Stress wave effects on the dynamic axial buckling of cylindrical shells under impact
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Abstract

Dynamic axisymmetric buckling of cylindrical shells, loaded dynamically by a Kolsky bar, is studied in order to analyse the influence of stress wave propagation on the initiation of buckling. The considered specimens are made of an aluminium alloy which has linear strain hardening and displays the Bauschinger effect. The acceleration wave speeds are obtained assuming the Tresca yield criterion. The deformation process is analysed by a numerical simulation using a discrete model for the shell and the theoretical predictions are compared with some published experimental data. It is shown that the different modes of failure depend on the impact velocity, and the pattern of buckling is a result of the stress wave propagation phenomenon, but the final shape depends on the inertia properties of the shell.

1 Introduction

Thin-walled metal tubes are used widely in industry, so that their static and dynamic behaviour has been studied both theoretically and experimentally. In particular, axially loaded tubes have been examined in order to obtain the critical forces, types and modes of buckling, the energy absorbing properties, etc. However, because of the complexity of the
problem, a detailed knowledge on the formation of the plastic mechanism is not completely understood due to the difficulties in obtaining the stress profiles in thin-walled members. In particular, the initiation of the buckling mechanism as a transient process has received little attention since the theoretical studies of Lee\textsuperscript{6,7}, except for and some recent results\textsuperscript{8-10}.

The experiments on aluminium alloy cylindrical shells reported by Chen Changen at al.\textsuperscript{4} and Li Ming at al.\textsuperscript{5} show some local buckling effects which are explained tentatively by stress wave effects. The effects of stress wave propagation on the elastic-plastic buckling of a rod subjected to an axial mass impact is studied numerically by Karagiozova & Jones\textsuperscript{9} using a discrete model, where the theoretical predictions are shown to be in agreement with some published experimental data. It is found that the inertia properties of the striking mass play an important role during the entire buckling process and different plastic waves propagate along the rod which cause various patterns of buckling initiation\textsuperscript{9}. The dynamic response of circular cylindrical shells, which have several geometrical characteristics and are subjected to an axial mass impact, is analysed by the same authors\textsuperscript{10} in order to clarify the influence of the elastic and plastic stress waves on the initiation of buckling and to provide some insight into the post-buckling behaviour. Different loading conditions are modelled to clarify the influence of the experimental techniques on the buckling process. Dynamic axisymmetric buckling of cylindrical shells loaded dynamically by a Kolsky bar is studied in this paper in order to analyse the influence of stress wave propagation on the initiation of buckling for this particular loading technique.

2 Loading Conditions

The Kolsky bar was used as a loading and a measuring devise by Li Ming at al.\textsuperscript{5} to study experimentally the dynamic buckling of circular cylindrical shells. This experimental set up is presented schematically in Fig.1. The striker bar is 300 mm long with a diameter of 14.9 mm, the input and the output bars are 1000 mm long with a diameter of 20 mm and are made of a high strength steel. The specimens are made of an aluminium alloy having a yield stress $\sigma_0 = 210$ MPa, Young’s modulus $E = 69$ GPa and a hardening

![Figure 1](image-url)
modulus $E_h = 828$ MPa. It is assumed in the numerical calculations that this aluminium alloy is strain rate insensitive. All specimens have a length $l = 45$ mm, an outer diameter 14 mm and specimens having two different thicknesses are examined - specimens T having $h = 1.5$ mm and TE having $h = 1$ mm.

The input and output bars are much longer than the specimens, so that the transmission of the load to the specimen should be considered as a transient process for impact velocities between 25 and 38 m/sec, which are analysed in this paper. It is assumed that for this particular range of impact velocities only elastic stress waves can propagate along the steel bars. Thus the velocity and strain distribution along the input bar can be obtained from the one dimensional elastic wave theory

$$\frac{\partial^2 u_B}{\partial x^2} = c_B^{-2} \frac{\partial^2 u_B}{\partial t^2},$$  \hspace{1cm} (1)$$

where $u_B$ is the axial displacement and $c_B$ is the elastic wave speed in the bar. Taking into account the corresponding initial conditions

$$u_B(x,0) = 0, \quad \dot{u}_B(0,0) = v_0; \quad \ddot{u}_B(x,0) = 0, \quad 0 < x \leq l_B$$ \hspace{1cm} (2a-c)

and the boundary conditions

$$\frac{\partial^2 u_B(0,t)}{\partial t^2} = c_B^2 (\beta l_B)^{-1} \frac{\partial u_B}{\partial x}, \quad \dot{u}_B(l_B,t) = 0 \quad \text{for} \quad 0 \leq t \leq l_B/c_B, \hspace{1cm} (3a,b)$$

the expressions for $v(x,t^*)$ and $\varepsilon(x,t^*)$ at $t^* = l_B/c_B$ are obtained

$$v(x,t^*) = v_0 e^{(x-l_B)/\beta l_B} \quad \text{and} \quad \varepsilon(x,t^*) = -v_0 c_B^{-1} e^{(x-l_B)/\beta l_B}, \hspace{1cm} (4a,b)$$

where $l_B$ is the length of the input bar, $\beta = G_{st}/G_B$ is the mass ratio between the striking and the input bar and $\beta = 0.163$ in the particular case examined.

The deformation process for $t > l_B/c_B$ involves an interaction between the stress waves through the contact surfaces of the bars and the shell but simplifications are made in order to focus the attention on the stress waves in the shell. The total energy of the system in Fig. 1 at $t = l_B/c_B$ is a sum of the kinetic energy of the striker, $T_{st}$, kinetic energy of the input bar, $T_k$ and the deformation energy of the input bar, $T_w$. Using eqns (4) and the characteristics of the particular loading device, the kinetic energy transmitted to the shell at $t = l_B/c_B$ is

$$T_k \approx G_{st} v_0^2 / 4.$$ \hspace{1cm} (5)

It is assumed in the numerical calculations that these loading conditions are equivalent to an impact on the shell by a mass $G = G_{st}/2$ having an initial velocity $v_0$.

The velocity of the shell end attached to the output bar is determined by
the elastic properties of the bar and the resulting stress in the shell end

$$\dot{u}_{sh}(l_{sh}, t) = \dot{u}_B(l_{sh}, t) = \sigma_B(t) / \rho_B c_B, \quad t > l_B / c_B, \quad (6)$$

where \(l_{sh}\) is the length of the shell, \(\rho_B\) is the material density of the bar, \(\sigma_B(t)\) is the stress at the bar cross-section, \(\sigma_B = N_x(l_{sh}, t)/A_B\), \(N_x(l_{sh}, t)\) is the axial force in the shell cross-section and \(A_B\) is the cross-section of the bar. It was found that the time of the impact event is comparable with the time \(t = 2l_B/c_B\), so that the assumed boundary conditions for the shell are described by eqns (3) and (6).

3 Discrete model of the shell

Metal tubes with small radius-to-thickness ratios are known to collapse into axisymmetric modes so that, for the purpose of analysis, the structure can be divided into longitudinal strips. Thus a lumped mass model can be used (Fig. 2) for the dynamic axisymmetric buckling of a circular cylindrical shell as, proposed in Reference 10.

The model in Fig. 2 consists of \(n\) rigid (with respect to bending), but compressible and weightless links of length \(L = L_i - \delta_i\) connected by springs, which simulate the elastic-plastic material properties, and \(\delta_i\) is the axial reduction of the \(i\)-th link. The springs in the axial direction cater for the axial forces and the bending moments, while the springs in the lateral direction model the forces caused by the circumferential membrane forces in the actual shell. The forces \(F_{i\alpha}^x\) and \(F_{i\alpha}^\theta\) are associated with the axial and the circumferential membrane stresses, respectively, which act in the \(\alpha\)-th “fibre” through the shell thickness, where \(\alpha = 2n_t + 1\), and \(n_t\) is the number of integration points across the thickness. The thickness of the model is \(2L_1 = h\), where \(h\) is the thickness of the actual shell. It is assumed that the total mass of the shell per unit hoop length, \(m_1\), is distributed as discrete masses \(m = m_1/2n\) at each end of the link.

Figure 2: A discrete model
Large displacements in axial and radial directions are considered for the model in Fig. 2, so that the equations of motion for $|\varphi_i| < \pi/2$ are

\[ G\ddot{u}_0 - N_0^x = 0, \]

\[ m\ddot{u}_i + mf_i - N_i^x + N_{i-1}^x = 0, \quad i = 1, \ldots, n \quad (7a,b) \]

\[ \frac{mL_i^2}{2} \ddot{\varphi}_i - \frac{mL_i}{2} f_i \sin \varphi_i + \frac{mL_i^2}{2} \cos \varphi_i - \frac{mL_i}{2} f_{i+1} \tan \varphi_{i+1} \cos \varphi_i = 0, \]

\[ \frac{mL}{2} \left( \ddot{\xi}_{i-1} + 2\ddot{\xi}_i + \ddot{\xi}_{i+1} \right) \cos \varphi_i + M_{i-1} - M_i \left( 1 + \frac{\cos \varphi_i}{\cos \varphi_{i+1}} \right) + M_{i+1} \frac{\cos \varphi_i}{\cos \varphi_{i+1}} = 0, \quad (7c) \]

where $G$ is the striking mass, $u_i$ is the total axial displacement, $f_i = L - L_i \cos \varphi_i$ is the axial displacement of the $i$-th link due to rotation $\varphi_i$ and the radial displacements, $w_i$, (taken positive in the outward direction) are referred to the model parameters as $w_i - w_{i-1} = \frac{z}{\sin \varphi_i}$. The generalised stresses $N_i^x, N_i^\theta$ and $M_i$ are the respective axial force, circumferential force and axial bending moment in the $i$-th cell.

The initial conditions for the problem studied are

\[ u_i(0) = 0, \quad i = 0, \ldots, n, \quad \dot{u}_0(0) = v_0, \quad \dot{u}_i(0) = 0, \quad i = 1, \ldots, n, \]

\[ w_i(0) = \dot{w}_i(0) = 0, \quad i = 0, \ldots, n \quad (8a-c) \]

and clamped boundary conditions are considered for the specimens

\[ w_0 = w_{n+1} = 0, \quad M_0^x(t) \neq 0 \quad \text{and} \quad M_n^x(t) \neq 0. \quad (9a-c) \]

An elastic-plastic, strain rate insensitive material with linear strain hardening and displaying the Bauschinger effect is modelled. It is assumed that the material obeys the Tresca yield criterion with kinematic hardening and that the temporal movement of the Tresca hexagon in the plane $(\sigma_x, \sigma_\theta)$, identified by the variation of its centre $(\sigma_x^c, \sigma_\theta^c)$, can be associated with the loading path of each fibre of the shell cross-section. In this case, explicit expressions for the stress increments as functions of the strain increments and the material properties can be obtained for elastic and plastic loading and elastic unloading associated with each side of the Tresca diagram as

\[ d\sigma_x = f_1(d\varepsilon_x, d\varepsilon_\theta, v, E, \lambda) \quad \text{and} \quad d\sigma_\theta = f_2(d\varepsilon_x, d\varepsilon_\theta, v, E, \lambda), \quad (10) \]

where $v$ is Poisson's ratio, $E$ is Young's modulus, $\lambda = E/E_h$, and $E_h$ is the
hardening modulus. The biaxial stress field is integrated across the wall thickness to give the corresponding axial and circumferential membrane forces and axial bending moments.

It can be shown that three loading stress wave speeds can be obtained for a media obeying the Tresca yield condition. The elastic loading wave travels with speed \( c^e = \sqrt{E / \rho(1 - v^2)} \) and the plastic waves can propagate with two different velocities, namely

\[
\begin{align*}
    c_1^p &= \pm \sqrt{\frac{E \rho}{2\lambda + (1 - \lambda)\sqrt{3}}} \left\{ \frac{2\lambda}{2\lambda(1 - v^2) + (1 - \lambda)\sqrt{3}} \right\} \frac{1}{\sqrt{2}} \\
    c_2^p &= \pm \sqrt{\frac{E \rho}{2\lambda + (1 - \lambda)\sqrt{3}}} \left\{ \frac{2(1 - v)\lambda(1 + v)(1 - \lambda)}{2\lambda(1 - v^2) + (1 - \lambda)\sqrt{3}} \right\}^{-1/2}
\end{align*}
\]

depending on the stress state, i.e. corresponding to the different sides of the Tresca hexagon.

4 Relationship between the stress waves and buckling shapes

Five particular examples of axial impact are analysed in the paper using the model in Fig. 2. These examples present some characteristic features of the deformation process of circular cylindrical shells when compressed between heavy masses which cause axisymmetric buckling. A comparison between the experimental data and the model predictions are presented in the table below, where \( \delta_y \) is the shortening of the shell, \( \Phi_i \) and \( \Phi_0 \) are the maximum diameter near the input and the output bar, respectively.

<table>
<thead>
<tr>
<th>Specimen</th>
<th>( \nu_0 ), m/sec</th>
<th>Experiment</th>
<th>Model prediction</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( \delta_y ), mm</td>
<td>( \Phi_i ), mm</td>
<td>( \Phi_0 ), mm</td>
</tr>
<tr>
<td>T11</td>
<td>25.01</td>
<td>4.9</td>
<td>-</td>
</tr>
<tr>
<td>T9</td>
<td>34.68</td>
<td>6.0</td>
<td>15.8</td>
</tr>
<tr>
<td>T2</td>
<td>38.02</td>
<td>11.4</td>
<td>15.8</td>
</tr>
<tr>
<td>TE9</td>
<td>25.24</td>
<td>9.0</td>
<td>15.4</td>
</tr>
<tr>
<td>TE2</td>
<td>29.67</td>
<td>9.5</td>
<td>19.0</td>
</tr>
</tbody>
</table>

The corresponding final buckled shapes are shown in Fig. 3. It is evident that the variation of the impact velocity causes different final buckling shapes for the same shell geometry. A relatively thick shell responds mainly by compression at a low impact velocity (Fig. 3(a)), while an increase of the impact velocity causes a dynamic plastic buckling when the entire length of the shell is involved in the buckling process and a
Figure 3: Final buckled shapes; (a)-T11, (b)-T9, (c)-T2, (d)-TE9, (e)-TE2

regular buckling shape is observed (Fig. 3(b)).

The development of the buckling shape of specimen T9 is presented in Fig. 4 at the times of 25, 51, 82 and 164 μsec (the time for the plastic wave to propagate along the length of the shell is 82 μsec). The stress-strain state of the shell at $t = 25$ μsec is shown in Fig. 5(a-c). Figures 5(a,b) present the movement of the yield loci identified by their centres, $q_x^c = \sigma_x^c / \sigma_{cr}$, $q_\theta^c = \sigma_\theta^c / \sigma_{cr}$, where $\sigma_{cr}$ is the critical elastic stress for the actual shell. It is evident that the plastic stress is almost uniaxial and the plastic wave originating from the impacted end has propagated a distance of $0.3l$. Another plastic wave produced by the reflected elastic wave has travelled a distance $0.25l$ from the opposite end. The axial strain distribution, $\varepsilon_x$, is presented in Fig. 5(c) and the distribution of the mean axial strain, for the particular times corresponding to the shapes in Fig. 4, are shown in Fig. 5(d). The buckling wave length is formed completely at $t = 51$ μsec (Fig.4), when the two plastic waves travelling towards each other have met and the entire length of the shell is plastic. For $t > 51$ μsec, the buckling shape does not change and only the radial displacements grow, but without causing
reverse plastic loading in any of the shell cross-sections, so that the final
buckling shape is almost regular.

Further increase of the impact velocity causes local deformations near
the output bar which is evident in Fig. 3(c) (specimen T2). The distribution
of the mean axial strain is shown in Fig. 6. Buckling starts to develop from
the impacted end. However, the radial inertia suppresses the growth of the
radial displacements until the reflected plastic wave from the output bar
causes larger strains and more rapid growth of the radial displacements
leading to large reverse plastic strains at the shell cross-sections near this
particular end. A similar buckling pattern is observed for the thinner shell
(specimen TE9) when is struck with \( v_0 = 25.24 \text{ m/sec} \) (Fig. 3(d)).

Figure 5: Stress-strain state of specimen T9
The increase of the impact velocity applied to specimen TE2 causes a different final buckling shape as shown in Fig. 3(e), where local deformations near both shell ends are observed. The initial phase of deformation is similar to the process described for the T specimens. Buckling starts to develop within a sustained uniaxial plastic flow. However, the radial inertia forces near the impacted end cannot support the unbuckled shape for \( t > 82 \) \( \mu \text{sec} \) and the radial displacements start to grow more rapidly (Fig. 7(a)), which leads to an elastic unloading and a reverse plastic loading at the cross-sections near both ends. The final stress profiles across the shell thickness at \( x/l = 0.067 \) and \( x/l = 0.9 \) are shown in Fig. 7(b,c).

The present results show that the initiation of buckling is governed by stress wave propagation effects, while the inertia properties of the shell play an important role in the post-buckling behaviour and the final shape.

Figure 6: Specimen T2 - mean axial strain distribution

Figure 7: Specimen TE9 - (a) buckling shapes, (b,c) stress distribution
5 Conclusion

The dynamic buckling of elastic-plastic cylindrical shells, loaded dynamically by a Kolsky bar, is governed by the stress wave propagation phenomenon and, in general, the entire length of the shell is involved in the deformation process. However, the final buckling shape depends strongly on the axial impact velocity and the geometry of the shell. Regular shapes occur in relatively thick shells when buckling develops within a sustained plastic flow. A localisation of buckling can occur with an increase of the impact velocity or a decrease of the shell thickness and the particular location of the maximum radial displacements depends on the magnitude of the initial axial velocity.

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References