Modeling of shock and impact behaviors of aluminum oxide
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Abstract

Aluminum oxide (AD995) is a leading candidate material in ceramic armor applications. The density of this material is 3.8 ~ 3.9 gm/cm³ with a porosity of about 2 percent. Its bulk and shear moduli are 231 and 156 GPa, respectively. From microscopic studies on recovered ceramic specimens from impact and penetration experiments, it has been established that the deformation under shock and impact loading is due to microcracking, microplasticity, pore collapse, and twinning. Rajendran [1] formulated a constitutive model to describe the stress-strain response of ceramics based on these various deformation modes. The Rajendran-Grove (RG) model has been implemented in the 1995 version of the EPIC code [2]. The shock response of AD995 is modeled through EPIC simulations of 1) planar plate impact at velocities below and above the Hugoniot elastic limit (HEL), 2) a metal plate impacting a slender AD995 rod, and 3) projectile penetration into a layered ceramic target plate. The model-generated results matched the measured and observed responses of AD995 in the experiments extremely well.

1 Introduction

Recently, Rajendran [1] reported a continuum mechanics based three dimensional constitutive model to describe the impact behavior of ceramic materials. This model is based on microcrack nucleation and
growth, as well as pore collapse mechanisms. Damage is defined in terms of an average crack density and is treated as an internal state variable. To keep the model formulation relatively simple, the damage nucleation is not modeled and the microcracks are assumed to be present prior to loading. This scalar damage model incorporates the effects of different damage processes under tension and compression using fracture mechanics based fracture criteria. References 3-5 describe the RG model’s ability to model the shock response of several ceramic materials.

The main objective of this paper is to model the shock and impact response of aluminum oxide through computer simulations of a variety of shock and penetration experiments. The ability of the Rajendran-Grove model to capture various salient features of the measured shock wave profiles is investigated. The simulations were made using the 1995 version of the EPIC code [2]. The experiments included: (a) low-velocity plate impact experiments of Dandekar & Bartkowski [6], (b) high-velocity plate impact experiments reported by Grady & Moody [7], (c) an aluminum plate impacting a long slender AD995 ceramic rod, performed by Grady & Wise [8], and (d) a long tungsten rod penetration into a target consisting of an AD995 ceramic tile backed by a thick steel plate, performed by Woolsey [9].

2 Rajendran-Grove Ceramic Model

The Rajendran-Grove (RG) ceramic model, as described in references 1 & 3, assumes the following: 1) preexisting randomly distributed flaws, 2) plastic flow when shocked above the HEL, 3) no plastic flow in tension, 4) degradation of elastic moduli under both compression and tension due to microcracking, and 5) pulverization only under compressive loading. Due to low fracture toughness in the ceramic, microcracking occurs at a relatively low tensile stress amplitude. When the ceramic plastically deforms, the model employs a strain rate dependent strength relationship. Plastic strains are calculated using a conventional viscoplastic radial return approach. The plastic strains are then subtracted from the total strains to obtain the elastic strains. The elastic stress-strain equations for the microcracked aggregate material are given by,

\[ \sigma_{ij} = M_{ijkl} \left( \epsilon_{kl} - \frac{P}{k_{l}} \right) \] (1)
where $\sigma_{ij}$ is the total stress, $\varepsilon_{kl}$ is the total strain, $\varepsilon_{kl}^p$ is the plastic strain due to viscoplastic flow and pore collapse, the strain difference $(\varepsilon_{kl} - \varepsilon_{kl}^p)$ is the elastic strain, and the components of the stiffness tensor $M$ are given by Rajendran [1]. The elements of this stiffness matrix are degraded through a crack density parameter. The RG model assumes that pore collapse is due to local microscopic flow in the matrix material surrounding the pores. The pore collapse strain components were derived from the pressure dependent yield surface of Gurson [10]. One advantage of employing Gurson’s flow surface is the absence of any adjustable model parameter in the analytically derived expression, given by:

$$\Phi(f, Y, S_{ij}) = \frac{\sigma_e^2}{Y^2} + 2f \cosh\left(\frac{3P}{2Y}\right) - 1 - f^2 = 0$$  (2)$$

where $f$ is the void volume fraction of the pores, $Y$ is the strength (flow stress) of the matrix (intact) material, $\sigma_e$ is the effective stress, $P$ is the pressure (mean stress), and $S_{ij}$ are the deviatoric stresses.

The total plastic strain rates are derived from the conventional plastic flow rule using the expression for $\Phi$. The void volume is determined from the following expression:

$$f = 1 - \exp(\varepsilon_v^p)$$  (3)$$

where $\varepsilon_v^p$ is the plastic volumetric strain of the matrix (void-free) material. The strength of the intact ceramic material is described by a strain rate dependent relationship:

$$Y = A \left( 1 + C \ln \dot{\varepsilon}^* \right)$$  (4)$$

where $A$ is initial yield strength, $C$ is the strain rate sensitivity parameter, and $\dot{\varepsilon}^*$ is the normalized (dimensionless) equivalent plastic strain rate.

In the ceramic model, microcrack damage is defined in terms of a dimensionless microcrack density $\gamma$, where $\gamma = N_o^* a^3$. $N_o^*$ is the average number of microflaws per unit volume and the maximum microcrack size, $a$, is treated as an internal state variable. The microcracks extend when the stress state satisfies a generalized
The microcrack extension causes the microcrack density $\gamma$ to increase, which results in stress relaxation in the cracked ceramic material. Since $N_o^*$ is assumed to be a constant, the increase in $\gamma$ is entirely due to the increase in the crack size $a$. The damage evolution law is described by,

$$\dot{a} = n_I C_R \left[ 1 - \left( \frac{G_{cr}}{G_I} \right)^{n_2} \right] \tag{5}$$

where $C_R$ is the Rayleigh wave speed, $G_{cr}$ is the critical strain energy release rate for microcrack growth, $G_I$ is the applied strain energy release rate, $n_I$ is the limiting crack growth factor, and $n_2$ is the crack growth index. $G_{cr}$ is calculated from the fracture toughness $K_{IC}$, which is treated as a model constant, while the two crack growth parameters ($n_I$ and $n_2$) can assume different values under compression and tension.

We employed a modified Mie-Gruneisen relationship for the equation of state (EOS) in which the calculated pressure is further reduced according to the ratio of the degraded bulk modulus to the intact bulk modulus. The corresponding EOS parameters, along with density and shear modulus, are given in Table 1 for the low density (90% alumina) and high density (99.5% alumina) aluminum oxide ceramic materials considered in this study.

**Table 1. Equation of state and material parameters.**

<table>
<thead>
<tr>
<th>EOS / material parameters</th>
<th>90% pure alumina</th>
<th>99.5% pure alumina</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta_1$ (GPa)</td>
<td>163</td>
<td>231</td>
</tr>
<tr>
<td>$\beta_2$ (GPa)</td>
<td>163</td>
<td>231</td>
</tr>
<tr>
<td>$\beta_3$ (GPa)</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$G$</td>
<td>2.3</td>
<td>2.3</td>
</tr>
<tr>
<td>Density (kg/m$^3$)</td>
<td>3555</td>
<td>3890</td>
</tr>
<tr>
<td>Shear Modulus (GPa)</td>
<td>106</td>
<td>156</td>
</tr>
</tbody>
</table>
3 Plate Impact Experiments

In plate impact experiments, a flat thin disk (plate) is made to impact against a target plate. Compressive stresses are produced and transmitted immediately from the plane of impact to the adjacent stress free areas of the material in the form of a stress pulse. Measurement of the free surface velocity at the rear of the target provides data for the loading (compression) and unloading (release) paths. Plate impact tests are also conducted to measure the Hugoniot Elastic Limit (HEL), which is the principal stress component under one dimensional strain at very high strain rates. When the release waves interact, tensile stresses of high amplitudes under a triaxial stress state are created, subsequently leading to failure of the material. In metals, such failure occurs only when the shock stress exceeds the HEL; in ceramics, however, tensile failure can occur even at stress levels below the HEL. This is because microcracks can be activated at relatively low amplitude stresses due to their low fracture toughness.

In the present study, plate impact experimental data from two different sources were considered: two low velocity experiments performed by Dandekar & Bartkowski [6] using high-pure AD995 alumina, and three high velocity experiments reported by Grady & Moody [7], two for a porous aluminum oxide (90% alumina), and one for AD995. In the high velocity experiments reported by Grady and Moody, the ceramic target plate was backed by a lithium fluoride window, and a laser velocity interferometer (VISAR) was used to record the particle velocity history at the target/window interface. Dandekar and Bartkowski used a PMMA window for both their stress gauge and VISAR measurements. Table 2 describes the dimensions and impact velocities of the plate impact experiments.

3.1 Modeling of the Plate Impact Experiments

Using the one-dimensional strain option in the 1995 version of the EPIC code, the RG model constants were initially adjusted to match the stress gauge measurement from the lowest velocity (83 m/s) plate impact experiment (test #1 in Table 2). Then, using the same model constants that were determined from test #1, the VISAR data from the remaining tests (at higher impact velocities) were predicted through additional EPIC simulations.
Table 2. Plate impact experimental details.

<table>
<thead>
<tr>
<th></th>
<th>Test #1</th>
<th>Test #2</th>
<th>Test #3</th>
<th>Test #4</th>
<th>Test #5</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Flyer Thickness (mm)</strong></td>
<td>4</td>
<td>4</td>
<td>4.677</td>
<td>4.673</td>
<td>5.013</td>
</tr>
<tr>
<td><strong>Target Thickness (mm)</strong></td>
<td>8</td>
<td>8</td>
<td>9.076</td>
<td>9.081</td>
<td>10.007</td>
</tr>
<tr>
<td><strong>Impact Velocity (m/sec)</strong></td>
<td>83</td>
<td>512</td>
<td>1542</td>
<td>2201</td>
<td>1943</td>
</tr>
<tr>
<td><strong>Alumina %</strong></td>
<td>99.5</td>
<td>99.5</td>
<td>90</td>
<td>90</td>
<td>99.5</td>
</tr>
<tr>
<td><strong>Measurement Technique</strong></td>
<td>Stress Gauge</td>
<td>Velocity (VISAR)</td>
<td>Velocity (VISAR)</td>
<td>Velocity (VISAR)</td>
<td>Velocity (VISAR)</td>
</tr>
</tbody>
</table>

Fig. 1 shows a comparison between the computed stress history and the experimental data for the low velocity (83 m/s) plate impact test. In this study, the RG model constants were determined through a series of numerical simulations wherein the values of the initial flaw size ($a_o$) and the number of flaws per unit volume ($N_o^*$) were systematically adjusted until the experimental spall signal from test #1 was successfully reproduced. A sensitivity study on the initial flaw size indicated that a value of 14 micrometers ($\mu$m) was required to match the measured spall signal at low velocities. To prevent premature fracture during the initial shock, a value of 0.1 was assigned to the limiting factor ($n_I$) for compressive (mode II) crack growth, while $n_I$ was assumed to be “1” for tensile (mode I) crack growth. In addition, the crack growth index ($n_2$) was assumed to be equal to “1” for both modes of crack growth. The calibrated RG model constants are: $K_{IC} = 3$ MPa$\sqrt{m}$, $n_I = 0.1$ (mode II crack growth), $a_o = 14$ $\mu$m, $N_o^* = 5 \times 10^9$ m$^{-3}$, and $\mu = 0.45$, where $\mu$ is the dynamic coefficient of friction that appears in the Griffith criterion.

Test #2 (impact velocity of 512 m/s) was then simulated using the set of constants determined by matching the low velocity data. The experimental data indicated that the AD995 retained its spall strength at this velocity. The measured particle velocity from this test was successfully predicted, as shown in Fig. 2.
Figure 1. Comparison of computed stress history with plate impact experimental data for impact velocity 83 m/s.

Figure 2. The RG model's velocity history prediction is compared with the VISAR data for the 512 m/s plate impact test.
The VISAR-measured velocity histories from tests 3 and 4 were also considered in the modeling. In these high velocity experiments, the thicknesses of the flyer and target plates were about 4.7 mm and 9.1 mm, respectively. Based on the elastic properties of the porous aluminum oxide (see Table 1), its longitudinal wave speed is about 9.25 km/s. Therefore, for tests 3 and 4, the expected pulse duration is about 1 microsecond ($\mu$s), with the initial shock wave arriving at the velocity measurement location (back surface of the target) at about 0.98 $\mu$s. The release wave from the flyer is expected to arrive at this location at about 2 $\mu$s. However, Fig. 3 shows that the release wave arrived at about 1.75 $\mu$s in test #3 (1.5 km/s impact velocity), indicating a faster release wave speed (>9.25 km/s).

Tests 3 and 4 (see Table 2) were simulated using the 1995 version of the EPIC code. The EOS parameters in Table 1 for the porous aluminum oxide (90% alumina, 6% porosity) were employed in the simulations. Initially, we simulated these porous alumina plate impact tests without any pore collapse, and the simulation results failed to match the measured velocity profiles. When modeling the shock response of the alumina, constitutive relationships which incorporate the effects of void collapse and microcracking are necessary to
capture the salient features of the measured wave profiles. The microcracking feature of the model controls the spall rebound portion of the signal, and the pore collapse feature improves the predictions of rise time and shock stress levels. With pore collapse, the RG ceramic model predicted the initial portions of the velocity histories fairly accurately, as shown in Fig. 3. The computed spall signals, which indicate "zero spall strength" in the alumina, also matched the measured spall signals reasonably well. In general, the model-predicted velocity profiles for tests 3 and 4 captured most of the salient features of the measured profiles.

Finally, test #5 (see Table 2) was simulated using the same set of RG ceramic model constants, except that we modeled the AD995 with only two percent (2%) porosity. A comparison between the calculated and measured particle velocity histories is made in Fig. 4. Due to the numerically introduced artificial viscosity (an accepted smoothing technique in explicit shock wave propagation hydrocodes), the elastic portion of the computed shock profile is less steep than the experimental measurement. However, the RG model predicted the "inelastic" portion of the shock wave and the elastic release very well, while the computed spall signal deviated slightly from the experiment. In general, the overall model prediction is good.

![Figure 4](image-url)
4 Bar Impact Experiment

Grady & Wise [8] used an impact configuration in which a flat 6061-T6 aluminum plate (disk) with a long trailing rod was made to impact a stationary long ceramic rod. Shown in Fig. 5, this configuration permits a time-resolved investigation of the dynamic response of materials under a multiaxial strain state. The rod section of the flyer plate maximizes the duration of high stresses at the impact end of the ceramic rod. During the experiment, time resolved particle velocity data were obtained at the free end of the target rod. We performed simulations of this experimental configuration and analyzed the measured axial particle velocity profile.

Figure 5. A schematic of the bar impact experiment showing the particle velocity (VISAR) measurement location.

4.1 Numerical Results for Bar Impact Configuration

We performed 2D axisymmetric EPIC simulations of an aluminum flyer plate (12.7 mm thick, 75 mm diameter) with a trailing rod (122 mm long, 20 mm diameter) impacting an AD995 ceramic rod (80 mm long, 10 mm diameter) at a velocity of 1035 m/s.

When the RG model constants (previously calibrated to match the lowest velocity plate impact data) were employed in the bar impact simulation, the model predicted excessive crack damage in the rod, which resulted in a computed peak velocity level that was significantly lower than the measured level. The simulation was repeated with \( n_1 \) (limiting crack growth factor) and \( n_2 \) (crack growth index) equal to 0.1 for both tensile (mode I) and compressive (mode II) microcrack growth. The calculated time history of particle velocity at the stress-free end of the ceramic rod compared extremely well with the measured data, as Fig. 6 indicates.
5 Penetration Experiment

Woolsey [9] performed depth of penetration (DOP) experiments in which a tungsten long rod projectile is launched at a nominal velocity of 1.5 km/s onto a 152 mm square ceramic tile that is laterally confined by a steel frame; the target assembly (tile and frame) is mechanically clamped to a thick steel backup block. The ceramic tiles that Woolsey tested were between 25 and 50 mm thick. In the experiments on 25-mm-thick AD995 ceramic tiles, the average measured residual DOP (into the steel block) was about 41 mm.

5.1 Numerical Results for DOP Configuration

Using the 2D axisymmetric geometry option in EPIC, we simulated a DOP configuration in which the target consisted of an AD995 ceramic disk (25 mm thick, 152 mm diameter) backed by a thick steel plate (127 mm thick, 203 mm diameter); the ceramic tile was also radially confined by a steel ring that was “clamped” to the
steel back plate. The projectile rod was 91 percent tungsten (78.7 mm long, 7.87 mm diameter), with an impact velocity of 1500 m/s. The RG model constants for AD995 (calibrated to match plate impact test #1) were employed in this simulation.

In the EPIC code, projectile penetration is achieved through numeric erosion (elimination) of elements whose equivalent plastic strains exceed 150%. Therefore, to permit erosion of the damaged ceramic material, the RG model algorithm defines the material as being in a pulverized state when $\gamma$ (the crack density parameter) exceeds a critical value (0.75) during compressive loading. Once pulverization has occurred, the strength $Y_p$ of the comminuted material varies linearly with compressive (positive) pressure $P$, as $Y_p = \beta_p P$, where $\beta_p$ is the pulverized strength model parameter.

This simple strength model allows the plastic strains of the pulverized elements (beneath the penetrating projectile) to build up to the critical erosion strain (150%). Based on DOP simulations for another ceramic (silicon carbide), $\beta_p$ was assumed to be 0.6.

The final configuration of the EPIC simulation is shown in Fig. 7, along with the experimental DOP (indicated by the dotted line). The RG model's prediction agrees almost exactly with the measured DOP.

![Figure 7](image-url)
6 Summary

This paper presents results from numerical simulations of a variety of shock and penetration experiments in which the Rajendran-Grove (RG) ceramic damage model was employed to describe the dynamic response of aluminum oxide subjected to shock loading conditions. Through simulations of plate impact experiments over a wide range of impact velocities, the effects of viscoplastic flow, microcracking, and porosity on the shapes and amplitudes of measured stress and velocity profiles were determined. The low velocity plate impact experiments (tests 1 & 2) were simulated to study the effects of microcracking on the spall behavior of the relatively high density, low porosity AD995. The simulation results from the high velocity plate impact experiments (tests 3, 4, & 5) on both the porous aluminum oxide (90% alumina, 6% porosity) and the AD995 (99.5% alumina, 2% porosity) clearly demonstrate the importance of modeling pore collapse in ceramics.

The RG model constants were initially calibrated by matching the simulated stress profile with the stress gauge data from an 83 m/s plate impact test. Using the same constants, the VISAR data from several tests at impact velocities well above 83 m/s were successfully predicted. In these experiments, the AD995 alumina retained its spall strength when shocked slightly above the HEL. The EPIC simulation results (using the RG model) showed similar behavior in AD995.

Based on the modeling of the porous aluminum oxide (90% alumina, 6% porosity), the present numerical study indicates that the inelastic deformation in alumina is dominated by pore collapse. Since the AD995 has a porosity of only about 2%, the loading portion of the wave profile from test #5 did not exhibit significant ramping. However, the pore-collapse-dominated ramping was clearly evident in the more porous alumina at high velocities (tests 3 & 4).

At present, it is not possible to describe the shock and impact behavior of AD995 (or any other ceramic), using just one set of RG model parameters. Since the model does not account for all the microscopic structural evolution in ceramics, it is extremely difficult to match all the measured wave profiles from different shock experiments. However, given the relative simplicity of this microcrack-based scalar damage model, it is very encouraging that this model is capable of accurately predicting the measured profiles in plate impact experiments at both low and high velocities, as well as the depth of penetration in a ballistic test.
References


