A strain history dependent model for concrete
J. Eibl, B. Schmidt-Hurtienne
Institute of Concrete Structures and Building Materials,
University of Karlsruhe, Postfach 6980, D-76128 Karlsruhe,
Germany
Email: gd60@rz.uni-karlsruhe.de

Abstract

Experimental evidence suggests that the complete stress-strain history has to be considered as a basis for constitutive modeling if concrete is subjected to high loading rates. An uniaxial dynamic constitutive law for concrete is presented. The uniaxial theory is extended towards the general three-dimensional case. The proposed dynamic continuum damage law as well as the uniaxial theory is verified on split Hopkinson bar tests with different load histories.

1 Introduction

The influence of the strain rate on the stress rate in concrete is important in all design situations involving short-duration, high amplitude impulsive loads, e.g. in accidental explosions and vehicle, railway or airplane impact on structures like bridges, offshore platforms, tanks, chemical factories or power plants. To explore the dynamic behaviour of concrete many experimental investigations have been performed.

Within theoretical investigations the strain rate dependency is usually taken into account by defining a stress-strain relation $\sigma = f(\varepsilon, \varepsilon = \text{const.})$ where the rate effect is assigned to a constant strain rate.

However the first author emphasized several times (Eibl [7], Bischoff
that there exists a much more complex interdependency between stress and strain in time. It can be shown that every external strain or stress history leads to a different internal stress-strain relation. This contribution explores the consequences on constitutive modelling when strain history effects are taken into account.

2 The experimental background

The experimental knowledge forming the basis for a constitutive law (CL) considering strain rate effects may be summarized as follows (Bischoff et al. [3], Weerheijm [11], Ross et al. [10], Zheng [13]):

- A strength increase due to high strain rate loading is inherent to many solid materials and therefore should be explained by a fundamental material property associated with dynamic fracture.
- Strength is usually more markedly influenced by the loading rate than the initial Young’s modulus.
- Heterogeneous materials show a more pronounced strain rate influence than less heterogeneous ones.
- Acoustic emission experiments (Bischoff et al. [3]) have shown that specimens dynamically loaded to a certain stress level show less damage accumulation than statically loaded ones. This is in accordance with the observation that axial strains under dynamic loading are smaller than those under static loading.
The strain rate range below and above approximately $\dot{\varepsilon} = 10^{-1}$ 1/s is dominated by different phenomena and mechanisms. At high strain rates damage formation seems to be primarily controlled by inertia effects.

### 3 A one-dimensional constitutive law (CL) for tension and compression

#### 3.1 Static part

On the basis of this knowledge a homogeneous damage model on macroscale has been derived from heterogeneous microscale behaviour. It consists of elastic springs with a strain-dependent failure limit and coupled friction elements which are activated after the springs are broken (Fig. 1). The degree of damage is given by $0 \leq D \leq 1$.

As the weakest link in a distribution of springs initiates quasi-brittle fracture a *Weibull*-extreme-value distribution is chosen for damage evolution $D(\varepsilon)$:

$$D(\varepsilon) = 1 - \exp \left[ -\frac{1}{2} \left( \frac{\varepsilon - \varepsilon_0}{a \varepsilon_0} \right)^2 \right] \quad \varepsilon \geq \varepsilon_0$$

(1)
with material parameters $a$ and $\varepsilon_0$ determining the onset of irreversible damage.

The broken elements $D(\varepsilon)$ still can resist external stresses due to internal friction which is finally reduced to zero with growing crackwidth. To cover the latter effect another Weibull-distribution is chosen:

$$F(\varepsilon) = 1 - \exp\left(-\frac{1}{2}\left(\frac{\varepsilon - \varepsilon_0}{b \varepsilon_0}\right)^2\right) \quad \varepsilon \geq \varepsilon_0$$ (2)

If $\mu$ characterizes the resistance of an element due to friction the static stress-strain relation finally reads:

$$\sigma(\varepsilon) = [1 - D(\varepsilon)] \cdot E \cdot \varepsilon + \mu \cdot D(\varepsilon) \cdot [1 - F(\varepsilon)]$$ (3)

Curbach [5] demonstrated the efficiency of this model with simulations of tests reported by Reinhardt & Cornelissen [9] (Fig. 3).

### 3.2 Dynamic part

The static constitutive formulation of eq. 3 has to be extended to include inertia effects causing the dynamic strength increase of concrete materials.

The following dynamic CL is based on the model in Figure 2 including double mass elements which separate at a specific strain. The sum over the impulses of the microcrack masses vanishes, so that the internal mass effect does not violate the equilibrium conditions of the homogeneous model on macroscale by falsely inducing artificial momentum.

If a mass element according to Figure 2b breaks at time $T$ an increment of damage occurs:

$$\Delta D = \frac{\partial D}{\partial \varepsilon} \cdot \frac{\partial \varepsilon}{\partial \tau} \cdot \Delta \tau$$ (4)

Due to the masses in these elements – inertia effect – the transferred stresses do not vanish immediately but with some time delay. The stress decrease may be given by a function $g(t - \tau)$ with values between 1 and 0. Regarding this time delay eq. 3 is extended towards the following CL:
\[
\sigma(t) = E \cdot \varepsilon(t) \cdot \left[ 1 - D(\varepsilon) \right] + \int_{\tau=0}^{\tau=t} \frac{\partial D}{\partial \varepsilon} \cdot \varepsilon \cdot g(t-\tau) \, d\tau \]
\[\text{where the convolution integral covers the dynamic inertia effects.}\]

Thus the external stress consists of two parts in eq. 5. The first part describes the growing damage of the elastic material due to the accumulation of micro-defects. The damage develops time dependent as the irreversible energy dissipation following the breakup of elastic elements and the ensuing stress relaxation meets combined frictional or viscous resistances in addition to the inertia effect. The remaining static part consists of the internal friction of already broken elastic springs which evolves independently from time. In case of an uniaxial CL the resistance \(1 - F(\varepsilon)\) fades away with growing crackwidth.

The dynamic decay function \(g(t-\tau)\) can be derived directly from a mechanical model (see Fig. 2b). For the sake of simplicity the basic decay characteristic may be approximated also by a line with negative slope using only one parameter \(t_0\) as the decay time:

\[
g(t-\tau) = 1 - \frac{t - \tau}{t_0} \quad \tau \leq t \leq \tau + t_0 \\
g(t-\tau) = 0 \quad t > \tau + t_0
\]


4 A three-dimensional constitutive law and its application to split Hopkinson bar test data

The ideas presented in the preceding sections are now being extended towards a general three-dimensional constitutive law (CL). The following paragraph gives a first approach to a multiaxial continuum damage model for the range of low confining pressures. By including a dynamic inertia component the CL reproduces split Hopkinson bar tests (Zheng [13]) in large scale axisymmetric finite element analyses.
4.1 Constitutive approach

The proposed constitutive formulation stays within the framework of strain-based scalar continuum damage mechanics, for a good discussion and review see Willam et al. [4]. The inelastic strain due to friction during the tensile damaging process suggests the following decomposition of the strain tensor into an elastic and an inelastic part:

\[ \varepsilon = \varepsilon_e + \varepsilon_i \]  

(7)

With eq. 7 the time-independent part of the damage law is written as:

\[ \sigma_{st} = (1 - D(\kappa_d)) E_0 : (\varepsilon - \varepsilon_i) \]

\[ \varepsilon_i = f[\kappa_i(q_i)] \]  

(8)

with  
\[ E_0 \] = initial elastic tensor  
\[ D \] = damage variable with 0 ≤ D ≤ 1  
\[ \kappa_d \] = equivalent damage strain  
\[ \kappa_i \] = equivalent inelastic strain  
\[ q_i \] = internal variable for inelastic strain evolution

The damage parameter depends on an equivalent strain measure derived...
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Figure 5: Split Hopkinson bar testing device

from the elastic strain tensor and the inelastic strain is determined according to the rules of plasticity.

The evolution of the equivalent damage strain $\kappa_d$ and the equivalent inelastic strain $\kappa_i$ is controlled by two loading conditions in strain space. The Hsieh-Ting-Chen [8] failure criterion is adapted for use as damage surface $F_d$ while the friction surface $F_i$ is based on the positive projection of the elastic strain tensor according to the rule that microcrack-damage develops perpendicular to the direction of the largest principal strain:

\[
F_d = c_1 J_2 + \kappa_d [c_2 \sqrt{J_2} + c_3 e_{e,\text{max}} + c_4 I_1] - \kappa_d^2 = 0
\]

\[
F_i = \frac{1}{2} e_e^+ : e_e^+ - \frac{1}{2} \kappa_i^2(q_i) = 0, \quad e_e^+ = P^* : e_e
\]

with:
- $c_1 ... c_4$ parameters of damage surface
- $I_1, J_2$ first invariant of elastic strain $e_e$ and second invariant of deviatoric part of $e_e$
- $e_{e,\text{max}}^{(k)}$ maximum (k-th) eigenvalue of $e_e$
- $P^* = \sum_{\alpha} H(\varepsilon_e^{(\alpha)}) \cdot d^{(\alpha)} \otimes d^{(\alpha)} \otimes d^{(\alpha)}$
- $P^*$: positive projection tensor of $e_e$
- $H$: Heaviside function
- $\varepsilon_e^{(\alpha)}, d^{(\alpha)}$: $\alpha$-th eigenvalue and eigenvector of $e_e$

A damage evolution law similar to eq. 1 is chosen
Figure 6: Compressive load histories (LHs)

\[ D(\kappa_d) = 1 - e^{\left(\frac{\kappa_d - \varepsilon_0}{\varepsilon_d}\right)^{\delta_d}} \quad \kappa_d \geq \varepsilon_0 \]  

(11)

and the internal variable \( q_i \) is related to the equivalent inelastic strain \( \kappa_i \) by the following empirical function:

\[ q_i(\kappa_i) = c \cdot (\kappa_i - \varepsilon_d) \cdot (1 - e^{-\varepsilon_{ii}}) \cdot e^{-\varepsilon_{i2}} \quad \kappa_i \geq \varepsilon_d \]  

(12)

where \( \varepsilon_0, \varepsilon_d, \kappa_d, c, \varepsilon_{ii}, \varepsilon_{i2} \) are material constants.

Furthermore secant unloading with the damaged elastic stiffness tensor is assumed. Figure 4 shows the biaxial compressive behaviour of the continuum damage CL for the concrete quality used in the split Hopkinson bar tests.

The dynamic extension of the static CL follows straightforward from eq. 5 in section 3. The strain-history dependence is developed from a slightly modified rheological model. The time decay function \( g(t-\tau) \) is directly evaluated by solving the differential equation for a double mass oscillator. Equation 5 in terms of the three-dimensional formulation finally reads:

\[ \sigma = \left(1 - D(\kappa_d) + \int_{\tau=0}^{t} \frac{\partial D}{\partial \tau} \cdot g(t-\tau) \, d\tau \right) \cdot E_0 : (\varepsilon - \varepsilon_i) \]  

(13)

4.2 Load histories generated by split Hopkinson bar apparatus

A systematic experimental investigation of dynamic stress-strain histories
was performed by Zheng [13] using a special modified Hopkinson bar device (Fig. 5). As concrete testing requires specimen lengths where the assumption of a homogeneous stress distribution inside the specimen does not hold anymore Zheng measured strains and accelerations directly on the specimen to evaluate stress-strain relations in three sections A, B and C (see Fig. 5c). The split Hopkinson pressure bar is shown in Figure 5a for the generation of compressive load histories, whereas Figure 5b depicts the experimental setup for tensile loading by prestressing a portion of the input bar according to Albertini & Montagnani [1] and Bachmann [2].

By varying the mass of the projectile (2 to 4 kg) and the impact velocity (5 to 20 m/s) the peak of the impulse could be adjusted from below static failure stress (load history LH7 in Fig. 6) up to immediate dynamic fracture (load history LH1). Inserting a sheet of cartoon between projectile and input bar increased the duration of the stress pulse (LH2, LH4 and LH6). All of the compressive stress histories measured in the input bar are shown in Fig. 6. In addition to these compression tests tensile load histories were generated by prestressing a length of 2.5 m of the input bar by 21 MPa.

4.3 Finite element verification

To verify the theory the CL described in section 4.1 was implemented into the finite element program ABAQUS Explicit. Full scale finite element models of the split Hopkinson bar tests were investigated to simulate the wave propagation and fracturing within the specimen. Figure 7 shows a view of the finite element mesh near the specimen with dashed lines indicating the symmetric part of the test setup. Aluminum bar and test specimen are constrained radially along the axis of symmetry.
Figure 8: Analysis of split-Hopkinson bar compression test LH1: a) strain on split Hopkinson bar, b) longitudinal strain on specimen (section A), c) stress on specimen (A) and d) stress-strain relation (A)

Figure 8 compares the analysis of stress history LH1 with the test result. The correspondence of strains measured on the mantle of the aluminum bar at points P1, P2, P3 with those generated by the analysis is satisfactory in Figure 8a. The strain and stress time recordings agree well with the computed results in Figure 8b, c. The stress-strain relation in section A of the specimen which suffered complete fragmentation during the test shows a dynamic strength increase factor of about 2.2. The maximum stress as well as the maximum strain on the specimen is met by the finite element calculation (Fig. 8d).

The second example presented in Figure 9a-d shows the tensile load history as described in Zheng [13]. The analysis was performed with the same choice of constitutive parameters as those used for the compressive load histories. The good agreement of the calculated results with the measurements underlines the general applicability of the dynamic formulation. The transmitted tension wave of the finite element solution in
Figure 9: Analysis of split-Hopkinson bar tension test: a) strains on split Hopkinson bar, b) longitudinal strain on specimen (section A, B), c) stress on specimen (A, B) and d) stress-strain relation (A, B)

Figure 9a at point P3 almost coincides with the measured results indicating the exact simulation of the internal fracturing process.

5 Conclusion

To properly represent the dynamic fracture of inhomogeneous materials the whole stress or strain history must be considered. Constitutive modeling simulates the accumulation of microcracks by means of damage mechanics. The time delay of damage evolution due to high strain rates is defined by a decay function that can be derived from a rheological micro-fracture model. The extension of the uniaxial strain-history dependent constitutive law towards the general three-dimensional case is verified on finite element simulations of split Hopkinson bar tests.
References


