Influence of concrete prestressing and gravitational loading on plastic energy dissipation and ductility of RC elements in seismic zones

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Abstract

The plastic energy dissipation (PED) and ductility are important parameters of RC structures in seismic zones. Several factors influence these parameters, mostly negatively, such as symmetrical reinforcing of RC section, over or under reinforcing, prestressing of concrete and gravitation loads. The influence peculiarities of reinforcing on its PED and ductility were analysed in a previous paper of this author. In this paper the influence of RC section initial stresses due to prestressing and gravitational loading on its plastic deformations is presented. Based on experimental results, it is assumed that the concrete modulus of elasticity increases linearly versus the compressive strength of concrete. Accordingly the ultimate elastic and ultimate plastic deformations were calculated for each concrete class. The values of the compression section ductility as a function of the concrete strength are obtained. It is shown that this function is parabolic. For the prestressed concrete section, the ductility factor \( \mu \) is limited to \( 3 \leq \mu < 6 \) (unconfined section). Since the compression steel strength can not exceed 400 MPa (as demonstrated by the steel-concrete common deformation equation), high-strength steel (HSS) for the prestressed concrete (PC) bending section does not contribute to its PED in the compression zone. In addition, when the tensile zone is analysed, the initial stresses in the HSS lead to significant decreases of the section PED and ductility. Also there is the problem of determination of the HSS conditional yield deformation. It is necessary to take into account that the PC section frequently is over-reinforced. The same problems arise when the RC section is under initial stresses due to gravitational (static) loading. The influence of gravitational stress on RC section PED under seismic forces is investigated applying the non-symmetric cyclic analyses. In this case the hysteretic loops area is reduced and accordingly the ductility factor decreases.

The results of this study are useful for seismic design and seismic codes for RC elements.

Keywords: prestressed concrete, gravitational loading, plastic energy dissipation, section ductility, RC element, seismic zones.
1 Introduction

Traditionally applying of prestressed concrete (PC) structures in seismic zones have been associated with the problem of a concrete brittle behavior, because, as a rule, high-strength concrete (HSC) is used in these structures (the concrete compressive strength $f_{ck} \geq 50$ MPa) [1, 2]. Possibility of plastic deformation development in PC structure section is the very limited, and therefore it has a lower ductility parameter. In addition, in prestressed concrete structures high-strength steel (HSS) reinforcing is used with the steel tensile strength $f_{sk}$ on the range 1000-2000 MPa. It is known, that this steel does not have a yield area on the “stress – strain” diagram. This leads to an additional reduction of the section plastic energy dissipation (PED) and its ductility parameter. However, these problems are known qualitatively, rather then quantitatively. Therefore, the main purpose of this presentation is to give a quantitatively correlation between HSC-HSS prestressed element section, on one hand, and its PED and ductility, on the other.

Another issue, which is closely related to the previous one, is the influence of axial stresses in columns and stresses due to bending of beams under gravitational loads caused any seismic behaviour of the RC structure [3, 4]. It was observed that compressive strength of concrete in structural elements was about 8 - 12 MPa, while the ratio $N / (b_c h_c f_{ck})$ for interior columns was about 0.1 - 0.5. In order to perform the dynamic (seismic) analysis of a structure, one needs to investigate the PED and corresponding ductility already existing in its element sections due to gravitational loading.

In this work we propose a method for estimating of the PC section PED and ductility, taking into account both issues detailed above. The major symbols used in this paper correspond to RC structural and seismic codes [5, 6, 7].

2 Prestressed concrete section PED and ductility

2.1 Influence of high-strength concrete (HSC)

The Young’s modulus of concrete, $E_c$, is characterised by elastic deformations, which increase proportionally to the concrete strength:

$$\varepsilon_{c\text{ el ul}} = \frac{f_{ck}}{E_c}$$

where $\varepsilon_{c\text{ el ul}}$ – concrete ultimate elastic deformations.

Therefore, it can be considered that the concrete modulus of elasticity will also increase proportionally with increasing of the concrete strength. In fig. 1 two diagrams of $E_c$ versus (vs.) $f_{ck}$ are given in accordance with the Israel Concrete Code (ICC) [7] for two kinds of aggregates – basalt and chalk (experimental results). We see an almost linear dependence of $E_c$ vs. $f_{ck}$. It is possible to express the $E_c$ mathematically for basalt and chalk aggregates accordingly by:

$$E_c = 21866.666 + 222.222 f_{ck}$$

$$E_c = 19433.333 + 204.444 f_{ck}$$
The maximum deviation of theoretical results from the experimental ones is equal to 1.95% (basalt aggregate) or 2.42% (chalk aggregate).

By using expressions (1), (2) and (3), we can get maximum concrete elastic deformations, $\varepsilon_{c\,el\,ul}$, for each type of concrete, including $f_{ck} = 70$ MPa (as shown in table 1).

Table 1: Maximum values of concrete elastic deformations, $10^3 \varepsilon_{c\,el\,ul}$, for two kinds of aggregates vs. $f_{ck}$ (MPa).

<table>
<thead>
<tr>
<th>$f_{ck}$</th>
<th>15</th>
<th>20</th>
<th>25</th>
<th>30</th>
<th>40</th>
<th>50</th>
<th>60</th>
<th>70</th>
</tr>
</thead>
<tbody>
<tr>
<td>Basalt</td>
<td>0.595</td>
<td>0.760</td>
<td>0.912</td>
<td>1.050</td>
<td>1.300</td>
<td>1.520</td>
<td>1.705</td>
<td>1.871</td>
</tr>
<tr>
<td>Chalk</td>
<td>0.667</td>
<td>0.850</td>
<td>1.020</td>
<td>1.170</td>
<td>1.450</td>
<td>1.686</td>
<td>1.893</td>
<td>2.074</td>
</tr>
</tbody>
</table>

It is known from the RC theory, that the maximum value of elastic-plastic strains $\varepsilon_{c\,el\,pl} \approx 0.002$, i.e. in HSC elements sections these deformations are rather small. For $f_{ck} \geq 50$ MPa the plastic part of strains is significantly smaller than the elastic one. Still, for PC structures only this kind of concrete is used. Therefore, it is necessary to estimate the influence of HSC on the section plastic energy dissipation (PED) and ductility.

Considering by the ultimate concrete plastic deformation $\varepsilon_{c\,ul} = 0.0035$, it is possible to calculate the PED and ductility of an ideal elastic-plastic section [4]:

$$U_{c\,el} = 0.5 \varepsilon_{c\,el\,ul} f_{ck} ; U_{c\,pl} = (\varepsilon_{c\,ul} - \varepsilon_{c\,el\,ul}) f_{ck} ; \mu = 1 + U_{c\,pl} / U_{c\,pl} \quad (4)$$

where $U_{c\,el}$, $U_{c\,pl}$ and $\mu$ – are an elastic energy potential, PED and ductility factor of the section accordingly.

The calculation results for basalt aggregate concrete are given in table 2. From this table the diagram of $\mu$ vs. $f_{ck}$ is built (fig.2, solid line).
Table 2: Values of $U_{c\text{el}}$, $U_{c\text{pl}}$ ($10^{-3}$ MPa) and $\mu$ vs. $f_{ck}$ (MPa).

<table>
<thead>
<tr>
<th>$f_{ck}$</th>
<th>15</th>
<th>20</th>
<th>25</th>
<th>30</th>
<th>40</th>
<th>50</th>
<th>60</th>
<th>70</th>
</tr>
</thead>
<tbody>
<tr>
<td>$U_{c\text{el}}$</td>
<td>4.5</td>
<td>7.5</td>
<td>11.25</td>
<td>15</td>
<td>26</td>
<td>37.5</td>
<td>51</td>
<td>66.5</td>
</tr>
<tr>
<td>$U_{c\text{pl}}$</td>
<td>43.5</td>
<td>55</td>
<td>65</td>
<td>75</td>
<td>88</td>
<td>100</td>
<td>108</td>
<td>112</td>
</tr>
<tr>
<td>$\mu$</td>
<td>10.66</td>
<td>8.33</td>
<td>6.78</td>
<td>6.00</td>
<td>4.38</td>
<td>3.66</td>
<td>3.12</td>
<td>2.68</td>
</tr>
</tbody>
</table>

It is assumed, that the “$f_{ck} - \mu$” diagram from fig. 2 correlates with a quadratic parabola, which is described the following equation:

$$\mu = 0.002639 f_{ck}^2 - 0.3694 f_{ck} + 15.6207$$  \hspace{1cm} (5)

The same method applies for chalk aggregates concrete. This is also shown in fig.2 (dotted line) according to eqn (5). The maximum deviation of theoretical results from calculated ones is equal to 13.3 % (for $f_{ck} = 40$ MPa). It should be noted that for RC structures this deviation is rather satisfactory, because the distribution of concrete strength values in experiments is 15-20 %. To the best of our knowledge, this is the first time that such a theoretical relationship between concrete strengths and the ductility factor of a RC section is obtained. This result is particularly important for PC elements’ sections, since for HSC the ductility factor is rather small. This factor is a negative for RC structures in seismic zones.

Figure 2: “$\mu - f_{ck}$” diagrams – calculation (1), parabolic correlation (2).
2.2 Influence of high-strength steel (HSS) reinforcing

2.2.1 Section tensile zone

It is suggested that in a tensile zone a prestressed reinforcement is deformed according to two-linear diagram (fig. 3). Intersection point coordinates are \( f_{pk01} \) and \( (f_{pk01} / E_{sp}) \), and the end point ones – \( f_{pk} \) and unknown ultimate deformation \( \varepsilon_{spul} \).

![Diagram](image)

Figure 3: The “stress – strain” diagram of prestressed tensile steel.

The elastic and plastic energy potentials of the HSS and its PED are equal to (accordingly):

\[
U_{sp el} = \frac{f_{pk}^2}{E_{sp}} \left[ \eta_p^2 - 0.5 \eta_p + \frac{E_{sp}}{2 f_{pk}} \varepsilon_{spul} \right], \tag{6}
\]

\[
U_{sp pl} = \left( \varepsilon_{spul} - \frac{f_{pk01}}{E_{sp}} \right) f_{pk01}; \mu = 1 + \frac{U_{sp pl}}{U_{sp el}}. \tag{7}
\]

In these eqns \( \eta = f_{pk01} / f_{pk} \).

If the PC structure reinforcement stress \( \sigma_{sp} \leq f_{pk01} \), the section tensile zone does not contribute to its PED and ductility, i.e. \( \mu = 1 \). However, if \( \sigma_{sp} > f_{pk01} \), \( \mu = f(\varepsilon_{spul}) > 1 \), but the ultimate steel deformation, \( \varepsilon_{spul} \), is unknown. It follows that we need an additional equation. This can be either the plain section hypothesis or the section energy equation [4]. According to the second,

\[
\varepsilon_{spul} = 0.002 + \frac{0.0015}{\rho_{sp}} \left( \eta_c - 1.5 \rho_{sp} + 0.5 \rho_{sp}' \right), \tag{8}
\]

where \( \eta_c = f_{cd} / f_{pd} ; \rho_{sp} = A_{sp} / A_c ; \rho_{sp}' = A_{sp}' / A_c \).
2.2.2 Section compression zone

If the PC section includes the compression prestressed reinforcement, the maximum stress in that reinforcement, $\sigma_{sp}'$, can be found from the concrete and steel deformations equation [4]:

$$\varepsilon_{sp ul} = \varepsilon_{c ul}; \max \sigma_{sp}' = \varepsilon_{c ul} E_{sp} \approx 0.002 200000 = 400 \text{ MPa} < f_{pk 01}. \quad (9)$$

It follows that the compression prestressed reinforcement does not contribute to the section PED and ductility ($\mu = 1$).

2.3 Influence of concrete and steel prestressing

Seismic forces influence the behavior of PC elements under stresses due to concrete prestressing. According to [7], compressive prestress in concrete $\sigma_{cp} \leq 0.33 f_{ck}$, i.e. an element section is in elastic stage. In this case, asymmetric oscillations occur under seismic loading. The seismic behavior of the section compression zone under initial stresses will be analyzed in the next part of this work (the influence of the gravitational loading), because both influences give the same effect from the viewpoint of hysteretic loops.

Steel prestressing in the PC structures, $\sigma_{sp}$, is an initial stress in the tensile reinforcement and also leads to the asymmetric oscillations. According to [7], the maximum value of the $\sigma_{sp} \leq 0.95 f_{pk 01}$, i.e. the initial stress in HSS reinforcement is smaller than the yield stress (see fig. 3), and still in elastic stage. It follows that the PC section, as a whole, is bending under eccentric compression and function in the elastic stage. Hence, the prestressed section does not have a PED potential up to a service loads action, and a ductility factor in this stage $\mu = 1$. If the PC section is over-reinforced, the prestressed steel in general does not contribute to the section PED, and the ductility factor depends on concrete plastic deformations only [4].

3 Gravitational loading influence on RC section PED and ductility

Seismic forces influence the behavior of building elements under gravitation stresses due to dead and live loads. Axial stresses in columns generally range between $(0.1 - 0.5) f_c$. In this case, asymmetric oscillations occur under seismic loading. It is assumed that RC section gravitation stress is equal to a maximum:

$$\sigma_{cg} = 0.5 f_c, \quad (10)$$

i.e. the concrete reaches elastic stress limit (OA on fig. 4).

We assume that under seismic loading this section reaches the plastic state (AB on fig. 4a). Unloading is indicated by BC parallel to OA. The second half of this cycle includes CO and OD (as a part of OB) with the residual strain 0.001. The first cycle energy dissipation is defined as the sum of two triangular areas (ABC + COD) and is equal to $0.125 10^{-3} (2 A_c + A_{c el ul}) f_c$. 

The second and third cycles are given in fig. 4b and 4c respectively. At each cycle \( 0.375 \times 10^{-3} A_{c} f_{c} \) of energy is dissipated. Thus, the total compression energy dissipation is [4]:

\[
U_{cg} = (A_{c} + 0.125 A_{c el ul}) f_{c} 10^{-3}. 
\]  

Fig. 5 shows hysteretic loops for compressive reinforcement. Hysteretic loops are calculated on the basis of equating of concrete and reinforcement strains.

For \( f_{c} = 30 \) MPa concrete with \( f_{s} = 400 \) MPa non-prestressed reinforcing steel, the stress in steel from gravitational loads follows AB and unloading – along BC. The first half of the second cycle loading follows by CBE and unloading – along EF parallel to CB, and the second one – along FM and MF. The first quarter of the third cycle loading goes along FEGL. The total energy dissipation of the compressed reinforcement becomes:

\[
U_{sg}' = 2 \times 0.5 f_{s} A_{s}' 0.75 10^{-3} = 0.75 10^{-3} f_{s} A_{s}'. 
\]
Figure 5: Stress – strain non-prestressed compressed reinforcement hysteretic loops taking into consideration the concrete stress 0.5 $f_c$.

The non-prestressed tensile reinforcement gravitational stress, $\sigma_{sg}$, can be analyzed in accordance with the plain section hypothesis. Full energy dissipation of this reinforcement is:

$$U_{cg} = (f_c - \sigma_{cg}) A_s (\epsilon_{su} - 0.002)$$  \hspace{1cm} (13)

The total energy dissipation of the section, $U_{g\text{tot}}$, is equal to

$$U_{g\text{tot}} = [(A_c + 0.125 A_{c\text{el ul}}) f_c + 0.75 A_s' f_s] 10^{-3} + (f_s - \sigma_{sg}) A_s (\epsilon_{su} - 0.002).$$  \hspace{1cm} (14)

Neglecting 0.125 $A_{c\text{el ul}}$ with respect to $A_c$, we can see [4]:

$$U_{g\text{tot}} \approx 0.5 U_{tot},$$  \hspace{1cm} (15)

i.e. the section PED decreases proportionally to gravitational stresses. This difference proves the importance of considering the RC member gravitational stresses. In this case, the ductility factor also decreases proportionally:

$$(\mu - 1)_g = (1 - \sigma_{cg} / f_c) (\mu - 1)_{g=0}.$$  \hspace{1cm} (16)

4 Conclusions

Applying of prestressed concrete (PC) structures in seismic zones have been associated with problems of a high-strength concrete (HSC) brittle behavior and high-strength steel (HSS) reinforcing, which does not possess a yield area on the “stress – strain” diagram. The possibility of plastic deformation development in PC structure section is the very limited, and therefore it has a lower ductility parameter. These problems are known qualitatively, rather then quantitatively. In
In this presentation a quantitative correlation between HSC-HSS prestressed element section, on one hand, and its PED and ductility, on the other, is given.

Based on experimental results, it is shown that the concrete modulus of elasticity increases linearly with a compressive strength of concrete. The values of the compression section ductility as a function of the concrete strength are obtained. It is shown that this function is parabolic. This is the first time that such a theoretical relationship between concrete strengths and the ductility factor of a RC section is obtained. For the PC unconfined HSC section, the ductility factor $\mu$ is rather lower and is limited to $3 \leq \mu < 6$.

If the PC section includes the compression prestressed reinforcement, the maximum stress in that reinforcement can be found from the concrete and steel deformations equation. It is shown that this stress is equal to 400 MPa. It follows that the compression prestressed reinforcement does not contribute to the section PED and ductility.

A PC element, which has initial stresses, behaves equally under seismic action and under gravitational loading. In both cases, the section PED and ductility factor decrease significantly. This decreasing is almost proportional to the relation of the initial stress to the concrete strength.

The results of this study should be useful for seismic design of RC elements and for RC structures seismic design codes.

**References**