penetrator and to the target ($Y_p$ and $R_1$ in eqn. (1)).

In the first stage of this investigation (see Reference [8]) we used simulations with only zero strength penetrators ($Y_p=0$) in order to investigate the properties of the target. Figure 3 shows the results from a typical simulation for an L/D=10 tungsten-alloy rod impacting a steel target at 2.2 km/s. One can clearly see that the tail velocity is practically constant for the whole penetration process, as expected, while the head velocity ($U$) stabilizes after about 20 microseconds (the entrance phase). Thus, to a first degree of approximation, the basic assumption of a steady-state process is justified for the case of zero strength penetrators.

![Figure 3. Simulation Results for the Tail and Head Velocities of a Zero Strength Tungsten-Alloy Penetrator (from [8])](image)

The main output of these simulations was the value of penetration velocity ($U$) which was used in the modified Bernoulli equation (eqn. (1)) to calculate $R_t$ values for each case. We found that the resulting $R_t$ values are independent of penetrator density in the range of 5-17 g/cc while a slight increase in $R_t$, of about 2.5%, was obtained for target densities changes in the range of...
5-15 g/cc. Using a range of materials for both targets and penetrators (steel, tungsten-alloy, copper and aluminum) we were able to find a correlation between the $R_t$ values as deduced from $U$ and eqn. (1) and the yield strengths of the targets ($Y_t$), which varied in the 0.4 to 2 GPa range. In all these simulations a simple elasto-plastic yield model (von Mises) was used for the targets, in order to avoid complication arising from strain rate or strain hardening effects. The expression we derived is:

$$R_t = B \cdot Y_t \left(1 + \frac{1}{2} \ln \frac{2E}{3Y_t}ight)$$  \hspace{1cm} (4)

where $E$ is Young’s modulus and $B$ is a slowly decreasing function of impact velocity between 1.4 to 1.15 in the 1-7 km/s range. This expression is similar to several analytical model predictions, based on cavity expansion analyses (see Reference [10]). Our conclusion from the first part of this study (Reference [8]) is that, as long as zero-strength penetrators are considered, the assumptions of the 1-D model are justified. Particularly, the penetration is a steady-state process for most of the time, and the assignment of a single strength parameter to the target, to describe its penetration resistance, is valid. This $R_t$ value can be considered as a true material property, depending only slightly on impact velocity. This fact is probably the main reason for the success of the hydrodynamic ($R_t = Y_p = 0$) and quasi-hydrodynamic ($Y_p = 0$) theories in accounting for the penetration depths of shaped charges into semi-infinite targets of various strengths.

The next step in our investigation (Reference [9]) was to introduce a strength to the penetrator ($\sigma_{yp}$) and check, through 2-D simulations, whether this results in a valid parameter ($Y_p$) for the AT model. This was done by solving the modified Bernoulli equation, as a quadratic equation in $U$:

$$U = \frac{V - \mu(V^2+A)^{1/2}}{1 - \mu^2}$$  \hspace{1cm} (5)

where $\mu^2 = \rho_p/\rho_t$ and $A$ is given by:

$$A = \frac{2(R_t - Y_p)(1 - \mu^2)}{\rho_t}$$  \hspace{1cm} (6)

Once the penetrator has a finite strength both its tail and head velocities ($V$ and $U$) decrease with time. A measure for the steadiness of the penetration process can be obtained by the time variation of $A$ when computed through eqn. (5). If $A(t)$ turns out to be relatively constant throughout the whole penetration process then one can use eqn. (6) to calculate the corresponding $Y_p$ knowing all the rest of the parameters. In particular, $R_t$ is known for each target from our previous work [8]. Our simulations showed that if the input yield strength of the penetrator ($\sigma_{yp}$) is not too large ($\leq$1.6 GPa) the penetration process is relatively steady and a meaningful value for $A$ can be extracted, from which $Y_p$ is determined through eqn. (6).
However, examining these $Y_p$ values as a function of impact velocity shows an appreciable increase, even for the relatively narrow range of 1.4 to 2.6 km/s, as shown in Figure 4.

![Figure 4: The Variation of $Y_p$ with Impact Velocity for a $\sigma_{yp} = 1.2$ GPa Tungsten-Alloy Penetrator and a Steel Target ($Y_t = 0.8$ GPa)](image)

Moreover, using various L/D rods we could also check for the uniqueness of $Y_p$ as a function of L/D. Figure 5 below shows that for an impact velocity of 1.8 km/s, $Y_p$ changes quite markedly with the penetrator's aspect ratio (L/D). This fact, together with the strong velocity dependence of $Y_p$, as shown in Figure 4, clearly show that $Y_p$ is not a valid parameter. Moreover, as Figure 5 shows, $Y_p$ decreases with increasing L/D which increases the term $R_L Y_p$ in eqn. (1). Thus, the effective strength of the target increases with L/D, resulting in lower normalized penetrations for the longer rods. Therefore, the reason for the experimental finding, of decreasing P/L with increasing L/D, is this increase in the effective penetration resistance (due to the decrease in $Y_p$). In summary, as far as the AT model is concerned, we could claim that the introduction of $Y_p$, as such, is the cause for discrepancies between the model on the one hand, and both simulations and experiments on the other.
2.2- 

1.84 

1.6- 

1.4- 

1.2 

0 10 20 30 40 

\(Y_p\) (GPa) 

L/D 

The dependence of \(Y_p\) on L/D for a \(\sigma_{yp} = 0.8\) GPa Tungsten Alloy Rod at 1.8 km/s (from [9])

5 Geometric scaling in terminal ballistics

The issue of geometric scaling is extremely important for experimenters in terminal ballistics since most of the research in this field is being done with scaled projectiles and targets (usually 1:3 and 1:4). To our best knowledge the first observation, in the open literature, of nonscaling effects, were those of Magness & Leonard [11] who show a 10% increase in the penetration capability of long-rods for each doubling of the scale (normalized penetration depths of 1:4 penetrators are less by 20% than full scale ones). Since these differences were found for both DU (Depleted Uranium) and tungsten alloys, impacting both armored and mild steels, the authors of [11] could not pin-point the reason for the non-scaling effect.

In order to better understand this phenomenon we performed a small number of experiments with L/D=10 copper and tungsten-alloy rods, at two different scales (6 mm and 12 mm in diameter). Targets were large 4340 steel cylinders treated to a hardness of 220 BHN. Prior to performing the experiments we have performed a simulation study on the effect of target size (diameter and thickness) on the penetration depth, see [12]. We found that in order to avoid lateral release effects targets should be 25 times larger than projectile’s diameter. Also, the thickness of the target should be at least 1.6
times the value of the penetration depth into a semi-infinite target, in order to avoid back surface influence.

Since some of the steel targets turned out to have a different hardness than planned we had to correct some of the data to the reference values of penetration into 220 BHN targets. This was accomplished by using the small scale (1:4) experiments for which the targets had lower (or higher) hardness via a correction factor giving the extra penetration per 10 units of BHN. Figure 6 shows the results of this study for both copper and tungsten-alloy penetrator. The numbers in squares, near each point, designate the hardness of the targets. The open squares correspond to the data corrected to the 220 BHN hardness. One can clearly see that while the large copper penetrators' data coincide with the smaller ones, for the tungsten-alloy two penetration curves are needed; one for each scale. The difference between the two lines is about 10% which agrees very well with Reference [11]. Thus, copper, which is a typical ductile metal, penetrates hydrodynamically with no scaling problems, while the tungsten alloy shows a clear non-scaling effect.

A possible explanation for these findings is based on fracture mechanics considerations, particularly on the onset of brittle failure with large specimens. Ivanov [13] discusses this effect quite extensively defining a characteristic length $l = 4\pi r_y$, where $r_y$ is the plastic zone extension ahead of a crack, which is a material parameter given by:

$$r_y = \frac{1}{2\pi} \frac{K_{IC}^2}{\sigma_y}$$

where $K_{IC}$ and $\sigma_y$ are the fracture toughness and yield strength of the specimen. For specimen dimensions greater than $l$, a brittle fracture is expected and vice versa. Using published values for these parameters result in $l$ values of 1 m for copper and 5 mm for tungsten-alloys. Thus, copper rods should penetrate hydrodynamically at all scales while tungsten-alloys could have a ductile-brittle transition in their failure mode when the scale is increased.

Thus, our interpretation of non-scaling effects in terminal ballistics is based on a transition from ductile to brittle failure of the penetrator head, when the scale is increased. The failure type can affect the shape of the penetrator which upon brittle failure can become more pointed, leading to a better penetration.

This self-sharpening mechanism is very similar to the one proposed by Magness and Farrand [14] to account for the difference between the penetration capability of DU and WA long-rods. It turns out that this 10% difference, for impact velocities in the range of 1.4 to 1.8 km/s, can be explained by the adiabatic shear failure which the DU penetrator experiences. We assume that a similar tendency to self-sharpening occurs with the large scale, semi-brittle penetrators while the small scale penetrator retains its mushroom shaped head.
Figure 6: Our Results for (a) Copper and (b) Tungsten-Alloy Penetrators at Two Scales
6 The role of maximum strain to failure

A recent simulation study, which we conducted, was aimed to find the sensitivity of the penetration depth of long-rods to their mechanical/physical properties. We varied yield and flow stresses, spall stress and shear moduli in a reasonable range of values to find only minor variations in penetration depths. However, the maximum equivalent plastic strain (the failure strain) resulted in appreciably deeper (and narrower) cavities. Reducing the value of this parameter to the range of 0.2-0.4 resulted in an increase of 20% in the penetration depth as compared with a simulation in which no constraint is put on this parameter. Figure 7 shows the results for typical simulations of a tungsten-alloy rod (L/D=20, L=300 mm) impacting an RHA target at 1.7 km/s. The experimental value for the penetration depth was 306 mm, which is simulated with \( e_F = 0.2 \). The curve has an asymptotic behavior, both for low and large values of \( e_F \), and the most drastic changes in penetration occur for \( e_F \) in the range of 0.5 to 1.0. The failure strain \( e_F \) of the penetrator is a measure of its ductility. Thus, one can link various experimental phenomena with \( e_F \). For example, we saw that fracture mechanics predicts a ductile to brittle transition as scale is increased (for some materials). This process can be quantified by adjusting decreasing \( e_F \) values to larger scale, resulting in more penetration, as found in the experiments. Also, the differences between the penetration capability of tungsten-alloy and depleted uranium, which have been attributed to the adiabatic shear phenomenon in [14], can be quantified by \( e_F \) in simulation work. The tendency to form adiabatic shearing should decrease \( e_F \), leading to a higher penetration capability.

![Figure 7. The Variation in Penetration Depth with the Strain-to-Failure for a WA Rod at 1.7 km/s and RHA Target](image-url)
It is understood that the failure process cannot be adequately described by a one parameter instantaneous failure model.

References