Propagation of a fatigue crack through a protective layer

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Abstract

The contribution describes a study of the behavior of a fatigue crack growing in a protective layer and penetrating through the interface and into the substrate. The study aims especially to quantify the influence of the interface on the rate and threshold value of a fatigue crack propagating perpendicularly to the interface. Special attention is devoted to the case of a crack touching the interface.

It is shown that the fatigue crack propagation rate and the corresponding threshold value are strongly influenced by the existence of an interface between two materials. A tentative procedure suggested in the paper makes it possible to quantify the effect and determine the dependence of a rate of propagation of a fatigue crack growing through the interface on the bi-material parameters α and β. The results generally contribute to a better understanding of the failure of structures with protective layers.

1 Introduction

Real bodies often contain regions that possess different mechanical properties. Components with protective coatings or other kinds of surface treatment provide an example. The proximity of a region with a different elastic modulus along with the presence of an interface have a pronounced influence on the fracture of composite bodies (e.g. Jones [1], Menčík [2]). The interface between two dissimilar media represents a weak point for many applications of structures. Fracture usually starts at a defect in the interface, especially at an interface
microcrack or at the edge of the interface. Another important problem is the influence of an interface on a crack penetrating through it.

This contribution describes a study of the behavior of a fatigue crack growing in one material and penetrating through the interface and into a second material. The study aims especially to quantify the influence of the interface on the rate of propagation of the fatigue crack. Special attention is devoted to the case of a crack touching the interface, see Fig. 1, and the problem of determining the threshold value for a fatigue crack with its tip at the interface is solved. The problem is studied under the assumptions of linear-elastic fracture mechanics.

A comprehensive theoretical treatment of the corresponding boundary value problem, see Fig. 1., exists in the literature, and formulas that describe the displacement and the stress distribution in the vicinity of a crack terminating at the interface have long been known (e.g. Fenner [3], Lin [4], Meguid [5]). The stress singularity for a crack terminating at the interface is of the type $r^{-p}$, where the value of the stress singularity exponent $p$ is in the interval $0 < p < 1$. In contrast to cracks in homogeneous bodies, where $p = 1/2$, for a fatigue crack that terminates at an interface, $p \neq 1/2$, and current fracture mechanics arguments and procedures cannot be used to describe the behavior of such a crack (e.g. Knesl [6]).

This study aims to quantify the influence of the interface on the rate of propagation of a fatigue crack.

The main goals are to suggest a fracture mechanics procedure that describes the rate of propagation of a fatigue crack with its tip at an interface and to find a procedure for estimating the threshold value for such a crack.

Because the most critical loading for the propagation of a crack is typically the opening mode I, a corresponding far field loading configuration with a crack perpendicular to the interface is considered in the following, see Fig. 1. Only the possibility of a fatigue crack penetrating into a second material is studied here, deflection of the crack along the interface is not considered. Furthermore, the
displacements and tractions at the interface are assumed to be continuous, i.e., the interface is of a welded type.

2 Propagation of a fatigue crack at an interface

If the tip of a crack is in a homogeneous material, the stress singularity exponent \( p \) equals \( 1/2 \) and a conventional fracture mechanics model can be used to describe the propagation of the fatigue crack. Under small scale yielding conditions (i.e., in the high cycle fatigue range) the rate of propagation of a fatigue crack is controlled by the value of the range of the stress intensity factor \( \Delta K \) and can be expressed in the form

\[
da/dN = f(\Delta K, K_{th})
\]

where \( K_{th} \) is the corresponding threshold value of the stress intensity factor. This is usually expressed in the form of the Paris law

\[
da/dN = C(\Delta K^m - K_{th}^m).
\]

where \( C \), \( m \), and the threshold value \( K_{th} \) are material parameters (e.g. Suresh [7], Klesnil [8]).

If a crack approaches and penetrates the interface, three different cases may be distinguished, see Fig.2. Case (a) corresponds to a crack that approaches the interface, case (b) is a crack that touches the interface (with its tip at the interface), and case (c) corresponds to a crack that has penetrated the interface.

As long as the crack tip is in a homogeneous material (cases a and c, Fig.2), the value of the stress intensity exponent has the value \( 1/2 \) and a conventional fracture mechanics approach can be used, as described above. However, the existence of the interface, i.e., the proximity of a region with a different stiffness, strongly influences the magnitude of the stress intensity factor (e.g. Atkinson [9], Chiang [10]) and thus the rate of the fatigue crack propagation. It follows from theoretical considerations [2] that for any ratio of \( E_1/E_2 \), the value of the stress intensity factor increases if the crack approaches a material with a lower elastic modulus, and decreases if it approaches a stiffer material. The stress intensity factor must be determined by using numerical methods.

![Figure 2: The possible forms of propagation of a crack perpendicular to the interface.](image-url)
If the crack touches the interface (Fig. 2(b)), the exponent singularity is $p \neq 1/2$. If the crack lies in a stiffer material ($E_1 > E_2$), the value of the stress singularity exponent $p$ will be greater than 1/2. Conversely if $E_1 < E_2$ then $p < 1/2$. There exists no general procedure for describing the rate of propagation of a fatigue crack with a general value of the stress intensity singularity exponent. In this paper it is suggested that the controlling variable for the fatigue crack propagation rate in such cases is size of the plastic zone around the tip of the crack. The effective value of the range of the stress intensity factor $\Delta K_{eff}$ calculated by using the equality of the sizes of the plastic zones in homogeneous and bi-material bodies is then used to estimate the rate of propagation quantitatively. The threshold value is determined in a similar way. The stress distribution around the crack tip must be known in order to estimate the corresponding size of the plastic zone.

2.1 Stress near the tip of a crack that terminates at an interface

Only the case of a crack perpendicular to an interface, see Fig.1, is discussed here. It can be shown [4,5] that the distribution of stress around the tip of a semi-infinite crack that terminates at an interface and is oriented perpendicularly to that interface is (for plane strain conditions and in material M2)

$$\sigma_{xx} = \frac{H_i}{\sqrt{2\pi}} r^{-p} \left\{ \lambda [(2f_R - g_R) \cos(\lambda - 1)\theta - (2f_I - g_I) \sin(\lambda - 1)\theta - \right.$$

$$\left. - (\lambda - 1)(f_R \cos(\lambda - 3)\theta - f_I \sin(\lambda - 3)\theta)] \right\}$$

$$\sigma_{yy} = \frac{H_i}{\sqrt{2\pi}} r^{-p} \left\{ \lambda [(2f_R + g_R) \cos(\lambda - 1)\theta - (2f_I + g_I) \sin(\lambda - 1)\theta + \right.$$

$$\left. + (\lambda - 1)(f_R \cos(\lambda - 3)\theta - f_I \sin(\lambda - 3)\theta)] \right\}$$

$$\sigma_{xy} = \frac{H_i}{\sqrt{2\pi}} r^{-p} \left\{ \lambda [g_R \sin(\lambda - 1)\theta + g_I \cos(\lambda - 1)\theta + \right.$$

$$\left. + (\lambda - 1)(f_R \sin(\lambda - 3)\theta + f_I \cos(\lambda - 3)\theta)] \right\}$$

$$\sigma_{zz} = \nu \left( \sigma_{xx} + \sigma_{yy} \right)$$

where $H_i$ is a generalized stress intensity factor, $\nu$ is Poisson's ratio, and

$$\lambda = p - 1, \quad f_R = 1, \quad f_I = g_I = 0$$

$$g_R = \lambda - \cos \lambda \pi \frac{\beta(\alpha + 2\lambda - (1 + 2\alpha - 4\alpha \lambda^2) \cos \lambda \pi + (1 + \alpha) \cos 2\lambda \pi)}{D(\lambda)}$$

$$D(\lambda) = 1 + 2\alpha + 2\alpha^2 - 2(\alpha + \alpha^2) \cos \lambda \pi - 4\alpha^2 \lambda^2$$

The parameters $\alpha$ and $\beta$ are bi-material parameters [4].
Figure 3: The plastic zone near the tip of a crack in the material M2.

\[
\alpha = \frac{E_1 \cdot \frac{1 - \nu_1}{1 + \nu_2} - 1}{4(1 - \nu_1)}, \quad \beta = \frac{E_1 \cdot 1 - \nu_2^2}{E_2 \cdot 1 - \nu_2^2}
\]

The exponent of stress singularity \( p \) is given by the solution of the equation

\[
2(1 - \cos 2p\pi) \{(1 + \alpha)^2 + (\alpha - \beta)^2 + 2(1 + \alpha)(\beta - \alpha)\cos 2p\pi\} = 0.
\]  

(4)

Thus the stress distribution around the crack tip can be expressed in the general form

\[
\sigma_y = H_I / \sqrt{2\pi f_0}(\phi, p, \alpha, \beta) / r^p
\]  

(5)

For given materials and loading conditions the stress distribution around the crack tip is given by the value of the generalized stress intensity factor \( H_I \). Note that for homogeneous materials \( H_I = K_I \).

The shape of the plastic zone predicted by linear elastic fracture mechanics for small scale yielding near the crack tip in the material M2, see Fig.3, can be obtained by substituting the stresses found by using eqn (3) into the Mises yield condition:

\[
\sigma_{ef} = \sigma_0,
\]

where \( \sigma_{ef} \) is the effective stress and \( \sigma_0 \) is the yield stress in simple tension for the material M2.

The location of the curve corresponding to the elastic-plastic boundary is then given in polar coordinates \((r, \theta)\) as the radius of the plastic zone \( r_p \):

\[
r_p = \left( \frac{H_I}{\sigma_0} \right)^{\frac{1}{p}} \left( \frac{1}{2\pi} \right) [6g_\theta p \left( \sin(p\theta) \sin((-p - 2)\theta) - \cos(p\theta) \cos((-p - 2)\theta) \right] + 16\nu \cos(p\theta) \left( \nu + \frac{1}{4\nu} - 1 \right) + 3 \left( g_\theta^2 + p^2 \right)]^{\frac{1}{2p}}
\]  

(6)
2.2 Estimation of the threshold value

If a fatigue crack stops when it reaches the interface, its stress distribution in the material M2 is given by eqn (3) and the condition for its propagation into the material M2 can be written in the form

\[ H_{i} > H_{\text{th}} \]  \hspace{1cm} (7)

where \( H_{i} \) is the corresponding value of the generalized stress intensity factor and \( H_{\text{th}} \) is its threshold value. Note that for a fatigue crack in a homogeneous body the corresponding condition has the form \( K_{i} > K_{\text{th}} \), where \( K_{\text{th}} \) is the threshold value of the material.

In accordance with the above considerations, the behavior of a fatigue crack with its tip at an interface is determined by the plastic zone created in material M2, see Fig. 3.

In the following, the area of the plastic zone \( R_{p} = R_{p}(H_{i}) \) in material M2 is considered to be the controlling variable.

For a homogeneous body, the value of the area \( R_{p} \) is given by (e.g. Unger [11])

\[ R_{p} = \left( \frac{K_{\text{th}}}{\sigma_{0}} \right)^{4} f_{\text{hom}}(\nu), \]  \hspace{1cm} (8)

\[ f_{\text{hom}}(\nu) = \left[ \frac{64 \left( 12 \nu^{4} \pi - 24 \nu^{3} \pi + 21 \nu^{2} \pi - 9 \nu \pi + 32 \nu^{3} + 52 \nu \pi - 20 \nu + 3 \right) + 123 \pi}{1024 \nu^{2}} \right] \]

where \( K_{i} \) is the corresponding value of the stress intensity factor and \( \sigma_{0} \) is the value of the yield stress of the material.

By analogy, in the case of a crack with its tip at the interface it can be written that

\[ R_{p} = \left( \frac{H_{\text{th}}}{\sigma_{0}} \right)^{4} f(\alpha, \beta, \nu) \]  \hspace{1cm} (9)

where \( f(\alpha, \beta, \nu) \) is a known function obtained by integrating of eqn (6).

In the following it is assumed that if \( K_{i} = K_{\text{th}} \) and \( H_{i} = H_{\text{th}} \), the areas of the plastic zones in homogeneous and bi-material bodies are the same. Thus the value of \( H_{\text{th}} \) can then be obtained using the equality of the areas of the plastic zones in homogeneous and bi-material bodies. Then

\[ R_{p}(H_{i} = H_{\text{th}}) = R_{p}(K_{i} = K_{\text{th}}) \]

and

\[ H_{\text{th}} = K_{\text{th}}^{2p} \sigma_{0}^{(1-2p)} \left[ \frac{f_{\text{hom}}(\nu)}{f(\alpha, \beta, \nu)} \right]^{\frac{1}{2}} \]  \hspace{1cm} (10)
where $\sigma_y$ is the yield stress of material M2. The values of the function $f(\alpha, \beta, \nu)$ are obtained by integration of eqn (6).

If $H_1 < H_{1h}$, the rate of propagation of the fatigue crack will zero, i.e., the fatigue crack will stop at the interface and will not propagate into the substrate. This condition makes it possible to determine the critical applied stress $\sigma_{\text{crit}}$ as a function of the composite parameters $\alpha$ and $\beta$. If the value of the applied stress is $\sigma_{\text{appl}} < \sigma_{\text{crit}}$, then the fatigue crack will not propagate into the substrate.

2.3 The rate of propagation of a fatigue crack with its tip at an interface

If $H_1 > H_{1c}$, the crack will grow into the material M2. It is supposed that the controlling variable which determines the rate of its propagation is again the area $R_p$ of the plastic zone in material M2.

In the previous case the threshold value was expressed in terms of the generalized stress intensity factor $H_1$. The rate of the fatigue crack propagation in material M2, $da/dN$, must be expressed in terms of the effective value of the range of the stress intensity factor $\Delta K_{\text{eff}}$ because it is possible to determine $da/dN$ directly by means of eqn (1). The effective value $\Delta K_{\text{eff}}$ is again given by the equality of the sizes of the plastic zones in homogeneous and bi-material bodies, i.e.,

$$R_p(\Delta K_{\text{eff}}) = R_p(H_1).$$

(11)

where the left side of eqn (11) corresponds to a homogeneous body and is given by eqn (8), whereas the right side is given by eqn (9). The relation between $\Delta K_{\text{eff}}$ and $H_1$ then has the form

$$\Delta K_{\text{eff}} = H_1^{\frac{1}{2p}} \sigma_0^{\frac{2p-1}{2p}} \left( \frac{f(\alpha, \beta, \nu)}{f_{\text{hom}}(\nu)} \right)^{-\frac{1}{4}}$$

(12)

The rate of propagation of a fatigue crack that touches the interface is then given by eqn (1), where $\Delta K = \Delta K_{\text{eff}}$, i.e.,

$$da/dN = C (\Delta K_{\text{eff}})^m.$$  

(13)

where $C$ and $m$ are material constants corresponding to the material M2 and measured for the homogeneous case.

The influence of the interface on the rate of propagation of a fatigue crack can be expressed as the ratio of the rates of propagation corresponding to bi-material and homogeneous bodies,

$$\frac{(da/dN)_{\text{bi-mat}}}{(da/dN)_{\text{hom}}} = \left( \frac{K_{\text{eff}}}{K_1} \right)^m.$$  

(14)
3 Numerical example

In order to describe the behavior of a fatigue crack with its tip at the interface between two different materials the values of the corresponding variables $H_{th}$ (the generalized threshold value, see eqn (10)) and $\Delta K_{th}^{eff}$ (the effective value of the range of the stress intensity factor, see eqn (12)) must be determined. The procedure for their estimation consists of the following steps:

1. Estimate the bi-material parameters $\alpha$ and $\beta$ and the value of the singularity exponent $p$, eqn (4).

2. Calculate the value of the function $f(\alpha, \beta, v)$ by integrating eqn (6).

3. Estimate the value of the generalized stress intensity factor $H_i$ for the given geometry and boundary conditions. This must be done numerically, e.g., by the finite element method.

4. Express $H_{th}$ eqn (10) for the given values of $K_{th}$ and $\sigma_{th}$ corresponding to the material M2.

5. If $H_i < H_{th}$, the crack will stop at the interface and the critical applied stress $\sigma_{crit}$ corresponding to the condition $H_i = H_{th}$ can be calculated. If $H_i > H_{th}$ then can be calculated $\text{d}a/\text{d}N$ by means of Eq.13. Material parameters $C$ and $m$ correspond to the material M2.

As an example of the approach presented, the results concerning a fatigue crack growing perpendicularly in a protective coating and penetrating through the interface and into a substrate material (Fig.4) are presented in the following. First, the critical applied stress $\sigma_{crit}$ corresponding to the fatigue threshold value is calculated.

Let us assume for simplicity that $v_1 = v_2 = 0.3$. Then both composite

![Figure 4: The dimensions and geometry of the tension specimen with a protective surface layer. $T = 15 \text{ mm}$, $t = 1 \text{ mm}$, $l = 30 \text{ mm}$, $\sigma_{appl} = 300 \text{ MPa}$.](image-url)
Table 1. The values of \( p \), \( H_t \), \( H_{th} \), \( \sigma_{\text{crit}} \) and \( K_t^{\text{eff}}/K_t \) as a function of the ratio \( E_1/E_2 \) for \( \nu_1 = \nu_2 = 0.3 \). The results correspond to the plane strain approximation.

<table>
<thead>
<tr>
<th>( E_1/E_2 )</th>
<th>( \alpha )</th>
<th>( \beta )</th>
<th>( \rho )</th>
<th>( H_t ) [MPa m(^{3})]</th>
<th>( H_{th} ) [MPa m(^{3})]</th>
<th>( \sigma_{\text{crit}} ) [MPa]</th>
<th>( K_t^{\text{eff}}/K_t )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>0.321</td>
<td>0.1</td>
<td>0.33277</td>
<td>17.45e-2</td>
<td>36.01</td>
<td>206.35</td>
<td>1.43</td>
</tr>
<tr>
<td>0.2</td>
<td>0.286</td>
<td>0.2</td>
<td>0.3662</td>
<td>13.90e-2</td>
<td>25.04</td>
<td>180.25</td>
<td>1.31</td>
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<tr>
<td>0.5</td>
<td>0.179</td>
<td>0.5</td>
<td>0.43389</td>
<td>9.02e-2</td>
<td>11.53</td>
<td>127.78</td>
<td>1.13</td>
</tr>
<tr>
<td>1.0</td>
<td>0</td>
<td>1.0</td>
<td>0.5000</td>
<td>6.28e-2</td>
<td>5.0</td>
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</tr>
<tr>
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<td>2.0</td>
<td>0.5745</td>
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<td>1.78</td>
<td>39.58</td>
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</tr>
<tr>
<td>5.0</td>
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<td>5.0</td>
<td>0.67885</td>
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<td>0.79</td>
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<td>10.0</td>
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<td>0.7536</td>
<td>3.15e-2</td>
<td>0.11</td>
<td>3.28</td>
<td>0.78</td>
</tr>
</tbody>
</table>

The results correspond to the plane strain approximation. Parameters \( \alpha \) and \( \beta \) depend only on the ratio of the values of Young's modulus \( E_1/E_2 \). The corresponding values of the stress singularity exponents are summarized for various values of \( E_1/E_2 \) in Table 1. Then the values of the generalized stress intensity factors \( H_t \) are calculated by using the finite element system ANSYS. It holds that

\[
H_t (\sigma_{\text{appl}}) = \sigma_{\text{appl}} H_t (1\text{MPa}),
\]

where the values of \( H_t (1\text{MPa}) \) are also given in Table 1. It is further assumed that the yield stress of the substrate is \( \sigma_0 = 600 \text{ MPa} \) and the fatigue threshold value is \( K_{th} = 5 \text{ MPa m}^{1/2} \). Then the generalized fatigue threshold \( H_{th} \) can be determined using eqn (10), see Table 1. In the end the critical stress \( \sigma_{\text{crit}} \) is calculated on the condition that \( \sigma_{\text{crit}} H_t (1\text{MPa}) = H_{th} \).

As the next step the rate of propagation of a fatigue crack growing into the substrate is estimated, see Fig.4. Let us assumed that the value of the applied stress is \( \sigma_{\text{appl}} = 300 \text{ MPa} \). Note that the corresponding value of the stress intensity factor for a homogenous body is \( K_t = 18.83 \text{ MPa m}^{1/2} \), which is greater than the assumed threshold value \( K_{th} = 5 \text{ MPa m}^{1/2} \).

4 Conclusion

A tentative procedure that deals with the propagation of a fatigue crack through the interface between two materials is suggested in the paper. Special attention is devoted to the case of a fatigue crack that touches the interface. This procedure makes it possible to quantify the effect of the interface on the threshold value and on the rate at which a fatigue crack propagates from the first material into the second perpendicularly to the interface.

It follows from the results presented that rate of propagation of a fatigue crack and the corresponding threshold value can be strongly influenced by the existence of an interface between two materials.
The procedure and corresponding numerical approach are illustrated using the case of a protective layer on a substrate. see Table 1. The results obtained contribute to a better understanding of the damage such a crack causes in composite materials.

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References