# Dynamic response of laminated glass under blast loading: effect of negative phase

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## Abstract

Most of the recent studies have emphasized the effect of the positive phase of blast loading while determining the structural response. The effect of the negative phase on the dynamic structural response has been ignored. This research focuses on the effect of the negative phase of a blast loading on the dynamic response of an architectural laminated glass. A closed-form solution based on the classical plate theory is developed and then the results are compared with a 3-dimensional nonlinear dynamic finite element analysis. Results indicate that the negative phase of a blast loading has a significant effect on the dynamic response of the laminated glass panel. In particular, mid-span panel deflection and stress due to negative phase pressure are almost double the corresponding quantities due to positive phase. This may mean that if the glazing does not fail in the positive loading duration, but it may do so in the negative loading phase because of higher dynamic stresses during this later phase.
*Keywords: blast load, laminated glass, negative phase, closed-form solution, finite element analysis, linear elasticity, linear visco-elasticity.*

## 1 Introduction

It is estimated that the majority of nonfatal injuries from bomb blasts result from airborne glass fragments from architectural glazing. The use of laminated architectural glass has been shown to mitigate this hazard.

The explosive blast wave has an instantaneous rise, rapid decay and a relatively short positive phase duration. The pressure-time history for an explosive blast wave is that the overpressure from positive phase drops to zero and then drops below zero with a negative phase within a finite time.

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Most of the recent studies have emphasized the effect of the positive phase of blast loading while determining the structural response. The effect of the negative phase on the dynamic structural response has been ignored [1-5]. This is true when used for the hardened structure response but may not true for the glazing panels. Smith et al. [6] reported that the laminated glazing pull out of the frame bite and fall in one piece outside the structure during the explosion tests for certain configuration cases. It may be the negative phase of the blast loading that causes the laminated glass panel failure. Based on such observation, Krauthammer and Altenberg [7] studied the effects of negative phase of the blast load on the probability of glass panel failure.

In this paper, an empirical quasi-exponential form has been extended to describe the explosive blast wave for both positive and negative phase. The effects of the negative phase of a blast loading on the dynamic response of an architectural laminated glass panel were studied. A closed-form solution based on the classical plate theory is developed and then the results are compared with a 3-dimensional nonlinear dynamic finite element analysis.

## 2 Blast load characterization

A typical pressure-time history curve for a blast wave is shown in figure 1. An empirical quasi-exponential form has been used to describe the wave, Kinney [8],

$$p(t) = p^0 (1 - t/t_d) e^{-\alpha\, t/t_d}, \tag{1}$$

where p(t) is the instantaneous overpressure at time t, $p^0$ is the peak overpressure observed when t is zero, $\alpha$ is the decay factor, and $t_d$ is the positive overpressure duration.

Based on the tabulated data given by Kinney [8], Dharani et al. [5] presented the following three equations for the three parameters in eqn (1).

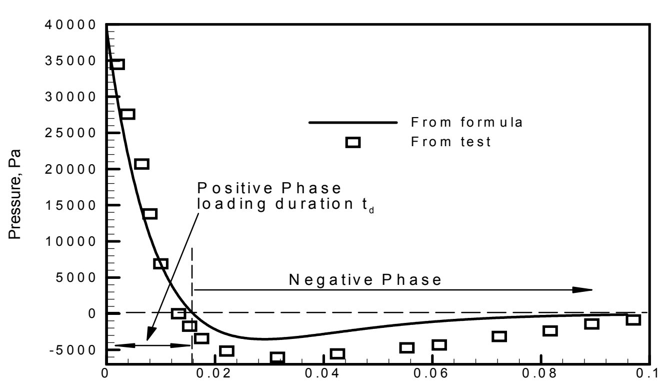


Figure 1: A typical pressure-time curve for an explosive blast.

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$$\frac{p^0}{p_x} = -1.75259\times10^{-3} + 22.112x^{-1} + 2121.79x^{-2} + 132456x^{-3}, \tag{2}$$

$$t_s = -40.9856 - 3.1605\times10^{-2}x + 24.4355x^{1/5}, \tag{3}$$

$$\begin{aligned}\alpha = 2.73453 - 2.19511\times10^{-3}x - 305.832x^{-1} \\ + 16022.4x^{-2} - 181202x^{-3}\end{aligned}, \tag{4}$$

where x is the scaled distance and $t_s$ is the scaled positive overpressure duration defined below.

By means of the scaling law, eqns (2)-(4) can be applied to other explosion cases. For a ground explosion not far from the explosion center, the scaled distance is expressed as, Kinney [8],

$$x = \left(\frac{w_0}{w}\right)^{\frac{1}{3}} d, \tag{5}$$

where w is the TNT equivalent of explosive energy release in the explosion to be described, $w_0$ is that of a reference explosion (2000 lb or 908 kg), and d is an actual distance from explosion center. The actual positive pressure duration is obtained as

$$t_d = \left(\frac{w}{w_0}\right)^{\frac{1}{3}} t_s. \tag{6}$$

The decay parameter α is not itself scaled, so that it can be obtained directly from eqn (4) at a properly scaled distance from eqn (5). When a blast wave encounters the building surface, it will be reflected, thereby amplifying the overpressure. Furthermore, if a blast source is placed on or near a reflecting surface, such as the ground, then the surface burst appears to have 1 to 2 times the source energy as the blast in free-air. Therefore, a scale factor needed to correct the difference between the calculation in free-air blast using formulae (1)-(6) and the experimental measure of ground blast. The scale factors for the peak overpressure between the test data and the calculated data are obtained with η = 3.36 by Dharani et al. [5]. Therefore, the blast wave generated by ground explosion imposes a dynamic load on objects given by

$$p_d(t) = \eta p(t), \tag{7}$$

where $p_d$ is the applied shock pressure on the laminated glass. Once the TNT equivalent of an explosion and the distance from the explosion center are known, the overpressure, the positive phase duration time and the decay factor can be calculated by using eqns (2)-(4). In these equations, the U.S. unit system should be used (1 lb= 0.454 kg, 1 ft = 0.305 m and 1 psi = 6900 Pa).

A complete pressure-time history that includes both the positive and the negative phase can be plotted using eqns (1)-(7). For w = 227 kg TNT equivalent and d = 51.8 m, predicted results from (1)-(7) are presented in figure 1. Also presented in figure 1 are the corresponding test results of Smith et al. [6]. Since agreement between the predicted and test data is good, eqn (1) can be used to simulate both the positive and the negative phases of overpressure from an

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explosion. It is interesting to note that Kinney [8] presented eqn (1) originally for the positive phase of blast loading.

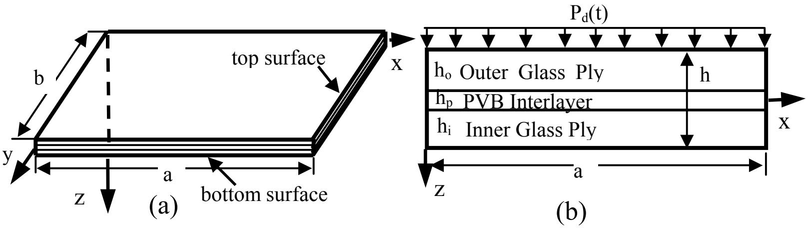


Figure 2: Schematic diagram of a rectangular laminated glass plate subjected to uniform blast loading.

## 3 Closed-form solution

Figure 2 shows a thin rectangular four edges simply supported laminated glass panel. The panel is composed of two architectural glass plies adhered by a polyvinyl butyral (PVB) interlayer. The glass ply is modeled as a linear elastic material and the PVB interlayer may be modeled as a linear viscoelastic material for which the deviatoric component of stresses is given by Dharani et al. [5]

$$S_{ij}(t) = 2\int_0^t G(t-\tau)\dot{e}_{ij}d\tau, \tag{8}$$

where $t$ denotes time, $\dot{e}_{ij}$ is the deviatoric strain rate and $G(t)$ is the stress relaxation modulus which is assumed to be of the form

$$G(t) = G_\infty + (G_0 - G_\infty)e^{-\beta t}, \tag{9}$$

where $G_\infty$ is the long time shear modulus, $G_0$ is the short time shear modulus and $\beta$ is the decay factor. The volumetric response is elastic, so the mean hydrostatic pressure, $p$, is computed by

$$p = -K\varepsilon_{kk}, \tag{10}$$

where K is the bulk modulus.

The short-time shear modulus (glassy modulus), $G_0$, is about 1 GPa magnitude while the long-time (rubbery modulus) shear modulus is about 1 MPa magnitude. Within a short time, PVB behaves like a solid glass material. Usually, the duration of the blast loading is about a mini-second magnitude. Therefore, to simplify the problem in the closed-form solution, we further assume the PVB material as a linear elastic solid via glassy modulus $G_0$ and bulk modulus $K$ as

$$E_0 = \frac{9KG_0}{3K+G_0}, \quad \nu_0 = \frac{3K-2G_0}{6K+2G_0}. \tag{11}$$

According to the classical theory of thin plate, the strain components in the x-y plane can be expressed by transverse deflection, w, of the plate as

$$\varepsilon_x = -z\frac{\partial^2 w}{\partial x^2}, \quad \varepsilon_y = -z\frac{\partial^2 w}{\partial y^2}, \quad \gamma_{xy} = -2z\frac{\partial^2 w}{\partial x \partial y}. \tag{12}$$

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The strain energy for the plate in plane stress state is

$$U = \frac{1}{2}\iiint_V \left(\sigma_x \varepsilon_x + \sigma_y \varepsilon_y + \tau_{xy}\gamma_{xy}\right) dxdydz \cdot \tag{13}$$

We obtain the strain energy of the laminated plate expressed by the transverse deflection, w, as

$$U = \frac{1}{2}(D_o + D_i + D_p)\int_0^a\int_0^b \begin{bmatrix} \left(\frac{\partial^2 w}{\partial x^2}\right)^2 + \left(\frac{\partial^2 w}{\partial y^2}\right)^2 + \\ 2\nu \frac{\partial^2 w}{\partial x^2}\frac{\partial^2 w}{\partial y^2} + 2(1-\nu)\left(\frac{\partial^2 w}{\partial x \partial y}\right)^2 \end{bmatrix} dxdy \,, \tag{14}$$

in which,

$$D_k = \frac{E}{3(1-\nu^2)}\left[h_k^3 + \frac{3h_p^2 h_k}{4} + \frac{3h_k^2 h_p}{2}\right],\ \text{k = o, i}$$

$$D_p = \frac{E_0 h_p^3}{12(1-\nu_0^2)} \cdot$$

Force work done is

$$w_s = \int_0^a\int_0^b p(t) w dxdy \cdot \tag{15}$$

The kinetic energy of the rectangular laminated panel can be written as

$$T = \frac{1}{2}\left(\rho_o h_o + \rho_p h_p + \rho_i h_i\right)\int_0^a\int_0^b \left(\frac{\partial w}{\partial t}\right)^2 dxdy \cdot \tag{16}$$

Using Hamilton's principle [9],

$$\int_{t_1}^{t_2}\left(\delta T - \delta U + \delta W_s\right)dt = 0 \,, \tag{17}$$

we get the equations of motion for a laminated plate as

$$D^*\left(\frac{\partial^4 w}{\partial x^4} + 2\frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4}\right) + M^*\frac{\partial^2 w}{\partial t^2} = p_o(1 - \frac{t}{t_d})e^{-at/t_d} \,, \tag{18}$$

where

$$D^* = D_o + D_i + D_p \,,$$

$$M^* = \rho_g h_o + \rho_p h_p + \rho_g h_i \,,$$

E, $\nu$, $\rho_g$ is Young's modulus, Poisson ratio and density of the glass plies, respectively; $\rho_p$ is density of PVB interlayer.

The simply supported boundary conditions are

$$w = 0,\quad \frac{\partial^2 w}{\partial x^2} = 0 \text{, at x = 0, a,} \tag{19}$$

$$w = 0,\quad \frac{\partial^2 w}{\partial y^2} = 0 \text{, at y = 0, b,} \tag{20}$$

and the initial conditions are

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$$w(x,y,t)=0, \qquad \frac{\partial w(x,y,t)}{\partial t}=0\text{, at t = 0.} \tag{21}$$

To get the natural frequency, we set

$$w(x,y,t)=W(x,y)e^{j\omega t}, \tag{22}$$

$$W(x,y)=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}A\sin\frac{m\pi x}{a}\sin\frac{n\pi y}{b}, \quad m,n=1,\ 3,\ 5,\ \cdots, \tag{23}$$

it gives

$$\left(\frac{m\pi}{a}\right)^4+2\left(\frac{m\pi}{a}\right)^2\left(\frac{n\pi}{b}\right)^2+\left(\frac{n\pi}{b}\right)^4-\frac{M^*}{D^*}\omega^2=0. \tag{24}$$

The natural frequency is

$$\omega_{mn}=\sqrt{\frac{K^*}{M^*}}, \tag{25}$$

where,

$$K^*=D^*\pi^4\left[\left(\frac{m}{a}\right)^2+\left(\frac{n}{b}\right)^2\right]^2.$$

The free vibration mode is

$$w_V(x,y,t)=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\left\{\begin{aligned}&A\sin\frac{m\pi x}{a}\sin\frac{n\pi y}{b}\cos\omega t\\&+B\sin\frac{m\pi x}{a}\sin\frac{n\pi y}{b}\sin\omega t\end{aligned}\right\}, m,n=1,3,5,\cdots \tag{26}$$

where, A and B are constants to be determined using initial conditions.

By using double Fourier expansion

$$p_0=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}a_{mn}\sin\left(\frac{m\pi x}{a}\right)\sin\left(\frac{n\pi y}{b}\right), \tag{27}$$

$$w(x,y,t)=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}A_{mn}(t)\sin\left(\frac{m\pi x}{a}\right)\sin\left(\frac{n\pi y}{b}\right), \tag{28}$$

we get

$$K^*A_{mn}(t)+M^*A''_{mn}(t)=a_{mn}(1-\frac{t}{t_d})e^{-at/t_d}\text{, m, n = 1, 3, 5, …, .} \tag{29}$$

The solution of the forced response of (18) is

$$w_F(x,y,t)=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\left\{f_1\cos(\omega t)+f_2\frac{\sin(\omega t)}{\omega t_d}+f_3e^{-\alpha t/t_d}\right\} \times f_4\sin\left(\frac{m\pi x}{a}\right)\sin\left(\frac{n\pi y}{b}\right), \quad m,n=1,\ 3,\ 5,\ \cdots \tag{30}$$

where

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$$f_1 = \frac{2\alpha}{T_{mn} t_d^2} - 1;\ \ f_2 = 1 + \alpha - \frac{2\alpha^2}{T_{mn} t_d^2};\ \ f_3 = 1 - \frac{t}{t_d} - \frac{2\alpha}{T_{mn} t_d^2};\ \ f_4 = \frac{16 p_0}{\pi^2 mn M^* T_{mn}}.$$

Using continuous conditions at t = $t_d$

$$w_V(x, y, t_d) = w_F(x, y, t_d), \tag{31}$$

$$\dot{w}_V(x, y, t_d) = \dot{w}_F(x, y, t_d), \tag{32}$$

we get the free vibration mode after positive phase duration as

$$w_V(x, y, t) = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} \left\{ \begin{array}{l} f_1 \cos(\omega t) + f_2 \dfrac{\sin(\omega t)}{\omega t_d} + \\ f_5 \sin(t - t_d)\omega - f_6 \cos(t - t_d)\omega \end{array} \right\}$$

$$\times f_4 \sin\left(\frac{m\pi x}{a}\right)\sin\left(\frac{n\pi y}{b}\right), \tag{33}$$

$$for\ \ t \geq t_d\ \ and\ \ m, n = 1,\ 3,\ 5,\ \cdots$$

where,

$$f_5 = \left(\frac{2\alpha^2}{T_{mn} t_d^2} - 1\right)\frac{e^{-\alpha}}{\omega t_d};\ \ f_6 = \frac{2\alpha e^{-\alpha}}{T_{mn} t_d^2};\ \ T_{mn} = \left(\frac{\alpha}{t_d}\right)^2 + \omega^2;\ \ \omega = \omega_{mn}.$$

After getting $w_F$(x,y,t) and $w_V$(x,y,t), we can calculate stresses by using the plane stress-strain relations.

## 4 Finite element modeling

The problem is also solved numerically using the commercial dynamic nonlinear three dimensional finite element code LS-DYNA3D developed by Livermore Software Technology Corporation [10]. The FEA meshes for the square laminated glass plate are $60 \times 60 \times 10$ corresponding to x-, y- and z-direction respectively. In the FE analysis, the PVB interlayer is modeled as a linear viscoelastic material for which the deviatoric components are given in (8-10).

## 5 Results and discussions

In the closed-form solution and the finite element calculation, the following constants are used: for the glass plies, $E$ = 72 GPa, $\nu$ = 0.25, mass density $\rho_g$ = 2,500 kg/m$^3$; for the PVB interlayer, $G_o$ = 0.33 GPa, $G_\infty$ = 0.69 MPa, $K$ = 20.0 GPa, $\beta$ = 12.6 s$^{-1}$, and $\rho_p$ = 1,100 kg/m$^3$. The dimensions are: $h_o = h_i$ = 4.76 mm, $h_p$ = 1.52 mm, $a$=$b$ = 1.325 m. The explosion parameters are $t_d = 7.7ms$, $\alpha = 0.55$ and $p_d = 6894.8 \times f$, where f varies from 0.05 to 1.0 to study the effect of the overpressure on the dynamic response. Simply supported boundary conditions are applied to the laminated glass plate. The laminated glass unit is discretized by using 3-D 8-node solid elements.

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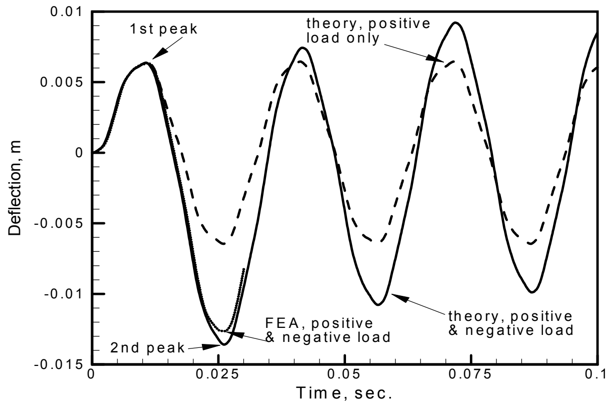


Figure 3: Mid-span deflection history of the laminated glass panel.

We studied two calculating models. One is the positive phase load only (PPLO) which first employs eqn (30) to calculate the dynamic response till the positive load duration $t_d$ and then the dynamic response is calculated using eqn (33) for the free vibration. Another is with negative phase load (WNPL) which calculates the dynamic response with full blast loading period by using eqn (30).

Figure 3 shows mid plane deflection-time history at mid-span of the plate with overpressure factor f = 1.0. In which the dash line is obtained by closed-form solution using the PPLO model which shows free vibration with identical amplitude after 7.7 ms, the positive phase load duration. The solid line is from the closed-form solution using the WNPL model and dotted line is the same but from the FEA result. The difference between the closed-form and the FEA at second peak is due to nonlinear response of large deflection obtained by the FEA. It is seen that the deflection at second peak is about double the corresponding quantities at first peak. Figure 4 shows the maximum principal stress-time history at mid-span of the plate. The dash line is obtained by closed-form solution on the top surface using the PPLO model and the solid line is the same but using the WNPL model.

As a comparison, the FEA results obtained from on top and bottom surfaces of the plate are denoted by the dash-dot and dotted lines, respectively. It is shown that the tensile stress at second peak is around twice as high as that at first peak. If the laminated glass plate is not to be damaged during the first peak tensile stress, the elastic recovery force of the plate plus the negative phase blast load will generate higher tensile stress on top surface. Therefore, the second peak tensile stress will be more dangerous to the plate than the first tensile peak tensile stress.

To account for the effect of the overpressure on the dynamic response of the plate, figure 5 and 6 show the deflections and maximum principle stresses at first and second peak, as shown in Figures 3 & 4, against the overpressure, respectively. It is seen that both mid-span deflection and stresses from the closed-form solution coincide very well with FEA results within the current load region.

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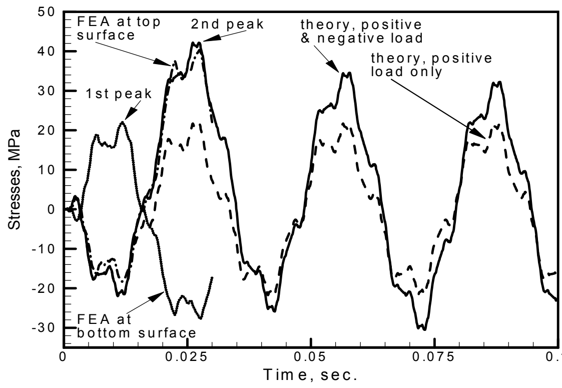


Figure 4: Mid-span maximum principal stress history of the laminated glass panel.

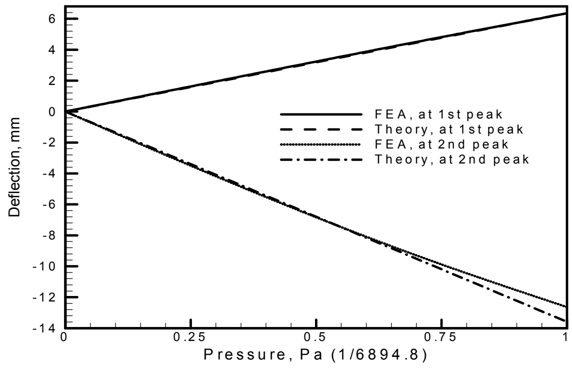


Figure 5: Effect of the overpressure on the mid-span deflection.

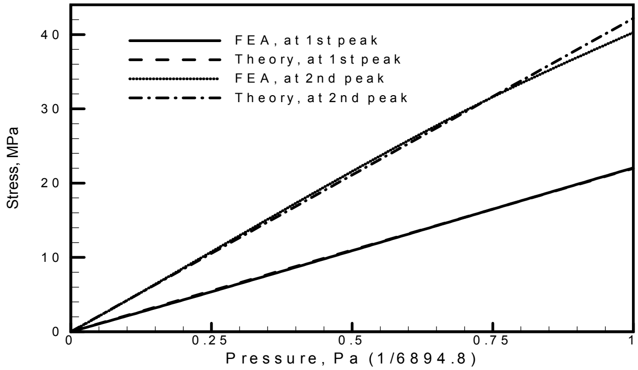


Figure 6: Effect of the overpressure on the mid-span maximum principal stress.

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## 6 Conclusions

The empirical quasi-exponential form of blast loading can approximately be used to simulate both the positive and the negative phase overpressure. The negative phase of blast loading has a significant effect on the dynamic response of the laminated glass panel. The mid-span deflection and stress due to negative phase pressure are around twice as high as the corresponding quantities due to positive phase at small deflection region. This may mean that the laminated glass panel may not fail in the positive loading duration, but it may be damaged in the negative loading phase because of higher dynamic stresses during this later phase. To precisely calculate the nonlinear dynamic response of a laminated glass plate at negative phase load under higher blast overpressure, a nonlinear solution is necessary and will be.

## Acknowledgment

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