A specific model for in-plane non-linear dynamics of masonry walls including some texture effects

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Abstract

Rigid elements are proposed for a specific discrete model that is developed with the aim to investigate the global dynamical response of masonry buildings subjected to strong earthquakes that cause mechanical degradation. In particular, the elements are plane quadrilaterals connected to each other by two normal springs and one shear spring at each side. The mechanical characteristics of these connections are defined paying attention to the micro-structure effects that arise when the mortar joints suffer high mechanical degradation. In particular, the micro-structure effect of inter-locking and local rotation of the blocks can be approximated by properly assigning the constitutive laws of the connection springs. A main computational advantage of the present approach consists in allowing a simplified description of the hysteretic response by defining separate phenomenological behaviours for the axial and the shear elastic connections that are based on experimental tests available in the technical literature.

1 Introduction

The present work originates from the observation that earthquake engineering studies of masonry buildings require the formulation of specific computational models that should be simplified enough to allow full dynamical analyses, but should also account for the peculiar behaviour of the masonry material under loadings that cause heavy mechanical degradation [7]. In this field, the main phenomenon that should be modelled in order to study the global performance of a building is the transmission and dissipation of seismic energy from the ground into the structure [1], with a process that often involves mechanical degradation. Thus,
specific models should work at a scale that lies between the detailed description of the micro-mechanical behaviour of the masonry materials and the global simplified description of the buildings’ response by means of models with very few degrees of freedom. In this perspective, some attention will be deserved here to the possibility of considering some textures’ effects on the resources of strength and stiffness of masonry walls.

The two idealized composite ‘masonry-like’ textures that have been investigated in the present study are shown in Figure 1. The rectangular units have the following dimensions: \( l \times d \times h = 25 \times 12 \times 5.5 \text{ cm}^3 \), while the dimensions of the square bricks are: \( l \times d \times h = 12 \times 12 \times 12 \text{ cm}^3 \). The thickness of the mortar joints is \( t = 1 \text{ cm} \). The vertical head-joints, interrupted by the bricks, are generally of poor quality and weaker than the horizontal continuous bed-joints, while the case of single leaf masonry is considered here for the sake of clarity, as a first approach.

The proposed discrete model adopts quadrilateral rigid elements that are connected at each side by three elastic devices that can be imaged as simple line springs. The elastic characteristics of these springs are defined by means of a specific procedure of identification with the objective to transfer the “memory” of some texture’s characteristics to the specific discrete model. The degradation of the mortar joints under monotonic loading cause an evolution of these texture characteristics that are approximated by assigning proper elastic-plastic constitutive laws to the shear connecting springs. Considering the response under cyclic loading, the hysteretic behaviour is defined without a specific attention to the micro-structure except for the fact that the initial skeleton curves are defined as to approximate the evolution of micro-structure effects under monotonic loading. At present, this limitation is related to the fact that the hysteretic constitutive laws must be assigned following a phenomenological approach, on the base of the experimental cyclic tests currently available in the technical literature.

2 Model kinematics

The plain domain \( \Omega \) is partitioned into \( m \) quadrilateral elements \( \omega^i \) such that no vertex of one quadrilateral lies on the edge of another quadrilateral. The deformed configuration of the discrete model is described as a function of the displacements...
of the local reference frames \( \{ o^i, \xi^i, \eta^i \} \) fixed to the moving elements. Three kinematic variables, the two translations \( u_i, v_i \) and the rotation angle \( \psi_i \), are associated to each element as shown in Figure 2 (left), and the whole kinematic configuration is described by the \( 3m \) Lagrangian coordinates assembled in the vector \( \{ u \} \):

\[
\{ u \}^T = \{ u_1, v_1, \psi_1, u_2, v_2, \psi_2, \ldots, u_m, v_m, \psi_m \}
\] (1)

The external loads, including the inertial forces, are condensed into three resultant, \( p_i \) and \( q_i \), and \( \mu_i \), that are applied to each element \( \omega^i \) as shown in Figure 2 (left).

The elastic devices that connect each couple of elements are placed in correspondence of three connection points named \( P \), \( Q \) and \( R \), as shown in Figure 2 (right). A shear elastic connection is placed in the mid-point \( Q \), while two normal elastic connections are placed in the external points \( P \) and \( R \), at a distance \( b \) from \( Q \). The elastic force in each device is proportional to a measure of mean strain associated to the corresponding connection point [8]. A shear strain \( \varepsilon^Q \), and two axial strains \( \varepsilon^P \) and \( \varepsilon^R \), are defined for each connection side, and they are assembled in the vector of generalized strains \( \{ \varepsilon \} \) as follows:

\[
\{ \varepsilon \}^T = \{ \varepsilon^P_1, \varepsilon^Q_1, \varepsilon^R_1, \varepsilon^P_2, \varepsilon^Q_2, \varepsilon^R_2, \ldots, \varepsilon^P_s, \varepsilon^Q_s, \varepsilon^R_s \}
\] (2)

being \( s \) the number of sides that connect the elements of the whole discrete model. In the small displacement theory, linearity allows to express the strain-displacement relations by considering a \( 3s \times 3m \) matrix \( [B] \):

\[
\{ \varepsilon \} = [B] \{ u \}
\] (3)

### 3 Material identification

#### 3.1 Elastic behaviour

The linear elastic characteristics of the connecting devices are assigned with the criterion of approximating the strain energy of the corresponding volume of pertinence considering five elemental loading tests, as shown in Figure 3. For simplicity, the method is presented with reference to the case of a regular mesh of...
square elements aligned with the principal axes of the material, and whose size corresponds to the side $2e$ of a square periodic cell. The direct numerical identification is performed by means of a finite element model of the heterogeneous material adopting periodic boundary conditions for axial and shear loading, and displacement boundary conditions for in-plane bending [3].

The definition of the stiffnesses of the normal line springs, as well as the distances $b$ between the springs that constitute each axial couple are obtained by simply equalling the corresponding strain energies between the finite element model of the heterogeneous material and the proposed rigid element model.

The texture effects are more evident when analyzing the shear test, shown in the centre of Figure 3. In fact, finite element analyses made on the two idealized “masonry-like” textures revealed a significant difference in the deformation patterns, as shown for example in Figure 4. The rectangular blocks tend to rotate as to align the long side with the horizontal joints, while square blocks tend to translate without significant rotation. Qualitatively, the deformation of rectangular blocks appears as a mix of shear and bending, plus a local rigid rotation, depending on the ratios between the elastic modulus of the block $E_b$, and of the elastic moduli of the horizontal and vertical mortar joints $E_h, E_v$. In particular, the blocks respond with a pure shear deformation for $E_b/E_h = 1$, while the local rigid rotation prevails for very high values of ratio $E_b/E_h$. These very high ratios has been considered with the aim to investigate the response when the mortar is degraded. The rigid rotation of the blocks can be considered as a characteristic “local rotation” of this masonry-like texture, and its importance tends to increase with the degree of heterogeneity. If we adopt square rigid elements whose side is $2e = 2l + 2h$ (see...
Figure 1), the local rigid rotation of the blocks can be directly assigned as the rotation \( \psi \) of the rigid elements. The proper measure of this local rotation is somewhat a compromise since it tends to be affected by the shear and bending deformations. At present, the following formula has been chosen, based on the displacements of the points \( A, B, C \) and \( D \) of the brick shown in Figure 5:

\[
\psi = \frac{u_C^y - u_A^y}{4 l} + \frac{u_D^x - u_B^x}{4 h} \tag{4}
\]

If the average symmetric shear strain is \( \varepsilon_s \), then \( \rho = \psi / \varepsilon_s \) is defined as the local rigid rotation ratio. The generalized shear strains for the vertical and the horizontal connecting devices are then [8]:

\[
\varepsilon_v = \varepsilon_s - \psi = \varepsilon_s (1 - \rho) \\
\varepsilon_h = \varepsilon_s + \psi = \varepsilon_s (1 + \rho) \tag{5}
\]

The equilibrium of the shear stresses implies that the stiffnesses of the shear connecting devices of the vertical and of the horizontal sides must be related by the following equation [8]:

\[
\frac{k_h^Q}{k_v^Q} = \frac{\varepsilon_v}{\varepsilon_h} = \frac{1 - \rho}{1 + \rho} \tag{6}
\]

A parametric study was performed on the two textures shown in Figure 1 in order to identify the mechanical characteristics of the connection devices. Young modulus of the blocks was fixed to be \( E_b = 10000 \) MPa, while the elastic moduli of the horizontal and vertical mortar joints, \( E_h \) and \( E_v \) respectively, assumed different values. In particular, ratio \( E_b / E_h \) varied in the range \([1 - 100000]\) while the effect of weakness of the vertical joints was studied by considering four different ratios \( E_h / E_v \) in the range \([1 - 30]\).

Figure 6 shows the trend of ratio \( \rho \) as a function of ratio \( E_b / E_h \). The difference of micro-structure behaviour between the two textures is noticeable, and it is worth noting that the rectangular blocks manifest a local counterclockwise rotation \( \psi \) that approaches the average symmetric shear strain \( \varepsilon_s \) (i.e. \( \rho \to 1 \)) for high values of \( E_b / E_h \), without a significant influence of the \( E_h / E_v \) ratio. In fact, when mortar is soft, the elongated shape of the blocks tends in any case to impose the geometric effect of alignment with the bed joints. Square blocks, on the other hand, manifest small values of local rotation that tend to increase clockwise for high \( E_h / E_v \) ratios, as a direct consequence of the higher stiffness of the horizontal shear connections respect to the vertical.
The performance of the present model in terms of approximation of the global stiffness can be evaluated by comparing the eigenvalues of some simple structures. Figures 7 and 8 refer in particular to the first eigen-solution calculated by means of three discrete models: an heterogeneous composite finite element model that is assumed as reference, the proposed rigid element model with shear springs defined according with Equation (6) (“no-symm rigid”), and a finite element model that adopt an orthotropic homogeneous Cauchy material (“homogeneous”). A fixed vertical left side was considered here as boundary condition. The graphs of the percentage errors $\Delta \omega \%$ in the computation of the first eigenvalue with respect to the heterogeneous finite element model reveal that the rigid element model with the shear springs defined according with Equation (6) (“no-symm rigid”) has a good performance in the whole range of $E_b/E_h$ ratio, while the finite element model
that adopt the orthotropic homogeneous Cauchy material ("homogeneous") shows a lack of rigidity in correspondence of high values of $E_b/E_h$ ratio. On the other hand, for the square block texture the approximation is good and substantially insensitive to the particular computational model adopted.

The effect of the texture when mortar joints are very degraded has effects also on the way in which the external loads are carried.

Figures 9 and 10, show two square masonry walls ($1.30 \, m \times 1.30 \, m$) made with different textures and subjected to the same vertical load distributed along $0.26 \, m$ on the middle top. It can be noted that the displacements of the rigid element mesh relative to the square blocks texture are concentrated in the middle of the masonry panel, just below the applied load, while on the other hand the mesh relative to the rectangular blocks texture manifests displacements that tends to be more diffused laterally. This fact shows the capability of the proposed model to deal with some “interlocking effects” between the blocks that become significant when mortar joints are very degraded. The two figures also show the different distribution of the “measures of stress and strain” evaluated in correspondence the shear connecting devices.

### 3.2 Monotonic plastic behaviour

The trend of evolution of the local rotation ratio $\rho$ during an elastic-plastic loading can be modelled with the proposed rigid elements by identifying different
0.000 ... 0.001 MPa
0.001 ... 0.002 MPa
0.002 ... 0.003 MPa
0.003 ... 0.004 MPa
0.004 ... 0.005 MPa
0.005 ... 0.006 MPa
0.006 ... 0.007 MPa

Measure of shear stress in the connections

Figure 10: Response of rigid elements to a vertical load considering a square block texture. Ratio $E_b/E_h = 1000$, $E_h/E_v = 10$.

0.000 ... 0.024
0.024 ... 0.048
0.048 ... 0.072
0.072 ... 0.097
0.097 ... 0.121
0.121 ... 0.145
0.145 ... 0.169

Measure of shear strain in the connections

Figure 11: Monotonic symmetric shear loading adopting Drucker-Prager for mortar joints. Rectangular blocks texture with $E_h/E_v = 10$.

constitutive-laws for the vertical and horizontal shear connection devices, as shown in the example of Figure 11.

For this case, a plane stress model was used for analyses in which the behaviour of the mortar followed the material model with linear Drucker-Prager yield criterion available within Abaqus finite element code [9].
3.3 Hysteretic assumptions

A phenomenological approach is followed in order to assign the hysteretic constitutive laws to the connecting devices [5, 6]. These laws are based on the experimental cyclic tests currently available in literature, and should be assigned to rigid elements whose size is approximately comparable to the test specimens. The adoption of this type of device is useful since it allows separate phenomenological descriptions or the hysteresis behaviour of the axial and shear connections that can be related by means of a simple Mohr-Coulomb criterion. The proposed laws for the axial and the shear springs are shown in Figure 12, inspired for example by [2, 4].

4 Final remarks

The present approach seems promising in view of the formulation of a model able to perform non-linear dynamical analyses with degrading characteristics of the masonry material. Some micro-structure effects related to the different masonry textures can be modelled in the elastic field as well as in the plastic field when considering simple monotonic loading conditions. At present, the proposed definition of the subsequent hysteretic behaviour follows a phenomenological approach without a specific attention to the textures.

References


