Calibration of a numerical model for the simulation of masonry under earthquake loading

F.M.B. Galanti, A. Scarpas, A.C.W.M. Vrouwenvelder

Structural Mechanics Division, Faculty of Civil Engineering
Delft University of Technology, Stevinweg 1, 2628 CN Delft, The Netherlands
E-mail: F.Galanti@CT.TUDelft.NL

Abstract

As part of a larger project involving the finite element analysis of infilled reinforced concrete frames this article concentrates on the modelling aspects of the masonry panel within the reinforced concrete frame. The main scope of the study was to reproduce the experimentally observed modes of failure of masonry panels and the propagation of damage in the masonry, distinguishing the conditions which can lead to one type of failure or another. Typical failure modes are the shear type (shear sliding along the bed joints) and the compression type (diagonal compression strut and corner crushing). The analysis has been carried out using the finite element program DIANA, and a number of modelling options have been considered.

Introduction

The use of masonry infills in reinforced concrete buildings is a common practice in many countries. Often, the design of such buildings concentrates on the bare reinforced concrete frame since the masonry infills are regarded as “non structural”. Even though this approach may be appropriate in areas with little or no seismic risk, it is relatively inappropriate for earthquake prone zones since the presence of any infill
can influence considerably the dynamic response of the building. The infill, which acts as a rigid diaphragm, can lead to an increase in both the in-plane lateral stiffness and strength of the frame in which it is placed. If the seismic loading is large enough, the masonry infill will fail in a relatively brittle manner with a sudden transfer of almost the entire load to the reinforced frame. A building designed on the “bare frame” assumption might thus have a completely different dynamic response than that which would be expected from the original calculations.

In order to study the interaction between the frame and the masonry infill, it is necessary to study the behaviour of masonry walls subject to lateral in-plane forces as typically encountered during an earthquake. Numerous experimental and analytical studies have been carried out in order to understand the mechanics of such walls. By comparison relatively little work is to be found based on numerical methods. This is mainly due to a long tradition of empirical methods for the design of masonry, and also due to the inherent difficulties in properly modelling numerically such a material.

In the context of clarifying such difficulties and to check the limits of numerical modelling, a number of analyses of masonry panels have been carried out using the finite element program DIANA. The analyses must be seen as an attempt to delineate the range of masonry material response that can be simulated by means of readily available finite element models. As such, they exclude the use of more specialised masonry material models such as that developed by Lourenço. Even though the utilised material properties were obtained from available experimental evidence, no attempt was made to simulate any particular experiment.

### Numerical aspects

The difficulties in the modelling of masonry mentioned in the introduction derive from the material’s low ductility and anisotropy. The brittle nature of the material implies localised snap-back behaviour making it difficult for the algorithms to find a correct solution, and the non-uniformity leads to the material having elastic and plastic properties that are different in various directions. As far as this latter point is concerned, a first rough approximation in the analyses is made by using an isotropic material. The problems arising from the low tensile strength of the material can be overcome by making appropriate choices of mesh size and solution procedure. Before discussing these aspects however, the modelling possibilities in DIANA will be briefly introduced.
DIANA

The FE analysis program DIANA used to perform the analyses, is a program developed by TNO Netherlands Organization for Applied Scientific Research. This program includes various models for plasticity and fracturing (cracking). Cracking can be modelled either discretely, using interface elements, or within solid elements using “smeared” cracks. In the latter case the deformation due to the opening of the crack is simulated within an element by introducing a crack strain $\varepsilon_{cr}$, which is to be added to the elastic strain $\varepsilon_e$. Without going into the formal mathematics of the crack model, the total strain $\varepsilon_t$, in the element in the normal direction to the crack can be written as

$$\varepsilon_t = \varepsilon_e + \varepsilon_{cr}. \quad (1)$$

The direction of the first crack is perpendicular to that of the maximum principal stress, and remains unaltered during further loading. More cracks may appear within the same element when a certain threshold angle is exceeded between the original crack and the new (to be created) crack. Moreover, as the cracks appear shear forces may still be transferred via the crack surface. This feature, referred to as shear retention, may be chosen to allow for constant shear stiffness across the crack plane or a variable stiffness that depends on the current crack strain. A number of optional features can be associated with this model such as softening, rate dependency and perfectly brittle behaviour. Further, it can be combined with a Drucker-Prager or Mohr-Coulomb yield criterion for the modelling of failure in compression.

![Graph 1](image1)

**Figure 1.** Linear tension softening: (a) softening diagram showing residual strength against crack opening $\delta_{cr}$ (note secant path for unloading and reloading shown by dotted line); (b) elastic response.
Tension softening and mesh size

An important aspect in the analysis of masonry is that of the modelling of the softening behaviour of the material as the cracks open. In DIANA the response of the cracked section is characterised by a softening curve, Figure 1, and a number of parameters, namely, the fracture energy \( G_f \), the crack band width \( h \), which represents an ‘average’ element dimension such as the width of the element, Young’s modulus \( E \), and the tensile strength of the material \( f_t \). If the results of the analyses are to be independent of the choice of mesh size, the value of \( h \) must meet a certain condition, which for a linear softening diagram is given by

\[
h \leq \frac{2G_f E}{f_t^2}.
\]

The fulfilment of the above condition basically ensures that the single element response does not present a snap-back behaviour. For the underlying theory see De Borst\(^1\) and De Witte et al\(^2\).

Solution procedure

There are several alternatives as to how to obtain a solution in the stepwise application of prescribed displacements in a FE analysis. A typical method is the full Newton-Raphson iteration procedure where a new stiffness matrix is set up at every iteration. The procedure is precise and converges to solution in a few number of iterations. However, each newly calculated stiffness matrix depends on the current state of the structure, making it difficult for the procedure to remain stable at critical points in the loading history. Another procedure is the initial stiffness method, which avoids setting up the stiffness matrix at every iteration point. At the cost of more iterations, this avoids also the possibility of the creation of negative or zero pivots in the stiffness matrix thus making it a more stable solution procedure.

The solutions of the analyses carried out in this study have been obtained using the initial stiffness method, and using displacement control as a loading method. This was out of the desire of being able to continue the analyses beyond critical points such as at the creation of new important cracks in the masonry panel, where the full Newton-Raphson method would otherwise stop. The results obtained in this way may thus give an idea of not only the maximum resistance that can be sustained, but also of the sustainable displacements beyond failure, along with the corresponding failure modes.
Modes of failure of masonry panels

Generally, three basic types of failure for a masonry wall fixed at the base and loaded horizontally in its plane can be distinguished: (i.) a flexural or rocking failure where cracking occurs at the base of the wall along the tension region; (ii.) diagonal cracking of the panel; (iii.) and a failure where shear sliding occurs along the bed joints. Further another mode of failure may occur, where crushing occurs in two diagonally opposite corners of the panel. The various modes of failure described are shown in figure 2. Varying the loading conditions and material properties of the wall can bring about one type of failure or another.

The expected strength of the masonry panel can be derived from an analysis of the wall using simple elastic beam theory. The stress field in the panel can be approximated by

\[
\sigma_{xx} = 0, \\
\sigma_{yy} = -\frac{N}{tw} - \frac{12(Fy + M)x}{tw^3}, \\
\tau_{xy} = \frac{3}{2} \frac{F}{tw} \left(1 - \frac{4x^2}{w^2}\right),
\]

where \( F \) is the horizontal force, \( N \) the vertical force and \( M \) the couple applied at the top of the wall as shown in figure 3;

\( x \) and \( y \) define a coordinate system with the origin at the middle of the top edge of the wall;

\( w \) and \( t \) are respectively the width and thickness of the wall.

The following expressions for the expected resistance of the panel to lateral force \( F \), have been derived assuming that the material fails at a point in the structure where the state of stress defined by eqns (3) violates either a tension cut-off criterion or a Mohr-Coulomb compression criterion. The expressions have been derived for three points where a critical state of stress has been assumed to take place, namely the bottom corner of the panel in tension (a), the one in compression (b) and the centre of the panel (c). In principle the expression that gives the lowest value of resistance determines the mode of failure and the minimum resistance that can be expected from the structure.
Figure 2. Typical modes of failure of masonry walls: 
(a) base cracking, (b) diagonal cracking, (c) corner 
crushing, (d) shear sliding along bed joints.

\[ F = \frac{1}{6} \frac{tw^2}{h_0} (f_t + p), \]  
\[ F = \frac{1}{6} \frac{tw^2}{h_0} \left( \frac{2c \cos \phi}{1 - \sin \phi} - p \right), \]  
\[ F = \frac{2}{3} twf_t \sqrt{1 + \frac{p}{f_t}}. \]

Here \( h_0 \) is the equivalent height of the wall which is defined as the height of the horizontal section with zero bending moment (\( h_0 \) is equal to the height of the wall when no couple is applied at the top, otherwise for a wall whose top edge is not free to rotate \( h_0 \) is equal to half the height); 
\( f_t \) is the tensile strength of the masonry; \( c \) and \( \phi \) are the Mohr-Coulomb cohesion and internal angle of friction respectively; 
\( p \) is the average pressure in the wall due to the vertical load (=\( N/tw \)).

The above expressions are similar to empirical formulas found in literature, see for example Magenes and Calvi\(^4\).
Description of the analyses carried out

Geometry of panels

The analyses have been carried out for a panel 2 m high by 3.6 m wide, with a thickness of 0.3 m. The panel is fixed at the bottom and is bounded at the top by a rigid steel beam. A mesh of 16 by 28 quadrilateral 8 noded plane stress elements has been used.

![Figure 3. Geometry and applied loads](image)

Material properties

Only two cases with differing material properties have been selected. The variation is in the tensile strength of the masonry, which has been chosen to be 0.5 MPa for a strong type of masonry (material P1), and 0.1 MPa for a weak masonry type (material P2). The other parameters, which are the same for both cases, are $E=5000$ MPa, $G=20$ J/m$^2$ and a uniaxial compressive strength of 4 MPa (which corresponds to a Mohr-Coulomb criterion with $\phi=30^\circ$ and $c=1.15$ MPa). A linear softening curve has been specified for the tension cut-off criterion. No hardening/softening has been specified for the compression criterion.
402 Structures Under Shock and Impact

Load combinations

The loads are applied as shown in Figure 3, that is, as concentrated forces applied at the middle of the rigid beam on top of the panel. To begin with, only the vertical load is applied in one single step. With displacement control the horizontal load consists of a prescribed horizontal displacement applied in small steps. Four basic load cases (LC1 to LC4) with varying vertical load will be used as shown in table 1, without any constraint on the rotation of the rigid beam attached on the top of the panel. In four other cases the beam will be fixed, in which case the basic designated load case name will be followed by ‘-F’ (e.g. LC1-F). Using eqns (4) the expected lateral strength of the panel can be determined for each different loading case along with the dominant mode of failure, see table 2.

Table 1. Load cases.

<table>
<thead>
<tr>
<th>Load case</th>
<th>Applied vertical load (kN)</th>
<th>Average Pressure, P (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>LC1</td>
<td>0</td>
<td>0.00</td>
</tr>
<tr>
<td>LC2</td>
<td>400</td>
<td>0.37</td>
</tr>
<tr>
<td>LC3</td>
<td>800</td>
<td>0.74</td>
</tr>
<tr>
<td>LC4</td>
<td>1200</td>
<td>1.11</td>
</tr>
</tbody>
</table>

Table 2. Expected lateral load resistance in kN using eqns (4).

<table>
<thead>
<tr>
<th>Constraint</th>
<th>Material type</th>
<th>Applied vertical load (kN)*</th>
<th>0</th>
<th>400</th>
<th>800</th>
<th>1200</th>
</tr>
</thead>
<tbody>
<tr>
<td>Free top rotation</td>
<td>P1, 0.5 MPa</td>
<td>162 (a)</td>
<td>282 (a)</td>
<td>402 (a)</td>
<td>522 (a)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>P2, 0.1 MPa</td>
<td>32 (a)</td>
<td>152 (a)</td>
<td>209 (c)</td>
<td>251 (c)</td>
<td></td>
</tr>
<tr>
<td>Fixed top rotation</td>
<td>P1, 0.5 MPa</td>
<td>324 (a)</td>
<td>475 (c)</td>
<td>567 (c)</td>
<td>646 (c)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>P2, 0.1 MPa</td>
<td>65 (a)</td>
<td>156 (c)</td>
<td>209 (c)</td>
<td>251 (c)</td>
<td></td>
</tr>
</tbody>
</table>

*: The letter in brackets indicates which of eqns (4) has been used to calculate the given value and thus the implied mode of failure: (a) base cracking; (c) diagonal cracking.

Results

The results presented here are those for monotonic static analyses using the different loading cases of Table 1. Figure 4 shows load displacement...
diagrams for different top edge restraining conditions, masonry tensile strength and implemented shear retention model (either variable or zero shear retention). The choice of type of shear retention seems to have a considerable influence on the maximum resistance of the walls. In the case of the variable shear retention model principal tensile stresses were observed well in excess of the maximum tensile strength of the material.

As expected, the walls that are restrained from rotating at the top respond more stiffly, however their maximum resistance is similar to that of the walls that were free to rotate.

It should be mentioned that the various curves represent peak response but not ultimate. The runs are being continued, focusing on the ductility characteristics of the walls.

Figure 4. Load displacement diagrams: (a) $f_r=0.5$ MPa, variable shear retention, (b) $f_r=0.1$ MPa, variable shear retention, (c) $f_r=0.1$ MPa, zero shear retention, (d) $f_r=0.1$ MPa, zero shear retention, fixed top rotation.
Figure 5. Deformations, principal tensile stresses and fully open crack patterns for $f_r=0.1$ MPa and low shear retention: (a) LC4, (b) LC4 cracks, (c) LC3-F, (d) LC3-F cracks.

Figure 5 shows the type of deformations and failure modes and stress patterns of the walls for two loading cases, one with the wall being fixed at the top. Resistance to lateral loading seems to occur principally via a diagonal compression zone with cracks that develop along its edges. It is interesting to note how this zone is delimited by two oblique cracks in the case of the fixed wall (figure 5d). This is a typical mode of failure of masonry with triangular sections of wall simply separating from the rest of the structure.

Conclusion

Numerical, finite element simulations of masonry walls subject to in-plane forces similar to those imposed by an earthquake has been carried out for a number of different loading cases. Several different modelling options have been utilised. Even though the masonry has been modelled as an isotropic material the finite element model seems to perform
satisfactorily. The model is capable of reproducing the typical response of masonry walls characterised by a stiff elastic branch and followed by a flattening part as the material degrades. Further, the model seems to reproduce the typical modes of failure of masonry, with horizontal cracks appearing along the top and bottom wall edges and diagonal cracking crack patterns forming in the body of the wall.

Current work aims at continuing the analyses under increasing imposed displacements so as to determine the ductility characteristics of masonry panels and possibly to determine the point at which collapse occurs. Further points of interest are the comparison of the results with experimental data, and the application of the model in a dynamic analysis.

References


