Multiple shot analysis in shot-peening using finite elements
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ABSTRACT

A computer program OBAID has been used to analyze complex target plate and allows for dynamic loading due to impinging single and multiple shot. Theoretical model using three dimensional isoparametric finite element has been presented, in which a provision is made for the surface hardening and cyclic impact loading caused by multiple shots. Theoretical model agrees reasonably well with other models.

INTRODUCTION

Shot peening is viewed as a process involving multiple and progressively repeated impact. The object is to include compressive residual stresses in the surface of the target. A general form of residual stress in a typical shot-peening operation is shown in Figure 1. The impact of spherical shots, which usually takes place at a high velocity - 100 m/sec, also causes plastic deformation which results in changes in the mechanical properties in the surface material.

An extensive work has been carried out by several authors [1-12] to model the process. An attempt to modelize the process, in viewing as process involving multiple repeated impact. When shots impinge upon a target of elastic/plastic material and the impact velocity is sufficiently high, the target material below each impact undergoes local plastic deformation and, upon rebound, the rest of the elastic material tends to push the plastically deformed zone resulting in compressively applying numerous impacts over the whole surface, the target acquires a shallow layer of superficial compressive residual stress below which it develops smaller equilibrating tensile stresses.

This paper attempts to present theoretical model using three dimensional isoparametric finite element, as will be seen, it surprisingly agrees reasonably well with other model. [1,2].

THREE-DIMENSIONAL FINITE ELEMENT MESH GENERATION

The finite element mesh for the target plate is shown in Fig. 2. In order to produce accurate residual stresses, it has been decided to divide the thickness of the target steel plate into nine layers (in the Y-direction). The plate thickness below the
neutral axis is divided into two equal layers each of 0.0079 m. Immediately above the neutral axis, the thickness of the first layer is 0.00397 m. Above this layer there are six equal layers each of 0.00198 m thickness. A flexibility exists for various plane meshes of variable dimensions. A quarter of the actual size of the target plate is chosen for the shot-peening analysis based on the finite element technique. Hence a mesh contains elements and nodes of the target, as shown in Fig. 3. The numbering scheme adopting is based on a right hand rule. Each element is numbered together with eight specific Gauss points (2 x 2 x 2) in the manner shown in Fig. 4.

THE IMPACT LOAD ANALYSIS

An attempt has been made to study the mechanics of a single and multiple shot indentation due to both static and dynamic loading using theoretical model and the computer program OBAID [1,3,4]. The shot is assumed rigid and undeformable while the target material receives indentations. Low and high velocity ranges have been considered together with the contact time.

The shot velocity ranges come from the data available from a series of test carried out by the author. These have been simulated in order to process specific results.

It is essential to dilate upon the basic criterion. The rigid steel shot is assumed to strike, with a gradually increasing load, on the steel target plate. This increasing load is imposed on each surface element, one at a time. The patch load in effect acts, during loading and unloading, on each surface element, exciting all the other elements through the connectivity model which links all the elements together. The second loading is then applied on the adjacent surface element, but this time all the residual displacements and residual stresses from the previous impact are allowed for. Therefore, the response of the target will be different. By storing and updating the state of the target the cumulative result is then plotted in a three-dimensional space. The process is repeated until the whole surface is covered more than once, so that each spot is loaded several times, the number of times depending upon the duration of the process.

GENERAL STEPS BY USING FINITE ELEMENT ANALYSIS

1. Apply a load increment, \( \{ \Delta F_n \} \), where \( n \) is the load increment
2. Accumulate total load \( \{ F_n \} = \{ F_{n-1} \} + \{ \Delta F_n \} \) and \( \{ R \} = \{ F_n \} \), where \( \{ R \} \) is the residual load vector.
3. Solve \( \{ \Delta u_i \} = [K]^{-1}\{ R \} \), where \( i \) is the iteration.
4. Accumulate total displacements
   \[ \{ U_i \} = \{ U_{i-1} \} + \{ \Delta U_i \} \]
5. Calculate strain increments as
\[
\{\Delta \varepsilon_i\} = [B]\{\Delta U_i\}
\]
and strains as
\[
\{\varepsilon_i\} = \{\varepsilon_{i-1}\} + \{\Delta \varepsilon_i\}.
\]

6. The stress increments are calculated using the current non-linear constitutive matrices. For steel target plate elastoplastic relations are expressed in the general form.
\[
\{\Delta \sigma_i\} = \{f(\sigma)\}\{\Delta \sigma\}.
\]
Accumulate stress as
\[
\{\sigma_i\} = \{\sigma_{i-1}\} + \{\Delta \sigma_i\}.
\]

ISP = stress point
= 0-elastic point
= 1-plastic point
= 2-unloading from plastic state

\(\sigma_y\) = uniaxial yield stress.

6.1 Firstly, the stress increment is calculated using the elastic material matrix as
\[
\{\Delta \sigma_i\} = [D]^e\{\Delta \varepsilon_i\},
\]
where \([D]^e\) is the elastic material matrix for steel target plate. First estimate of total stress:
\[
\{\sigma_i\} = \{\sigma_{i-1}\} + \{\Delta \sigma_i\}.
\]

6.2 Calculate
\[
\{\bar{\sigma}_i\} = \{f(\bar{\sigma})\}
\]
\[
\{\bar{\sigma}_{i-1}\} = \{f(\bar{\sigma}_{i-1})\}
\]
using von Mises yield criterion or another yield criterion.

6.3 If plastic point (i.e. ISP=1), go to step 6.5.

6.4 If \(\bar{\sigma} \geq \sigma_y\) -plastic point (ISP=1), transition from elastic to plastic, calculate factor \(f_{TR}\):
\[
\begin{align*}
f_{TR} &= \frac{\sigma_y - \sigma_{i-1}}{\bar{\sigma} - \sigma_{i-1}}. \\
\end{align*}
\]
Stress at yield surface
\[
\{\sigma_i\}^y = \{\sigma_{i-1}\} + f_{TR} \times \{\Delta \sigma_i\}.
\]
Calculate elasto-plastic stress increment
\[
\{\Delta \sigma_i\} = [D]^ep\{\sigma_i\}^y \times (1-f_{TR})\{\Delta \varepsilon\}.
\]
Total stress
\[
\{\sigma_i\} = \{\sigma_i\}^y + \{\Delta \sigma_i\}.
\]
Surface Treatment Effects

Set ISP=1; go to 6.7.
If \( \sigma_i < \sigma_y \) -elastic point, \( \sigma_i = \sigma_i' \); go to step 6.8.

6.5 Plastic point in the previous iteration, check for unloading, i.e. \( \sigma = \sigma_y \) (see Fig. 5) go to step 6.6. Unloading at this point, set ISP=2, total stress.
\[ \{\sigma_i\} = \{\sigma_i-1\} + \{\Delta\sigma_i\} \]
and set
\[ \{\sigma_y\} = \{\sigma_i-1\}; \]
go to step 6.8

6.6 Loading at this point
\[ \{\Delta\sigma_i\} = [D]^g_{ep}\{\sigma_i-1\}\{\Delta\varepsilon\}. \]
Total stress
\[ \{\sigma_i\}_T = \{\sigma_i-1\} + \{\Delta\sigma_i\}. \]

Let \([D]\) be the stiffness matrix of an element in the global direction at the beginning of the increment of the \(n\)th iteration within the increment; then
\[ \{\Delta\sigma_n\}_{xyz} = [D]^t_{xy}\{\Delta\varepsilon_n\}_{xyz}, \]
where \(\{\Delta\sigma_n\}_{x}\) and \(\{\Delta\varepsilon_n\}_{x}\) are the resulting stresses and strains of the element in this iteration. The total stresses at the end of the iteration in this element are given by
\[ \{\Delta\sigma_n\}_{xyz} = \{\sigma_n-1\}_{xyz} + \{\Delta\sigma_n\}_{xyz}, \]
where \(\{\sigma_n-1\}_{xyz}\) is the total balanced stress at the previous \((n-1)\)th iteration. Let the principal stresses corresponding to global total stress be \(\{\sigma_n\}_{p}\), i.e.
\[ \{\sigma_n\}_{p} = [T_e^{-1}]^t\{\sigma_n\}_{xyz} \]
where
\[ [T_e] = \begin{bmatrix}
  c^2 & s^2 & sc \\
  s^2 & c^2 & -sc \\
  -2sc & 2sc & c^2 - s^2
\end{bmatrix} \]
\[ [T_e]^{-1} = \begin{bmatrix}
  c^2 & s^2 & -sc \\
  s^2 & c^2 & sc \\
  -2sc & -2sc & c^2 - s^2
\end{bmatrix} \]
\(s = \text{sine and } c = \text{cosine} \)

From the constitutive relations in the principal direction the stresses are given by
\[ \{\sigma_n\}_p = \{\{\varepsilon_n\}\}. \]
The unbalanced stresses in the principal direction are given by

\[ \{\Delta \sigma_n\}_p = \{\sigma_n\}_p - \{\sigma_n\}_p \]

and balanced stresses in global direction \((x)\) are found as

\[ \{\Delta \sigma\}_x = [T_e]_x^T \{\Delta \sigma_n\}_p \]

These unbalanced stresses in the global direction are treated as initial stresses and are subtracted from the total stresses \(\{\sigma_n\}_x\) to obtain balanced stresses as

\[ \{\sigma_n\}_x = \{\sigma_n\}_x - \{\Delta \sigma_n\}_x \]

To maintain equilibrium, the equivalent balanced nodal forces are calculated as

\[ \{F_n\}_x = \int_v \{B\}_x^T \{\Delta \sigma_n\}_x \, d \text{vol}, \]

and these forces are applied to the material surrounding the target. Having determined the incremental initial stresses at the integration points, the computation of incremental initial loads, and the iterative procedure of the resulting set of equations, is carried out in the manner explained above. To evaluate stresses and strains at the Gaussian integration points throughout the whole target an eight-point integration point mesh for isoparametric solid elements is adopted.

6.7 Stresses calculated using the elastoplastic material matrix do not drift from the yield surface, as shown in Fig. 5. The following correction is taken into consideration, based on the equivalent stress-strain curve:

\[ \bar{\sigma}_{\text{corr}} = \bar{\sigma}_{i-1} + H \Delta \bar{\varepsilon}_p \]

where

\[ \Delta \bar{\varepsilon}_p = \sqrt{\frac{2}{3}} \Delta \varepsilon_{ij}^p \Delta \varepsilon_{ij}^p = \frac{2}{3} \lambda^T a \lambda \]

= equivalent plastic strain increment.

\(H\) is the strain hardening parameter, such that \(\Delta \varepsilon_p = \lambda\). Equivalent stress is calculated from the current stress state.

\[ \{\bar{\sigma}_i\} = f(\{\sigma_i\}) \]

\[ \text{factor} = \frac{\bar{\sigma}_{\text{corr}}}{\sigma} \]

Therefore the correct stress on the yield surface is

\[ \{\sigma_i\} = \text{factor} \times \{\sigma_i\} \]

6.8 End
7. The total stresses are converted into equivalent internal loads from
\[
\int_{\Omega} [B]^T \{\sigma_i\} \, d \text{vol}
\]
and the residual load vector is calculated from
\[
\{R\} = \{F_n\} - \int_{\Omega} [B]^T \{\sigma_i\} \, d \text{vol}.
\]

8. Check for convergence. Two types of convergence criteria are used. They are the residual and the displacement convergence. The Euclidean norms are tested as
\[
\|R\| \leq \text{TOL}
\]
\[
\|\Delta U\| \leq \text{TOL},
\]
where
\[
\|R\| = \sqrt{\langle R^T, R \rangle}
\]
is the Euclidean norm of the residuals;
\[
\|F_{\text{ext}}\| = \sqrt{\langle F_{\text{ext}}^T, F_{\text{ext}} \rangle}
\]
is the Euclidean norm of the externally applied load;
\[
\|\Delta U\| = \sqrt{\langle \Delta U^T, U^T \rangle}
\]
is the Euclidean norm of the incremental displacement;
\[
\|U\| = \sqrt{\langle U^T, U \rangle}
\]
is the Euclidean norm of the total displacements and the tolerance limit for impact is taken as 0.01.

If the convergence is not achieved, go to step 3 and repeat all the steps for the next iteration. If the convergence is achieved, then go to step 1 and repeat the process with the next load increment.

RESULTS

All results have been plotted in three dimensions for three increments of loading from the output. The graphs plotted are the results of residual stresses; they show a three-dimensional mesh and three planes, namely the X-Y plane, the X-Z plane and Y-Z plane, are indicated on them. All graphs are along the thickness of the layers viewed in the Y-Z plane.

For the same mesh grading system under increment 2 both static and dynamic residual stresses viewed in the Y-Z plane are plotted (Figs 6 and 9). The magnitude
and positions of such stresses vary in both cases. Similarly, under increment 3 the residual stresses are plotted viewed first in the Y-Z plane (Figs 8 and 9). In the case of increment 3 much higher residual stresses are predicted. For comparison the residual stresses for the increment viewed in the X-Y plane are also dawn.

Such information is better visualized in the form of a distribution at the central plane through the thickness, as shown in Fig. 10. For comparison, the distribution as predicted from other models [1,2] is also shown.

It is concluded that the real plot of three-dimensional residual stresses can be plotted on the three-dimensional mesh as when viewed in specific planes they would, in general, form a distribution curve of a cosine function.

CONCLUSIONS

A three-dimensional dynamic finite element analysis has been carried out to take into consideration elastoplasticity. Computer program OBABID has been used together with theoretical model to obtain the results. The residual stress distributions - due to multiple shot - obtained from finite element analysis are in a good agreement with those obtained by others [1,2].

REFERENCES


Surface Treatment Effects


Fig. 1. General form of residual stress distribution in shot-peening operation

Fig. 2. Finite element mesh for target plate
Fig. 3. Elements and nodes of the target plate

Fig. 4. 20-noded Isoparametric element together with eight specific Gauss points
Fig. 5. (a) Equivalent-stress-strain curve for steel plate
   (b) Yield surface in the principal stress axes
Fig. 6. Residual Stress distribution. Increment 2. Section A-B

Fig. 7. Residual stress distribution. Increment 2. Section B-C
Fig. 8. Residual stress distribution. Increment 3. Section A-B

Fig. 9. Residual Stress distribution. Increment-2. Section B-C
Fig. 10. Comparative study of residual stress for increment 2 in X-Y plane