Modern geopenetrators and relevant revision of concrete penetration models

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Abstract

Penetration experiments on concrete targets are interpreted with empirical formulae and with the cavity expansion theory. The empirical models predict significantly too low penetration depths. The cavity expansion theory gives an agreement between theory and experiment within less than 4%. The applied material model parameters for concrete are discussed.

1. Introduction

The performance of penetrators against concrete can be significantly increased by an improved contour of the penetrator. During the last years TDW was successful in developing penetrator designs with an increased penetration capability in concrete accompanied with absolutely stable flight trajectories. The higher penetration could not be predicted with available empirical models from the literature because these models are based on experiments with conventional penetrator designs. It was therefore necessary to look for other models describing the penetration process. The cavity expansion theory (CET) is one possibility to improve the predictive capabilities without too much numerical expenses.

As an example impact tests against concrete (strength between 35 to 50 Mpa) in the impact velocity range from 510 to 560 m/sec were analysed with empirical formulae and the cavity expansion theory as well.
2. Penetration experiments

The tests were performed with penetrators having a high strength steel body with an outer diameter of 60 mm. The penetrator is a special TDW design developed for penetration in geological materials as soil, rock and concrete. The total length is about 510 mm and the mass 6 kg. The impact velocities are shown in table 1; one shot (No. 2) was performed with 509 m/s and the other five shots in the range 556 to 563 m/s.

The targets are built up of unreinforced cylindrical concrete blocks with a diameter of 1 m and a length of 1 m. The strength of the concrete material is measured at different locations within the target. An average value for the compressive strength is given in table 1. The greatest grain diameter of the concrete aggregate is 16 mm. The measured penetration depths are also listed in table 1. A good comparison can be made between trials 1 and 5 (nearly same velocities and same concrete strengths give comparable depths) and trials 3, 4 and 6 by the same reason.

Table 1: Test data and results

<table>
<thead>
<tr>
<th>Test Ref. No.</th>
<th>Penetrator mass [kg]</th>
<th>Impact velocity [m/s]</th>
<th>Concrete strength [MPa]</th>
<th>Penetration depth [m]</th>
<th>Young modulus [GPa]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>6.013</td>
<td>556</td>
<td>34.8</td>
<td>1.34</td>
<td>29.5</td>
</tr>
<tr>
<td>2</td>
<td>6.039</td>
<td>509</td>
<td>34.8</td>
<td>1.145</td>
<td>29.5</td>
</tr>
<tr>
<td>3</td>
<td>5.980</td>
<td>563</td>
<td>46.3</td>
<td>1.11</td>
<td>33.5</td>
</tr>
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<td>4</td>
<td>6.023</td>
<td>563</td>
<td>46.3</td>
<td>1.105</td>
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</tr>
<tr>
<td>5</td>
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<td>560</td>
<td>34.3</td>
<td>1.28</td>
<td>29.5</td>
</tr>
<tr>
<td>6</td>
<td>5.986</td>
<td>562</td>
<td>51.3</td>
<td>1.08</td>
<td>35.3</td>
</tr>
</tbody>
</table>
3. Penetration models

3.1 Empirical models

3.1.1 Empirical formulae

This section gives a collection of different concrete penetration formulae which are taken from the literature.

The modified PETRY formula, referenced in [1], calculates the penetration depth $P$ [inch] in a thick concrete target:

$$P = 12 \cdot K_p \cdot A_p \cdot \log_{10} \left(1 + \frac{V^2}{215000}\right) \quad (1)$$

The meaning of the symbols is as follows: $K_p$ is a dimensionless constant depending on the concrete strength with a value of 0.00426 for normal strength concrete (about 35 MPa). $A_p$ is the impactor mass divided by its caliber area, given in [lb/ft$^2$]. $V$ is the impact velocity in units of [ft/s].

The next formula is from the Army Corps of Engineers (ACE) (ref. [1]):

$$P = \left(\frac{282 \cdot D^{12.15} \cdot \rho_c}{Y^{0.5}} \cdot \frac{V}{1000}\right)^{1.5} + 0.5 \cdot D \quad (2)$$

This calculates also the penetration depth $P$ [inch]. $D$ is the penetrator diameter [inch], $\rho_c$ is the so-called caliber density, the penetrator mass divided by the third power of the diameter [lb/in$^3$]. $Y$ is the concrete compressive strength [psi]. $V$ is again the impact velocity, given in [ft/s].

The third formula is a modified version of the famous NDRC formula given in ref. [2]. For penetration depths lower than $2D$ the penetration depth $P$ [cm] is calculated by

$$P = 0.21 \cdot \frac{N \cdot M}{Y^{0.5}} \cdot \left(\frac{V}{D}\right)^{1.8} \quad (3a)$$

$N$ is a dimensionless nose factor depending on the penetrator’s nose shape. $M$ is the mass of the penetrator in [kg]. $Y$ is the concrete
Compressive strength given in [bar]. The impact velocity $V$ has herein the dimension $[\text{m/s}]$ and the penetrator caliber $D$ is measured in $[\text{cm}]$. For penetration depths greater than $2D$ the following formula is used:

$$P = (0.0525 \cdot \frac{N \cdot M}{Y^{0.5}} \cdot \left(\frac{V}{D}\right)^{1.8}) + D \quad (3b)$$

with the same symbols and dimensions as given above.

Another concrete penetration formula was given by in ref. [3]. As first step an auxiliary function is calculated:

$$G = \frac{120328}{Y^{0.5}} \cdot \frac{N \cdot M}{D^{2.8}} \cdot \left(\frac{V}{1000}\right)^{1.8} \quad (4a)$$

$N$ is the dimensionless nose factor, $M$ the penetrator mass $[\text{kg}]$, $Y$ is the concrete compressive strength in $[\text{kPa}]$, $D$ is the penetrator diameter $[\text{cm}]$. The impact velocity $V$ has the dimension $[\text{m/s}]$. In a second step, the penetration depth $P$ $[\text{cm}]$ is given by

$$P = 2D \sqrt{G} \quad \text{if} \quad P \leq 2D \quad \text{or} \quad \sqrt{G} \leq 1 \quad (4b)$$

$$P = D (1+G) \quad \text{if} \quad P > 2D \quad \text{or} \quad \sqrt{G} > 1 \quad (4c)$$

The last penetration formula in this collection was established by the Waterways Experiment Station (WES), see ref. [4]. The penetration depth $P$ $[\text{m}]$ is given by

$$P = \frac{N \cdot M}{\rho \cdot A} \cdot \left(\frac{V \cdot R_y}{3} - 0.4444 \cdot \ln(1 + 0.75 \cdot V \cdot R_y)\right) \quad (5a)$$

The relevant quantities are: the dimensionless nose factor $N$, the penetrator mass $M$ $[\text{kg}]$, the density of the target (concrete) material $\rho$ $[\text{kg/m}^3]$, the caliber section area of the cylindrical part of the penetrator $A$ $[\text{m}^2]$ and the impact velocity $V$ $[\text{m/s}]$. $R_y$ $[\text{s/m}]$ is a quantity describing the strength of the target material and is calculated from
\[ R_y = \sqrt{\frac{\rho}{YR}} \]  

wherein \( YR \) is a function of the concrete compressive strength \( Y \) [Pa] and the ratio of the greatest diameter of the aggregate DAG \([m]\) to the penetrator caliber \( D \) [m]:

\[ YR = Y \left( \frac{3 \cdot DAG}{D} \right)^{0.2} \]  

### 3.1.2 Comparison with experimental results

For the penetrator presented in section 2 (diameter 6 cm, mass 6 kg) penetration predictions are made using the five empirical models of section 3.1.1, varying the compressive strength of concrete (see fig. 1 with \( Y = 35 \) MPa and fig. 2 with \( Y = 46 \) MPa). In these figures the penetration depth \( P \) [m] is plotted vs. the impact velocity in the region from 400 to 700 m/s. Concerning the penetrator, a nose factor of 1.33 is used as a realistic value for a conical nose with a nose length / caliber ratio of about 3. In the diagrams also the experimental results of the tests (see table 1) are included. The experimental penetration depths are located considerably above the empirical curves. An analysis of fig. 1 shows that even the best formula (WES model) is 30 \% below the experimental results. These discrepancies show the importance of other models for an adequate description of concrete penetration processes.

### 3.2 Cavity expansion model

#### 3.2.1 Theoretical background

The cavity expansion formalism, presented in [5], fills the gap between the empirical penetration formulae and numerical simulation. It is based on the assumption that the resistive pressure exerted on a penetrator is equivalent to the pressure on the surface of an expanding spherical cavity. This concept is applied to the cratering process of the projectile and it is assumed that the crater is formed by a series of spherical cavity expansions initiated at the tip of the moving penetrator. Recovered penetrators show at the nose and tail portion a considerable amount of abrasion caused by contact with the target.
material, but a relatively clean portion around the central part of the penetrator body. This seems to suggest that the cavity produced by the high speed penetrator is the result of the pushing action of the nose portion of the intruding penetrator against the target material. By elastic recovery this material reattaches to the tail portion of the penetrator.

The equation of motion of the projectile is given by

\[
M \cdot \frac{d^2 Z}{dt^2} + \int_0^Z 2 \cdot \pi \cdot p \cdot c \cdot \sin^2 \phi \, dc = 0
\]

(6)

\(M\) is the penetrator mass. \(Z\) is the penetration depth as the distance between the penetrator’s nose tip to the target surface. \(p\) is the pressure exerted on the surface of the cavity. \(c\) is the distance from the penetrator tip to the cavity initiation point. \(\phi\) is the half cone angle of the penetrator nose. The pressure \(p\) is expressed as

\[
p = p_s + 2 \cdot \rho_l \cdot \sin^2 \phi \cdot \left( B_1 \cdot c \cdot \frac{d^2 Z}{dt^2} + 2 \cdot B_2 \cdot \left( \frac{dZ}{dt} \right)^2 \right)
\]

(7)

\(p_s\) means the shear resistance and the second term gives the dynamic pressure from inertial effects. \(\rho_l\) is the locked density of the target material. \(B_1\) and \(B_2\) are constants depending on the material properties of the target.

The target is modeled as a homogeneous material with an elasto-plastic behaviour, linear strain hardening under shear stress and ideal compressible locking characteristics. For such a material the shear resistance \(p_s\) and the constants \(B_1\) and \(B_2\) have the form

\[
p_s = \frac{2}{3} \cdot Y \cdot \ln \delta + \frac{4}{9} \cdot E_t \cdot \left( \frac{\pi^2}{6} - \sum_{m=1}^{\infty} \frac{\delta^m}{m^2} \right) + \frac{4}{3} \cdot \beta \cdot E
\]

(8)

\[
B_1 = 1 - \frac{\alpha}{\delta^{2/3}}
\]

(9)

\[
B_2 = \frac{3}{2} - \frac{\alpha}{\delta^{2/3}} \cdot \left( 2 - \frac{(1 - \delta)^2}{1 - \alpha} \right) + 0.5 \cdot \delta^{4/3} \cdot \left( 1 - \frac{(1 - \alpha / \delta)^2}{1 - \alpha} \right)
\]

(10)

with
\[ \alpha = -\varepsilon_1, \quad \beta = \frac{Y}{2 \cdot (E - E_t)}, \quad \delta = \alpha + 3 \cdot \beta \]

\(\varepsilon_1\) is the locking strain accompanying the transition from the elastic to the plastic state and is taken to be constant in the present model. \(E\) is the Young modulus for the concrete material, which depends on density and compressive strength. \(E_t\) is the tangent modulus in the plastic region.

The cavity formation is assumed to take place up to the penetration depth \(Z_c = L + [(D/2) \tan \phi]\), where \(L\) is the length of the penetrator's nose. Only in this range eq. (7) is valid. For \(Z > Z_c\) the variable \(c\) is constant in time and \(c\) in eq. (7) has to be replaced by \(Z\).

The equations were applied to the nose profile of the actual penetrator and the equation of motion which gives an ordinary but nonlinear differential equation can be solved numerically using a Runge-Kutta procedure.

3.2.2 Calculation and comparison with experiments

For the application of the cavity expansion theory some data concerning the target material have to be specified. At first, the locking strain \(\varepsilon_1\) is given in [6] to be -0.08. (This corresponds to a locking density \(\rho_1 = \rho_1(1 + \varepsilon_1)\) of 2609 kg/m\(^3\), assuming the uncompressed density \(\rho\) to be 2400 kg/m\(^3\)). For the modulus of elasticity of concrete an empirical relationship from ref. [7] is used:

\[ E = 0.33 \cdot \rho \cdot \sqrt{Y} \]

where the uncompressed density \(\rho\) has to be inserted in [lb/ft\(^3\)] and the compressive strength \(Y\) in [psi]. The corresponding values for the test concrete targets are shown in table 1. The tangent modulus \(E_t\) has been varied in the range 0.2-0.3 GPa.

It turned out that the tangent modulus is a relatively sensitive parameter for the penetration depth calculation. Fig. 3 shows the penetration depth vs. the tangent modulus \(E_t\) using the target data from the tests (see table 1).

The experimental results are indicated in the diagram. The analysis of the results shows that the tangent modulus \(E_t\) depends on the target strength, too.
The $E_t$ values corresponding to the penetration depths of the tests No. 1, 2 and 5 lie together ($E_t = 0.233$ to $0.262$ GPa) and have equal compressive strength (about $34.5$ MPa); those of test No. 3, 4 and 6 form another group ($E_t = 0.290$ to $0.298$ GPa) having compressive strengths from $46.3$ to $51.3$ MPa.

4. Summary

Experimental impact tests of penetrators against concrete targets have been presented. The resulting penetration depth could not be interpreted with the empirical formula available in the literature. The predicted penetration was considerable lower than experiment (30% lower for the best empirical formula from WES). A much better agreement between theory and experimental values could be achieved with a model based on the cavity expansion theory (CET). The applied material model description for concrete showed that the tangent modulus in the plastic region is very important. It turned out that the tangent modulus correlates with the compressive strength of the concrete (similar to the Young modulus). The CET model reproduces the experimental penetration depths with an accuracy of less than $4\%$, which is a significant improvement with respect to the empirical equations.

References:


Fig.: 1 Concrete Penetration, experiments 1, 2, 5 and empirical equations