Modeling of impact load characteristics for dynamic response analysis of concrete structures
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ABSTRACT
A method of simulating the impact load characteristics acting on concrete structures during accidental collisions of vehicles, aircraft, ships, etc. is considered. The impacting body is modeled as a system consisting of interconnecting lumped masses, springs and dashpots. Each lumped mass has the longitudinal and rotational degrees-of-freedom in order to simulate not only the rigidity but also eccentricity in the impacting body. Verification of the analytical method is carried out through comparisons with experimental results. Consequently, it is found that the impact load characteristics as well as acceleration response can be simulated quite accurately. Parametric evaluations of the resulting impact load characteristics based on the effects of mass distribution, spring constants, collision speed and eccentricity are carried out. These parameters are considered to be the main factors that influence the final impact force-time function. Furthermore, the analysis is linked through an interactive process to a separate analytical procedure that is capable of predicting the dynamic response of concrete slab structures. Finally, the response of concrete handrails under vehicular impact collision is performed.

INTRODUCTION
Understanding the impact force-time function and its pertaining characteristics during an impact collision is a major step towards design of concrete structures that are subject to accidental collisions. In order to understand the load function characteristics, it is considered necessary to be able to analytically duplicate or simulate the possible load function for an arbitrary collision. The physical constitution of both the colliding body and the impacted structure remain large factors that dictate the final outcome of the resultant load function. Other related factors are the collision speed and also angle of impact. Once the loading function during the collision of two arbitrary bodies can be accurately predicted, further steps, such as application of probabilistic methods for determining a dynamic design impact load, can be performed. The authors have already proposed an analytical procedure for the dynamic response analysis of ultimate behaviors of reinforced concrete beams\cite{1,2} and slabs\cite{3,4} subjected to soft
impact loads[3,5]. By integrating the dynamic response analysis to an analytical procedure for predicting the resultant impact force function, a total integrated analytical prediction of soft impact problems can be derived.

A multi-lumped-mass model is employed for modeling the collision of deformable impacting bodies into concrete structures. The accuracy of the analytical procedure is verified by comparisons with test results from a series of tests performed. The effects of different physical constitutive elements in the impacting body are analytically studied and the degree of contribution to the resulting impact force function is considered. The multi-mass model is then linked (coupled) to a dynamic analytical process for analysis of reinforced concrete (RC) slab structures through an interactive process. As an example, the effects of automobile collisions into RC handrails are considered.

In this paper, the feasibility of predicting the impact force-time function for an arbitrary soft impact collision is considered by applying a multi-mass model to represent an impacting body. The characteristics in the resultant load function for different compositions of impacting body are also considered. The model is then linked to a dynamic response analysis for RC slab structures through an interactive process to enable a global analysis of the entire impact phenomenon to be performed. The procedure proposed is considered to be of value in the process of seeking a proper dynamic approach to the design of concrete structures that are subjectable to impact loads.

MODELING OF IMPACT LOAD CHARACTERISTICS

Definition of Impact Load Characteristics

The terminology "impact load characteristics" employed here for an arbitrary collision can be defined as consisting of the following physical properties; impact force-time relation, maximum impact force, duration of impact force, loading rate, shape of the impact force function and impulse. A schematic illustration of the impact load characteristics is shown in Fig.1. By quantifying these values regarding to different cases of soft impact loads, a guideline for drafting the necessary design impact loads for different collisions can be performed. Furthermore, application of probabilistic methods can also be carried out efficiently once these values are expressed quantitatively.

Outline of Analytical Procedures for Prediction of Impact Load Characteristics

Various analytical procedures have been proposed for calculating the resulting impact force function for impact collisions. One of the most well-known models is the Riera approach[6,7], applied in predicting the impact force function for collision of aircraft into concrete nuclear reactor containments. Various research have been reported in the literature especially regarding the impact of aircraft or airborne projectiles into reinforced concrete structures[8,9,10,11]. Most of the methods available require a detailed knowledge of the distribution of mass along the axis or also the buckling strength within different portions of the impacting body.

In this particular study, main attention is placed in modeling especially the impact force function derivable from the collision of an automobile into a concrete barrier or structure. One of the main objectives of the present research is the design of reinforced or prestressed concrete handrails that are subjected to vehicular impacts. Unlike aircraft or projectiles, where the mass distribution is rather small with reference to the total length, the mass of an automobile is concentrated along
Structures under Shock and Impact

Fig. 1 Schematic representation of impact load characteristics

a rather short distance and the effects of eccentricity along the impacting axis can be considered to be large. Therefore, determining the distribution of mass and also buckling strength would be rather difficult. Furthermore, it has also been reported that a multi-mass model can predict the resultant impact force function for an aircraft colliding into a concrete barrier rather accurately[8]. Considering these facts, it can be considered that a multi-mass model would have a more practical application for simulating the collision of different types of impacting bodies.

Moreover, the authors are at present carrying out research on prediction of the original physical constitutive model of the impacting body based on the derived impact load characteristics. By applying the “System Identification (SI)” procedure[12] to a known impact force function, it is possible to predict the type and physical constitution of the impacting body. But the SI method requires a large amount of calculation when a rather complex model is applied. Therefore, instead of adopting a complex and presumably more accurate model, the present multi-mass model is adopted here. The present paper deals only with the application of the multi-mass model, while the application of the SI method for “backfiguring” of the physical constitution of an impacting body together with concepts for drafting design impact loads will be dealt with in a following paper.

Analytical Procedure

The impacting body, which is deformable under soft impacts, is modeled as a system consisting of lumped masses interconnected by nonlinear visco-elasto-plastic axial and rotational springs as shown in Fig.2. Each lumped mass has the longitudinal and rotational degrees-of-freedom in order to simulate not only the rigidity but also eccentricity in the impacting body. The effects of eccentricity in the impacting body are considered necessary as the mass distribution along the central axis, especially in automobiles, is not uniaxial and in most cases, collisions occur at various angles.

To simulate impact collisions into concrete structures (deformable targets), it is considered that the concrete target is connected to a rigid wall with a spring (spring stiffness, $k_f$) and a dashpot (coefficient of viscous damping, $c_f$). By altering the values of $k_f$ and $c_f$, it is possible to simulate deformation and also the failure process within a concrete structure. The impact force-time function during
an impact collision can be given as a sum of forces in the spring located next to the target and the total relative deceleration of the lumped-mass point next to the target\cite{8}. Therefore, the impact force-time function acting on the concrete target can be expressed as,

\[ F(t) = R_1 + I_1/\Delta t \]  \hspace{1cm} (1)

where, \( R_1 \) = force in the spring which is located next to the target (spring stiffness, \( k_1 \)), \( I_1 \) = impulse from collision of 1st mass to target and \( \Delta t \) = time in which transfer of momentum occurs.

The dynamic equation of motion is solved using the Newmark-\( \beta \) method (\( \beta = 1/6 \)). The equation of motion for the entire system can be treated as,

\[ [M] \{\ddot{U}\}_t + [C] \{\dot{U}\}_t + [K] \{U\}_t = \{R\}_t \]  \hspace{1cm} (2)

where, \([M]\) represents the mass and moment of inertia matrix, \([C]\) and \([K]\) are the damping and stiffness matrices for the axial and rotational springs while \( \{R\}_t \) represents the external load vector. \( \{\ddot{U}\}_t \), \( \{\dot{U}\}_t \) and \( \{U\}_t \) are the acceleration, velocity and displacement vectors, respectively. The subscript “\( t \)” denotes the quantities at time \( t \) while a dot denotes a derivative with respect to time.

The response history is divided into discrete time increments \( \Delta t \), which are of equal length. The system is calculated for each time increment with properties determined at the beginning of the interval. The discretized equation for Eq.(2) during a discrete time increment of \( \Delta t \) is given in the following equation,

\[ [M] \{\Delta U\}_{t\rightarrow t+\Delta t} + [C] \{\Delta \dot{U}\}_{t\rightarrow t+\Delta t} + [K] \{\Delta U\}_{t\rightarrow t+\Delta t} = \{\Delta R\}_{t\rightarrow t+\Delta t} \]  \hspace{1cm} (3)

As Eq.(3) is simply an approximating equation of motion, it is solved using the Newmark-\( \beta \) method, as given by the following equations,

\[ \{\Delta U\}_t = \{\ddot{U}\}_t \cdot \Delta t + \{\dot{U}\}_t \cdot \frac{\Delta t^2}{2} + \{\Delta \dot{U}\}_t \cdot \frac{\Delta t^2}{6} \]  \hspace{1cm} (4)

\[ \{\Delta \dot{U}\}_t = \{\dddot{U}\}_t \cdot \Delta t + \{\Delta \ddot{U}\}_t \cdot \frac{\Delta t}{2} \]  \hspace{1cm} (5)

Substituting Eqs.(4) and (5) into Eq.(3) brings about the following equation,
The above equation of motion is solved for each time step interval, resulting in the acceleration, velocity and displacements at each time step.

**EXPERIMENTAL PROCEDURE**

Verification tests are carried out to check the applicability and accuracy of the analysis. Fig.3 shows the test apparatus employed for impact force simulation. The impacting body consists of 3 separate cubic metal rigid bodies connected by rubber pads (7.5x7.5x6.0 cm), which act as springs and dashpots. A 200 kgf (1.96 kN) weight, with a load cell attached to the front face, is used as the target. Both the target and the impacting body are suspended from the ceiling by metallic wires and can move freely in the motion similar to a pendulum. By raising both the target and the impacting body to different heights, different collision speeds can be attained. The impact force at the impacting face and acceleration response at the center of gravity (CG) of each lumped mass are recorded using an analog data recorder. The testing apparatus allows large collision speeds to be obtained without requiring much horizontal clearance. Furthermore, by allowing the target to be movable, it is possible to simulate a semi-rigid target, such as concrete structures, and also a moving target. The main difficulty here is to provide adequate mass ratio for the target in relative comparison with the impacting body.

Fig.4 shows the details of the impacting body employed in the tests. Each lumped mass consists of several metal plates being held together by bolts at four corners, as shown in the figure. By altering the number of metal plates in each lumped
mass, different variations of masses can be obtained. The effects of eccentricity within the impacting body can be simulated by displacing the metal plates relative to the central axis. Tests are carried out on three different combinations of masses to study the effects of combination of masses and also eccentricity. A summary of the tests performed is listed in Table 1. The notation “L” is used to indicate a small mass (approximately 15kgf (147N) in weight) while “H” represents a comparatively larger mass (approximately 25kgf (245N) in weight). In the tests with eccentricity, “M” denotes a mass on the central axis while “U” represents a mass with eccentricity relative to the central axis. In Series 1, the weight of each lumped mass, the number of mass and collision speed are the parameters for the test while the distribution of mass and collision speeds are the parameters for Series 2. The effects of eccentricity are considered in Series 3 of the tests.

VERIFICATION OF ANALYTICAL PROCEDURE

The validity of the analytical procedure can be confirmed by comparing the test results with the analysis results. Since direct collisions between each lumped-mass is not possible in the tests carried out due to the presence of rubber pads, it is considered that the effects of transfer of momentum between the lumped mass and concrete target, which is given by the second term in Eq.(1), can be neglected here. Thus, the impact force function can be assumed as,

$$F(t_1) = R(t_1) + k_1(u_1(t_1) - u_T(t_1))$$

where, $k_1$ = spring stiffness of spring attached to 1st mass, $u_1$ = displacement of 1st mass and $u_T$ = displacement of target.

Table 2 shows a comparison between the impact load characteristics for the test and analysis. The typical results for impact force function and acceleration response for each series of tests are shown in Figs.5 through 10, respectively. The results in Table 2 indicate that the duration of load in the analysis of Series 1 and 2 are larger than the tests, while the time to maximum load is larger in the whole series of analysis. On the whole, the predicted maximum impact force in the analysis can be considered to be quite accurate.

Fig.5 shows a comparison of the impact force functions for a uniaxial 3-mass system. The denotation $h$ represents the height of fall of the impacting body. Two different heights of fall, i.e., $h = 5\text{cm}$ and $h = 20\text{cm}$ are carried out. The results show that the analysis gives quite good predictions of the resulting impact forces, particularly the results until the first peak values. But the results after the
### Table 1 Details of tests

<table>
<thead>
<tr>
<th>Test series</th>
<th>Test code</th>
<th>Number of masses</th>
<th>Distribution and weight of mass (kgf)</th>
<th>Distribution of eccentricity (cm)*</th>
<th>Mass ratio (impactor, target)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Series 1</td>
<td>L</td>
<td>1</td>
<td>(5-0-0)</td>
<td>(0)-(0)-(0)</td>
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<td></td>
<td>LL</td>
<td>1</td>
<td>(5-15-0)</td>
<td>(0)-(0)-(0)</td>
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<tr>
<td></td>
<td>LLL</td>
<td>3</td>
<td>(5-15-15)</td>
<td>(0)-(0)-(0)</td>
<td>0.225</td>
</tr>
<tr>
<td></td>
<td>H</td>
<td>1</td>
<td>(25-0-0)</td>
<td>(0)-(0)-(0)</td>
<td>0.126</td>
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<tr>
<td></td>
<td>HH</td>
<td>2</td>
<td>(25-25-0)</td>
<td>(0)-(0)-(0)</td>
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<tr>
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<td>HHH</td>
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<td>(25-25-25)</td>
<td>(0)-(0)-(0)</td>
<td>0.369</td>
</tr>
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<tr>
<td></td>
<td>HH</td>
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<td>(25-25-25)</td>
<td>(+3.76)-(0)-(0)</td>
<td>0.363</td>
</tr>
<tr>
<td></td>
<td>MM</td>
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<td>(25-25-25)</td>
<td>(+3.76)-(0)-(0)</td>
<td>0.363</td>
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<td>MMU</td>
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<td>(25-25-25)</td>
<td>(+3.76)-(0)-(0)</td>
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</table>

Note: Spring stiffnesses are slightly different in HHH, HHF, and MM. '+' sign represents upward eccentricity with respect to central axis.

The maximum impact forces differ slightly from the test results especially for the results of $h=20$cm. The curve shapes in the unloading regions are slightly different in the analysis. The main reason for the difference in the unloading regions can be attributed to the difference in material properties of the rubber pad during loading and unloading. In the analysis performed, the material characteristics for the rubber pad are modeled using only the loading portion of the force-deflection relation. On the whole, the analysis predicts a larger duration of impact force.

Fig. 6 shows the corresponding comparison of acceleration response for the case of

### Table 2 Comparison between test and analytical results

<table>
<thead>
<tr>
<th>Test code</th>
<th>Collision speed (m/sec)</th>
<th>Maximum load (tf)</th>
<th>Loading rate (tf/msec)</th>
<th>Time to max. load (msec)</th>
<th>Duration of load (msec)</th>
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<td>Test</td>
<td>Analysis</td>
<td>Test</td>
<td>Analysis</td>
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<td>L</td>
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<td>1.625</td>
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<td>0.865</td>
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<td>1.874</td>
<td>2.230</td>
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<td>0.299</td>
</tr>
<tr>
<td>HHH</td>
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<td>0.911</td>
<td>0.121</td>
<td>0.167</td>
</tr>
<tr>
<td></td>
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<td>2.321</td>
<td>2.351</td>
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<td>MMM</td>
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<td>UMM</td>
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<td>0.132</td>
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</table>

Note: Spring stiffnesses are slightly different in HHH, HHH, and MMM.
Fig. 5 Comparison between impact force functions for test and analysis (Series 1 - HHH) \((1\text{tf} = 9.807 \text{kN})\)

Fig. 6 Comparison between acceleration response for test and analysis (Series 1 - HHH; \(h=20\text{cm}\))

\(h=20\text{cm}\). A time lag is present between the response of each mass in both the tests and analysis. The response is first noticed in the first mass, and then appears consecutively in the following masses. In the present analysis, the coefficient of viscous damping for the impacting body is set at zero due to lack of the relevant information. Consequently, the effects of damping in the acceleration response is not dominant in the analytical results. The period in the analysis is therefore larger than in the tests. The amount of acceleration and also the curve shapes predicted in the analysis are in good accordance with the test results.

The results for a 3-mass system with a small mass at the center are shown in Figs. 7 and 8. The results in Fig. 7 show that the analysis gives good predictions of the resulting impact forces particularly until the first peak. But the analytical results after the maximum impact force differ from the test results. The curve shape during unloading is roughly similar in both the test and analysis but the duration of impact force is larger than in the tests. As for the acceleration response given in Fig. 8, the analysis results are also in good agreement with the tests except for the lack of decay in the analysis.
Fig. 7 Comparison between impact force function test and analysis (Series 2 - HLH) \((1tf = 9.807 \text{ kN})\)

Fig. 8 Comparison between acceleration response for test and analysis (Series 2 - HLH; \(h=20\text{cm}\))

The results for effects of eccentricity are given in Figs. 9 and 10. The analytical results are different from the tests, especially in the acceleration response of the 2nd and 3rd masses. The amount of acceleration predicted is smaller than in the tests, together with a larger period. As for the impact force function, the entire loading duration is in good agreement with the test results. The shape of the impact force function, particularly for \(h=5\text{cm}\) is similar in both cases. The reason for the difference of the results in this series of test can be attributed to the fact that the rotational stiffness of the rubber pads applied in the analysis are different from the actual stiffness. Unlike the axial stiffness, the rotational stiffness of the rubber pads are difficult to be obtained experimentally and thus, assumed values are employed in the analysis.
On the whole, the multi-mass model is capable of predicting the resultant impact load characteristics accurately up to a practical degree, especially the initial curve shapes. Furthermore, the difference in the acceleration response of each mass is correctly simulated in the analysis. It is considered that the impact failure modes and principal dynamic behaviors of concrete structures are usually dominated by the initial part of the resulting impact force function. Since the present analysis is capable of predicting accurately the initial peak of the resulting impact force function, the analysis can be practically employed for studying the dynamic behaviors of concrete structures.

**EFFECTS OF CONSTITUTIVE ELEMENTS IN THE MULTI-MASS MODEL**

The effects of the physical constitution of an impacting body is studied analytically here. An idea of the amount of contribution of each element to the resultant impact force function would be of help when considering the design impact loads for concrete structures subjectable to various collisions.
**Effects of Mass Distribution**

The effects of mass distribution in a model with constant spring stiffnesses and also constant collision speeds are analytically considered. Fig. 11 shows the impact force function for a system with different weights (20 - 100kgf (196 - 981N)) of the front mass. From the figure, it is clear that the value of the first peak and also the duration until maximum impact force are heavily dependent on the weight of the first mass. Therefore, it can be concluded that when other factors are kept constant, the maximum impact force and duration until maximum impact force increase with an increase in the weight of the front mass.

**Effects of Number of Masses**

For a constant amount of total mass, the effects of the number of equally discretized lumped masses are analytically considered here. Fig. 12 shows the analytical results of the impact force function for an impacting body with total weight of 75kgf(736N). The spring constant as well as collision speeds are kept constant while the number of equal lumped masses are varied as one, three and five. From the figure, it is clear that as the number of masses are increased, the maximum impact force as well as duration until maximum impact force decreases, but the total loading duration increases. The main reason for the decrease in maximum impact force and also the duration until maximum impact force may be attributed to the decrease in weight of the front mass as the number of masses are increased. Moreover, the number of conflation points in the load function increases as the number of masses are increased, thus causing the loading duration to be larger.

**Effects of Spring Constants**

Altering only the spring constant (0.5 - 5.0tf/cm (4.9 - 49.0N/cm)) for the front spring in a 3-mass system produces the results shown in Fig. 13. It is clear that the loading rate is governed by the size of the spring constant in the foremost spring. The maximum impact force increases while the duration until maximum impact force decreases with an increase in the value of the spring stiffness in the front spring. Therefore, the duration of load for soft impacts are relatively larger compared to rigid (hard) impacts.

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![Fig. 11 Analytical results of impact force functions (Effects of mass distribution)](image1)

![Fig. 12 Analytical results of impact force functions (Effects of number of masses)](image2)
Effects of Collision Speed

Fig. 14 shows the analytical results of the impact force function for a 3-mass system. All the factors are kept constant except for the collision speed. The collision speed is varied between 2 to 12 m/sec. It can be considered that the collision speed affects both the maximum impact force and the loading rate, but the loading duration and the duration until maximum impact force are not dependent on the collision speed. The main reason that the duration is not affected can be attributed to the fact that the natural frequency of a multi-mass model is not related to the collision speed.

Effects of Eccentricity

Fig. 15 shows the analytical results of a 3-mass system with equal eccentricity within different portions of the system. It is clear that the duration of impact force is dependent on the amount and position of mass eccentricity. The difference can be attributed to the change in natural frequency of the system as the eccentricity is altered. Another important result is that eccentricity at the front portion of a system (UMM) can cause a large difference in the impact force function. Therefore, not only when eccentricity is present at the front portion, but also when collisions occur at different angles (relative eccentricity), the shape of the resultant function can be expected to be different.

LINKING PROCEDURE FOR DYNAMIC RESPONSE ANALYSIS OF IMPACT COLLISION

Linking (coupling) the multi-mass model to a dynamic response analysis of concrete structures is necessary to give a complete analysis of an entire impact phenomenon. It has been reported that uncoupling of both analyses would produce adequate results of the total behavior of concrete structures subjected to soft impact loads[13]. But the real effects of the spring constant \( k_1 \) and coefficient of viscous damping \( c_1 \) used in the multi-mass model to represent the deformation process of the concrete structures would not be properly simulated when the analyses are uncoupled.

The authors consider that it is feasible to uncouple the analyses when determining
the impact load characteristics and also the design impact load. But when a detailed study of the dynamic behavior of the concrete structure is necessary, coupling of both analyses is important. An outline of the procedure employed for coupling of both analyses is shown in Fig. 16. When considering the dynamic behavior of concrete structures, there is a necessity of evaluating the effective mass as well as effective stiffness of the structure that is directly involved in the impact phenomenon. By applying an interactive method as shown in Fig. 16, the total mass of the structure as well as the effective stiffness from the dynamic behavior analysis can be directly input into the multi-mass model, thus resulting in a proper and updated evaluation of the effective mass as well as effective stiffness of the concrete structure.

ANALYSIS OF REINFORCED CONCRETE HANDRAILS

A practical example of the application of the analytical procedure is the analysis of vehicles colliding into concrete handrails of expressways. An ideal design procedure of concrete handrails for expressways is rather difficult. An ideal handrail should be able to withstand the impact from a colliding vehicle. The handrail should not act as a solid barrier to stop the collision but more as a flexible wall that is capable of absorbing most of the impact collision energy. Therefore, it is necessary to design concrete handrails to fail under bending as energy absorption is better during ductile type of failure[4].

For simplicity and also due to the lack of experimental data on full-scale vehicles, six classifications of vehicles can be assumed as shown in Table 3. The vehicles are assumed as a system consisting of three lumped masses. This is the minimum amount of mass necessary to produce the main difference in the impact load characteristics for various vehicles. The present study here is aimed at providing a guideline of the impact load characteristics for different classifications of vehicles. Other methods, such as the FEM, would be more appropriate when a proper response of vehicles are needed. But in the case of setting guidelines for the impact load characteristics for structural design, the present method is considered to be adequate. The collision speed and total weight are based on the design specifications of an expressway in Japan[14] while the spring constant is based on
Initial conditions
Material characteristics

Nonlinear dynamic analysis of RC slab structures

① DATA CHECK
② Dynamic analysis of impact load characteristics
③ Slab stiffness
④ Impact force

* Failure criteria or end of impact collision
① Displacement, acceleration, strain, stress, slab stiffness

Fig. 16 Linking (coupling) procedure for global analysis of impact collision

Table 3 Vehicle types and conditions employed in analysis

<table>
<thead>
<tr>
<th>Vehicle type</th>
<th>Total weight (tf)</th>
<th>Distribution of weight (tf)</th>
<th>Collision speed (km/hr)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Automobile (Front engine)</td>
<td>1.2</td>
<td>0.6 - 0.3 - 0.3</td>
<td>80</td>
</tr>
<tr>
<td>Automobile (Midship engine)</td>
<td>1.2</td>
<td>0.3 - 0.6 - 0.3</td>
<td>80</td>
</tr>
<tr>
<td>Automobile (Rear engine)</td>
<td>1.2</td>
<td>0.3 - 0.3 - 0.6</td>
<td>80</td>
</tr>
<tr>
<td>Small truck</td>
<td>1.6</td>
<td>0.6 - 0.5 - 0.3</td>
<td>80</td>
</tr>
<tr>
<td>Large truck</td>
<td>5.5</td>
<td>2.5 - 1.3 - 1.7</td>
<td>65</td>
</tr>
<tr>
<td>Large truck (loaded)</td>
<td>14.0</td>
<td>2.5 - 5.5 - 6.0</td>
<td>50</td>
</tr>
</tbody>
</table>

* Spring constant: 0.7 (tf/cm), Collision angle: 90 deg.

the average stiffness of a normal automobile obtained from static failure (crush) tests[15]. In this present study, the effects of collision of the front-engine car and small truck into RC handrails will be analytically considered. Furthermore, full-scale tests were performed on actual RC handrails where a single rigid metal mass of 2.5tf(24.5kN) was impacted from a height of 105.2cm (potential energy = 2.63tf·m (25.8kN·m)) in a motion similar to a pendulum. Details of the latter test are given in Reference 16. These three loading conditions will be analytically considered in this Section.

A nonlinear dynamic layered finite element procedure is applied in the analysis of RC handrails. The finite element meshes for the handrail is shown in Fig. 17. The Newmark-β method is employed to solve the equations of motion during discrete time intervals. The effects of plasticity, cracking in concrete elements, triaxial yield and failure criterion for concrete, transverse shear stresses, and also the loading and unloading phenomena in the plastic regions are incorporated into the procedure. Details of the analytical procedure are given in Reference 3 and 4. The impact failure mode for an impact collision can be determined based on the deformation characteristics in the handrail. Fig. 18 shows the results of impact force-deflection relation for analysis of the full-scale tests (denoted as "Test"), the front-engine car (denoted as "Car(FE)"") and small truck (denoted as "Truck(S)""). The distribution of deflection in the cross-sections at failure are shown in Fig. 19. Failure is defined in the analysis as the point where concrete crushing occurs.

Fig. 18 shows that the impact force-deflection relation for both the "Car(FE)" and
"Truck(S)" are similar while the results of the Test is totally different. The loading rate in the Test is larger because the impacting body is rigid and therefore, resulting in a higher initial stiffness as well as loading rate. This phenomena can also be attributed to the effects of inertia in the structure. A larger amount of inertia can be expected under higher loading rates. In the case of both vehicles, deformation of the vehicles occur during the collision and thus resulting in a slow loading rate. Even though the maximum impact force is larger for the "Test", the deflection at failure is small when compared with both vehicles. This phenomenon can be attributed to the short duration of loading in the "Test". The final amount of deflection in the test handrail can be expected to be larger, i.e., appearing only after failure (concrete crushing) occurs. From Fig.19, it is clear that the deflection is concentrated only at the middle of the slab for the "Test" while the deflections are distributed all over the structure for the "Car(FE)" and "Truck(S)" results.

Fig.20 shows the expected crack patterns in the front and rear faces from the analysis. The crack pattern for the "Car(FE)" load and "Test" are considerably different. The results from Figs.18 and 19 indicate that in the analysis of the "Test", the effects of inertia and higher modes of vibration are evident and this
causes local failure to occur. This is the main cause of the complicated crack patterns in Fig.20(a). In Fig.20(b), total structural failure can be expected and therefore the crack patterns are uniformly distributed. These crack patterns almost agree with the actual crack patterns obtained from the actual full-scale tests[14].

Fig.21 shows the deformation mode at failure in the handrails from analysis. It is clear that the deformation mode for the "Test" indicate punching shear failure at the middle of the handrail while for the "Car(FE)" load, bending or total structural failure can be expected.

(a) Deflection in D-D section of Fig.17
(b) Deflection in C-C section of Fig.17

Fig.19 Distribution of deflection for RC handrail at failure (Analysis)

Fig.20 Direction perpendicular to main principal stress of RC handrail (Analysis)
CONCLUSIONS

The applicability of the multi-mass model to simulate the impact load characteristics of deformable impacting bodies is studied. The model is then linked to a dynamic response analysis of concrete slab structures through an interactive process to enable a complete analysis of the interaction between the impacting body and concrete target during soft impacts. The linked analysis is considered to be a powerful tool for application to the dynamic design of concrete structures that are subjectable to soft impact collisions.

The main conclusions from the present study can be summarized as follows:

1. The multi-mass model is capable of simulating the resultant impact load characteristics for soft impact collisions accurately up to a practical degree. In particular, the maximum impact force and also initial curve shape can be accurately predicted. Since the impact failure modes and principal dynamic behaviors of concrete structures are usually dominated by the initial part of the resulting impact force function, the analysis can be practically employed for studying the dynamic behaviors of concrete structures.

2. The resulting impact load characteristics for an impact collision is affected by the following physical properties: mass distribution, number of masses, spring constants, collision speed and degree of eccentricity. Eccentricity at the front portion of a system can cause a large difference in the impact force function. When eccentricity is present at the front portion, or when collisions occur at different angles, the shape of the resultant function can be expected to be different.

3. Linking the multi-mass model to dynamic response analysis of concrete structures through an interactive process would enable a complete global analysis of soft impact collisions to be performed. Moreover, the effective stiffness and effective mass of the concrete target can be simulated accurately using such a procedure.

4. The impact failure modes in RC handrails are affected by the rigidity of the impacting body. The punching shear failure mode is dominant during the collision
of a rigid body while a deformable body (vehicle) would more likely cause flexural failure to occur.

REFERENCES