Dynamic analysis of slender towers made of no-tension material with limited compressive strength

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Abstract

This paper deals with a simplified non-linear model of continuous cantilever beams that, while it can make it easier to represent the dynamic response of masonry columns and slender structures with simple geometry and flexural behaviour, is able to account for the main mechanical characteristics of the material.

A non-linear elastic constitutive law is defined in terms of generalized stress and strain, assuming the material has no resistance to tension and has limited compressive strength, which has been formulated in previous work for rectangular cross-section beams and generalized, in the ongoing work, to the case of hollow rectangular sections, in order to study the dynamic behaviour of various types of towers, bell towers and similar structures. Therefore, this relationship has been used to develop a numerical method that allows one to analyse the non-linear dynamic behaviour of masonry isolated towers under multi-component earthquake excitations or different dynamic loads.

Some numerical examples are presented in order to show: (i) the difference in prediction of dynamic response obtained by means of the developed model with respect to the linear elastic one, (ii) the influence of the material’s compressive strength on non-linear response, (iii) information provided by the model on damage spread in masonry structures.

1 Introduction

The problem of structural analysis of masonry buildings and monuments, and among them, of ancient isolate towers subjected to static and dynamic loads has been appeared very interesting in recent years and large research effort have been
devoted to this field of study. Nevertheless, difficulties have been encountered mainly in describing the actual mechanical behaviour of the material. As a consequence, most of the studies especially on dynamic behaviour of masonry structures have been conducted by means of linear elastic models. Recently, masonry-like (no-tension) material models, based on a constitutive law more realistic in modelling the differences between tensile and compressive behaviour of the material, have been developed and used successfully in static analysis (e.g. Del Piero [1], Lucchesi et al. [2]) but they have been recognized not easily applicable to dynamic analyses (Degl’Innocenti et al. [3]). As a consequence, interest has been directed at developing simplified models that, while being able to account for the main mechanical characteristics of the material, should be easily used (Zani [4], Bennati and Barsotti [5]).

With the above-mentioned aim, recent studies (Lucchesi and Pintucchi [6, 7]) have been devoted to define and to enhance a numerical model to perform non-linear dynamic analysis of masonry structures with simple geometry and flexural behaviour, such as columns, isolate towers, bell towers and in general slender structures, which can be represented by means of mono-dimensional element.

Particularly, the model, firstly, set forth for masonry columns, has been developed using a non-linear constitutive equation expressing the generalized stress (normal force and bending moment) as a function of the generalized strain (the extensional strain and the curvature of the beam’s longitudinal axis) for rectangular cross-section beams, assuming material’s no resistance to tension in the longitudinal direction but neglecting its limited compressive strength. Besides, in order to study the dynamic behaviour of various types of towers and similar structures, the above-mentioned constitutive law has been generalized to the case of hollow rectangular sections beams.

As it has been done in previous works for rectangular beams, in the ongoing work, the constitutive relation for hollow rectangular cross section beams has been refined taking into account the material limited compressive strength. The obtained numerical method has been implemented into a computer code in order to study coupled transverse and axial motion of slender towers, under very general loading conditions, including both axial and horizontal dynamic excitations applied at their base and their own weight.

Results obtained by means of this model have pointed out significant qualitative differences in response with respect to that from linear-elastic models. Moreover, comparison with masonry-like material models shows a good agreement of results and shorter computational time required by the proposed model.

Finally, it should be noted that since the model takes into account material non linear behaviour in all sections along the height allowing to obtain some measures of local and global damage, it may be suitable to predict towers seismic behaviour, often affected by higher modes of vibration and, consequently, by damage not concentrated at their base only. Moreover, coupling phenomena between transverse and axial vibrations, which are widely recognized as very influencing the seismic behaviour of slender towers, are also taken into account and vertical components of earthquake excitations, often
significant sources of additional damage for this kind of structures, can be introduced as input ground motions.

2 Constitutive equation

In order to describe masonry towers behaviour, a non-linear elastic constitutive relation for hollow rectangular cross-section beams made of no-tension materials, with limited compressive strength, has been developed.

By making the usual assumption of plane sections, according to the classical Euler-Bernoulli hypothesis, and accounting for axial stresses only, the beam strain is described by the extensional strain \( \varepsilon \) and the curvature \( \kappa \) of the longitudinal axis, whereas the stress state is represented by axial force \( N \) and bending moment \( M \).

Since it is assumed that material has no resistance to tension and limited compressive strength, under the condition of uniaxial stress, the considered constitutive equations is represented by the diagram of Figure 1, where \( \sigma_o \) denotes the maximum compressive stress.

The same Figure shows the pattern of the axial stress component \( \sigma \) in any given transverse section of the beam for \( M > 0 \). Cases differ each other for the position of the neutral axis and of the transitional axis (where \( \sigma \) reaches the limit value \( \sigma_o \)) with respect to the section geometry.

Each illustrated case, such as the analogous for \( M < 0 \), corresponds to a different region in the plane \((\varepsilon, \kappa)\), in which different expression of the constitutive relation holds. Twenty-nine regions, corresponding to the 13 cases of Figure 1 (denoted respectively as \( E_1, \ldots, E_{13} \) in Figure 2), to the analogous cases for \( M < 0 \) (\( E'_1, \ldots, E'_{13} \)) and to the trivial ones, where \( \sigma \) vanishes and where \( \sigma = \sigma_o \) in any point has been identified by means of the eight straight lines of equations:

\[
\kappa = \pm \frac{2\varepsilon}{h}, \quad (1)
\]

\[
\kappa = \pm \frac{2(\varepsilon - \varepsilon_o)}{h}, \quad (2)
\]

\[
\kappa = \pm \frac{2\varepsilon}{h - 2t}, \quad (3)
\]

\[
\kappa = \pm \frac{2(\varepsilon - \varepsilon_o)}{h - 2t} \quad (4)
\]

where \( h \) and \( t \) are the section’s geometric parameters (shown in Figure 1), and \( \varepsilon_o \) is the strain at which the stress \( \sigma_o \) is reached.

In the region corresponding to Case 1, where the section is uncracked and uncrushed, the constitutive relation is linear elastic. In the remaining regions, where section results cracked or crushed or, finally, both situations occur at the same time, the constitutive law is not linear and the extensional and flexural problems are coupled (see Appendix).
Figure 1: Stress $\sigma$-strain $\epsilon$ diagram and $\sigma$ patterns over the cross-section.
The above-described constitutive law has been used to develop a numerical model which, implemented into a computer code, allows to perform non linear dynamic analysis of slender masonry towers subjected to both vertical and horizontal excitations applied at their base or different dynamic loads.

The finite-element method has been used to discretize the beam, by assuming three degrees of freedom at each node. In order to guarantee the continuity of both transverse displacement of beam axis and rotation, conforming elements and Hermite shape functions have been selected, while linear shape function have been used for axial displacement. Successively, it has been required to calculate the tangent stiffness matrix. As the consistent mass, stiffness matrixes and the loading vector have been obtained, standard numerical techniques have been used to integrate the coupled transverse and longitudinal equations of motion obtained through the discretization. (Lucchesi and Pintucchi [6, 7]).

Figure 2: Partition of the plane $\varepsilon$, $\kappa$. 
3 Numerical examples

Referring to some typical characteristics of ancient tower in Italy, the analyzed tower is chosen 40 m height with a square section 5.5 m wide and 1.0 m constant tick wall. A value of 3600 MPa has been assumed for Young’s compression modulus $E$, while the mass density $\rho$ is taken to be 1800 kg/m$^3$. Regarding the viscous damping, the widely assumed value of 0.05 for the damping ratio of the tower first two elastic flexural modes has been considered.

Results obtained applying a horizontal acceleration at the structure’s base, varying according to a sine law, are presented in order to evidence what we consider to be some particularly interesting qualitative aspects of the non-linear elastic response. By way of example, firstly, the period $T$ has been assumed equal to 1.5s and the amplitude has been set at 0.2g. The harmonic loading is applied for 1s, after which the oscillations are free.

Figure 3 shows comparison of the time histories of the relative horizontal displacements at the top of the tower for: (i) linear elastic material, (ii) no tension material with limitless compressive strength, (iii) no tension material with three different values of compression strength ($\sigma_o = 2.5, 2.0, 1.7$ MPa). In Figure 4, time histories of vertical displacements at the top of the tower, due to coupling in the inelastic range are also presented.

Figure 3: Time history of horizontal displacement at the top of the tower.

Results obtained for this example evidence the consistent difference in response with respect to the linear elastic behaviour, if material no-resistance to tension is taken into account. No-resistance to tension leads to amplifications of horizontal displacement and to elongation of lateral period, induced by the formation of the first cracked zones and the consequent loss of stiffness. When
limited compressive strength is also considered, a further amplification in displacements occurs and elongation in axial and transverse periods of vibration.

It should be noted that the presented model, taking into account the material’s non-linear behaviour in all sections through the tower’s height, allows one to obtain some information on global and local structural damage. As regard global parameter of damage, the cracked portions of volume of the structure as well as the crushed ones may be likely indicators. Even if they have not been validated by means of experimental evidences, their definition appears to be suitable since damage levels are certainly correlated to the spread of cracking and crushing throughout the tower volume.

Figure 4: Time history of vertical displacement at the top of the tower.

Figure 5: Time history of the cracked volume over the tower’s total volume.

Figure 5 and 6 give values of the cracked and crushed volumes, non-dimensionalized to the tower’s total volume, as a function of time, for the considered non-linear cases. As expected, when limited compressive strength is
considered and it decreases, portions of cracked volume become smaller, while portions of crushed volume growth. It’s interesting to note that no allowance for degradation due to load reversals can be made, as the model is a non-linear elastic one.

![Figure 6: Time history of the crushed volume over the tower’s total volume.](image)

Since the first example shows an amplification in response due to non-linear behaviour, the next one is presented to evidence that it may be also the opposite. The harmonic loading’s period has been assumed equal to the structure's first transverse period of elastic vibration (T=1.07s), while the amplitude is 0.2g. For material’s compressive strength, medium value of 2.0 MPa has been maintained.

![Figure 7: Time history of the horizontal displacement at the top of the tower.](image)

Non-linear response is compared with the linear one. As shown in Figure 7, the elastic structure presents the well known damped resonant response, since the first natural period is equal to that of the applied load. Instead, in the case of the no-tension tower, with cracking and consequent loss of stiffness, the system's
vibration period becomes longer, inducing the response to deviate from that typical of resonance. The displacements, therefore, decrease to moderate values.

4 Conclusions

A general numerical model to perform non-linear dynamic analysis of masonry slender towers, under multi-component earthquake excitations or different dynamic loads, has been presented. In developing the model, a non-linear constitutive equation giving the generalized stress as a function of the strain ones, has been defined for a beam with a hollow rectangular cross section.

Some numerical examples evidence clearly consistent qualitative differences in response with respect to the linear elastic analysis. Moreover, preliminary results suggest the suitability of the model for studying seismic behaviour of slender towers, under real earthquake excitations. In fact, since non linear behaviour is allowed in all sections along the height, the model may be useful to obtain some measures of local and global damage and be suitable to predict tower seismic behaviour, often affected by higher modes of vibration. Moreover, coupling phenomena between transverse and axial vibrations, which are widely recognized as very influencing the seismic behaviour of slender towers, are taken into account and vertical components of earthquake excitations, often significant sources of additional damage for this kind of structures, can be introduced as input ground motions.

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Appendix

Referring again to the section’s geometry of Fig.1, and denoted as $h = h - t$, $h_1 = h - 2t$, $b = b - s$, $b_1 = b - 2s$, $E$ the Young modulus, for $M > 0$, the constitutive law is

\[ E_2: \quad N = E(2bt + sh_1) - \frac{Eb(2e + \kappa h)^2}{8k}, \]

\[ M = Eb \left( 4e^3 - 3sh^2\kappa^2 - \kappa^3 [h_1^3 - 2t(2h_1 + 2ht_1)] + \frac{E \kappa s h_1^3}{12} \right), \]

\[ E_3: \quad N = E(e(\kappa - \kappa t_1)) - \frac{E s (2e - \kappa h)^2}{8\kappa}, \]

\[ M = E \kappa \left( 2bt(h_1^2 + 2ht_1) + sh_1^3 \right) + \frac{E e^3 s \kappa^2}{24} - \frac{Eb t_1 + s h_1^2}{8}, \]

\[ E_4: \quad N = - \frac{Eb(e + \kappa h - 2e)^2}{8\kappa}, \quad M = \frac{Eb (e + \kappa h)(\kappa h - 2e)^2}{24 \kappa^2}, \]
\[ E_5: N = E (2bt + sh_2) \varepsilon + \frac{Eb(2\varepsilon - 2\varepsilon_o - 3h\kappa)}{8\kappa}, \]  
\[ M = \frac{Exsh_2^3}{24} - \frac{Eb}{8}\kappa [h_1^3 - 2t(h_2^2 + 2hh_1)] + 3h^2\kappa^2 (\varepsilon_o - \varepsilon) + 4(\varepsilon - \varepsilon_o)^3, \]  
\[ E_6: N = \frac{Eb(2\varepsilon - 2\varepsilon_o + 3h\kappa)}{8\kappa}, \]  
\[ M = \frac{2bt(h_2^2 + 2hh_1) + sh_2}{24} - \frac{2(\varepsilon - \varepsilon_o)^3 + \kappa h_1}{6\kappa^2} + \frac{Eb(2\varepsilon - 2\varepsilon_o + 3h\kappa)}{8\kappa}, \]  
\[ E_7: N = \frac{Eb(2\varepsilon - 2\varepsilon_o + 3h\kappa)^2}{8\kappa} + \frac{E_8(2bt + sh_2)}{24}, \]  
\[ M = \frac{E_8}{24\kappa^2} \]  
\[ E_8: N = E \frac{b\varepsilon_o(h_2 + 2\varepsilon_o) - 2(\varepsilon_o b + h_2)}{8\kappa}, \]  
\[ M = E \frac{3\varepsilon^2 - 3\varepsilon_o^2 + \varepsilon_o^2}{6\kappa^2} - \frac{Exh_2^3}{12} - \frac{Eb\varepsilon_o h_2^3}{8}, \]  
\[ E_9: N = E \frac{s(\varepsilon - \varepsilon_o)^3 - 3h\kappa}{24\kappa^2} + \frac{Eb\varepsilon_o h_2^3}{12} + \frac{Ek(\varepsilon - \varepsilon_o)^3}{8}, \]  
\[ M = \frac{E_9}{6\kappa^2} - \frac{Eb\varepsilon_o h_2^3}{12} + \frac{sh_2^3(\varepsilon - \varepsilon_o)}{8}, \]  
\[ E_{10}: N = \frac{2\varepsilon_o^2 + h_2}{24\kappa^2}, \]  
\[ M = \frac{E_9}{24\kappa^2} + \frac{Ek(\varepsilon - \varepsilon_o)^3}{8}, \]  
\[ E_{11}: N = \frac{b^2[(2\varepsilon - 2\varepsilon_o - \kappa h_2)^2 + 8\varepsilon_o h_2]}{8\kappa}, \]  
\[ M = \frac{Es\varepsilon_o h_2^3}{24\kappa^2} + \frac{Ek(\varepsilon - \varepsilon_o)^3}{8}, \]  
\[ E_{12}: N = \frac{Eb\varepsilon_o (2\varepsilon - 2\varepsilon_o - \kappa h_2)}{24\kappa^2}, \]  
\[ M = \frac{E_9}{24\kappa^2} + \frac{Ek(\varepsilon - \varepsilon_o)^3}{8}, \]  
\[ E_{13}: N = E \frac{h_2 + 2\varepsilon_o}{24\kappa^2}, \]  
\[ M = E \frac{12\varepsilon^2 - 12\varepsilon_o^2 + 4\varepsilon^2}{24\kappa^2} + \frac{3h\kappa^2 - 3\varepsilon^2}{24\kappa^2}, \]  

**Reference**


