Interactions of shock waves with a sphere and arrayed spheres

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Abstract

This paper deals with the results of experimental and numerical studies of shock wave interaction with a single sphere and arrayed spheres. An 80 mm dia. sphere suspended vertically along the centerline of a 300 mm x 300 mm vertical shock tube was loaded with a planar shock wave at a shock Mach number of 1.2 in air. Unsteady drag forces working on the sphere were measured with two accelerometers installed inside it. The measured result was compared with that of numerical simulation solving the Navier-Stokes equations, which agreed well with experiment. It was found that a maximum drag force appeared at the time instance slightly after the transition of the reflected shock wave from regular to Mach reflection and a negative drag force was created only for a very short duration of time when the transmitting shock wave merged at the rear stagnation point of the sphere. The shock/arrayed spheres interaction was experimentally investigated in a 60 mm x 150 mm diaphragmless shock tube. The process of the interaction was visualized sequentially by using double exposure holographic interferometry. Due to the three-dimensional distributions of arrayed spheres the shock wave attenuation was much more pronounced than in arrayed cylinders.

1 Introduction

When a planar shock wave is reflected from a wedge, its reflection pattern takes either regular reflection or Mach reflection depending upon its Mach number Ms, wedge angle, and value of specific heats ratio. Although reflected shock waves patterns were supposed to be self-similar [1], due to the existence of
boundary layers developing behind the incident shock wave, even over the wedge angle which warrants Mach reflection, regular reflection appears. With its propagation for some distance from the leading edge the transition to Mach reflection takes place. Such a delayed transition from regular to Mach reflection is typical to wedges with apex angles close to the critical transition angle [2].

When a planar shock propagates over a convex cylindrical surface, its reflection pattern is at first regular, and then the transition to Mach reflection takes place with decreasing the wall inclination angle [3]. Itoh and Takayama [4] visualized quantitatively, by using double exposure holographic interferometry, flows over a 40 mm dia. cylinder placed in a 40 mm x 80 mm shock tube. Analyzing interferometric fringes, they estimated unsteady drag force of the shock-laden cylinder and found that a maximum drag force appeared when the reflected shock transition from regular to Mach reflections took place. The value of unsteady drag coefficient decreased gradually with the elapse of time to the steady one in the flow behind the incident shock wave.

It is not known whether or not the critical transition angle over spheres is identical with that over convex cylinders. Bryson and Gross [5], so far the authors know, were the first who visualized shock/sphere interaction in a horizontal shock tube and compared the trajectory of triple points of diffracted shock waves over the sphere with Whitham's ray shock theory [6]. However, the unsteady drag force was not measured.

The attenuation of weak shock waves propagating through three-dimensionally distributed obstacles has been one of the fundamental topics of shock dynamics, which was recently revived related to the suppression of tunnel sonic booms. Abe [7] experimentally investigated shock interaction with arrayed spheres in a 60 mm x 150 mm diaphragmless shock tube having a higher degree of reproducibility. Sequential holographic interferograms were displayed in computer-assisted animation. It is found that three-dimensional shock wave interactions effectively attenuate transmitting shock waves.

2 Shock/sphere interaction

2.1 Experiment

Experiments were carried out in a vertical shock tube in the Interdisciplinary Shock Wave Laboratory of the Shock Wave Research Center (SWRC) of the Institute of Fluid Science, Tohoku University.

2.1.1 Vertical shock tube
The vertical shock tube consisting of a 300 mm dia. and 2,500 mm long high pressure chamber made from stainless steel, a double diaphragm section, and a 300 mm x 300 mm and 4,000 mm long low pressure channel. Each of the two test sections made from aluminum alloy had a 300 mm x 500 mm view field. A 700 mm dia. and 1,500 mm long dump tank was connected to the low pressure channel. The selection of Mylar diaphragm thickness ranging from 0.05 to 0.188 mm and driver gases either helium or nitrogen, controlled shock Mach numbers.
ranging from 1.1 to 1.6 in air. The reproducibility of this shock tube is not as good as diaphragm-less ones. Hence, in the recent series of experiments cellophane diaphragms are used which are more brittle than Mylar ones and rupture more quickly.

![Figure 1: A 80 mm dia. sphere and its installation in the test section: (a) Sphere model; and its installation in the test section.](image)

### 2.1.2 Sphere model

Figure 1 shows a 80 mm dia. sphere made of aluminum alloy, weighing approximately 350 g, inside which two accelerometers (Endevco Piezoelectric accelerometer 2250A-10; sensitivity 1.02V/m/s²; eigen resonant frequency 80 kHz) were installed. The accelerometers were backed up with charge amplifiers (Endevco Model-133). Upon loading a Ms = 1.2 shock wave the force acting on the model was estimated to be at most 400N and hence a 2 mm dia. stainless steel wire was strong to suspend the sphere from underneath the diaphragm section along the shock tube center axis. Output signals from the accelerometers were taken out through co-axial cables from the rear stagnation of the sphere, processed with a strain gauge charge amplifier (Kyowa CDV-700A Endevco Model-133), and displayed on a digital memory (Hioki 8855-memory Hicorder; 2µs/word 500kHz sampling). These data were processed by means of moving averaging of 23 data points.

In preparatory tests the sphere did not move violently as imagined before the trial. The present aluminum sphere had its eigen resonant frequency of approximately 30Hz so that its resulting response was so slowly that its force was distinctly removable from high frequency response of shock loading.

### 2.1.3 Visualization

Shock waves propagation over the sphere was visualized with double exposure holographic interferometry [8] and a high speed video camera (Shimazu Prototype ISIS maximum 10⁶ frames/s) based on object beam path of holographic interferometry. Such visualizations were helpful for understanding dynamic response of the model against shock loading and also to confirm the
level of disturbances caused from the wire suspension and also the signal take out lead wires. High speed video recording of 100 frames is one of unique features of the present experiment. Light source was a relatively intense flash lamp of a 1 ms pulse duration. High speed video images were displayed in the form of animation with combination of holographic interferograms.

Light source of double exposure holographic interferometric was Q-switched holographic ruby laser (Inolus; pulse duration of 25 ns; wavelength of 6,943 nm) and holographic sheet films of 100 mm x 125 mm were used. Reconstruction was carried out by using an Argon-ion laser and recorded on 100 mm x 125 mm Neo Pan SS sheet films. Holographic system so far used has been developed for shock wave research in SWRC since 1975. However, it may need to be updated by introducing phase shift electronic recording systems in which films will no longer be used.

2.1.4 Pressure measurement

Figure 2 shows frontal view and cross sectional view of the pressure transducer distribution. 11 piezo-resistive pressure transducers (Entran EPI-58; sensitivity 100 mV/100 psi; eigen resonant frequency of 220 kHz) were distributed from the frontal stagnation point at every 15 degree interval. The 11th transducer was shifted by 30 degrees from the rear stagnation point. The single output cable was covered with a plastic tube and the signals were processed with a strain gauge amplifier (Kyowa CDV-700A), and recorded in a digital memory (Hioki 8855-memory Hicorder; 2µs/word 500kHz sampling).

![Figure 2: Installation of pressure transducers.](image)

2.2 Numerical simulation

Numerical simulations were carried out with a finite difference scheme by solving the three-dimensional Navier-Stokes equations with combination of Goldberg-Ramakrishnan one-equation turbulent model [9]. Three-dimensional unstructured meshes of 120 x 120 x 300 are covering a computational domain of a quarter of 300 mm x 300 mm x 500 mm test section and a 80 mm dia. sphere which is fixed in the middle computational domain. Meshes were densely
distributed over the surface so as to resolve boundary layer imposing non-slip condition on its surface boundary. However, the presence of suspension wire and signal output cable was neglected. On the side wall surfaces the same mesh sizes were used as inviscid flow applying the slip condition. The computation was continued until the reflected shock waves from the side walls arrived at the sphere surface.

3 Shock/arrayed sphere interaction

3.1 Experiment

Experiments were conducted in a 60 mm x 150 mm diaphragmless shock tube in the Shock Wave Research Center of the Institute of Fluid Science, Tohoku University.

3.1.1 Diaphragmless shock tube

The shock tube consisted of a 100 mm dia. and 1,000 mm long leak section, a 100 mm dia. and 300 mm long auxiliary high pressure chamber, a 290 mm dia. and 2,900 mm long high pressure chamber, and a 60 mm x 150 mm low pressure channel. A 150 mm dia. low pressure channel was connected to a continuously area variable section from a 150 mm dia. to a 60 mm x 150 mm and was inserted co-axially inside the high pressure chamber. A test section had a 150 mm x 250 mm view field connected to the low pressure channel. The shock tube was designed and manufactured in house in order not only to achieve higher degree of reproducibility but also to minimize surface irregularities with well polished low pressure channel wall and to decrease gaps at the individual flange sections.

A rubber membrane was inserted at the end of the auxiliary high pressure section being backed up with a convexly shaped steel support. It was bulged with high pressure nitrogen, sealing the entrance of a 150 mm dia. low pressure channel so as to separate a high pressure driver gas from low pressure air.

![Figure 3: Arrangement of arrayed spherical models.](image)
This arrangement is so mechanical and controllable that pressures are very accurately adjusted and the movement of the rubber membrane became very repeatable. Hence a higher degree of repeatability was achieved. The scatter of shock Mach numbers for Ms = 1.015 to 2.82 in air was found to be 0.25 % [10].

3.1.2 Arrayed sphere model
Figure 3 shows the arrangement of arrayed spheres inside the shock tube test section. The combination of three truncated spheres were sandwiched with two 2 mm thick acrylic plates and placed over the side walls of the shock tube test section in a staggering arrangement with horizontally 26 mm interval and vertically 30 mm interval. The blockage ratio of this arrangement was about 54 %.

3.1.3 Visualization and animated display
Holographic interferometric visualization was carried out [7] with the same arrangement as described in the section 2.1.3. Under the identical initial condition visualizations were repeated only by varying the delay time at every 3 μs from the time instant when the incident shock arrived at the first row of the arrayed spheres. Reconstructed images were recorded on 100 mm x 125 mm Neo Pan SS sheet films, which were later scanned with a 300 dpi digital scanner. The scanned images were stored on a PC and their contrast and image sizes were edited by using a graphic software package Adobe Photoshop (Adobe System Inc.) and a Scan Image (Scion Co.). Animated display of these processed images was conducted on SGI Onyx2 by using a graphic software package Mediaconvert (SGI Inc.).

4 Results and discussion

4.1 Shock/sphere interaction
Shock Mach number was 1.2 in ambient air and its corresponding Reynolds number for the characteristic length of the sphere diameter was approximately 300,000.

4.1.1 High speed video and interferometric observations
Shock/sphere interaction was visualized based on schlieren optics by using a high speed video camera at 500,000 frames/s and also double exposure holographic interferometry. In figures 4a-d, these results are compared at 152, 312, 448, and 808 μs, respectively. At the earlier stage of shock reflection, patterns show typical transition from regular to Mach reflection. After the incident shock arrival at the frontal stagnation point at 70 μs, the transition appears between time instants of 128 and 136 μs which correspond to the elapsed time of 48 to 56 μs. Since the speed of Ms = 1.2 shock wave at 293 K is 0.408 mm/μs the distance between the frontal stagnation point and the transition point ranges from 19 to 23 mm. Therefore, the critical transition angle over it is approximately 33.5 degrees, whereas in the case of a 80 mm dia. cylinder it is nearly 33.0 degree [3]. It should be noticed that the critical transition angles on a
80 mm dia. sphere nearly agreed with that on a 80 mm dia. cylinder for $M_s = 1.2$ in air.

Once Mach reflection appears at the critical transition angle, its triple point or a ring shaped loop of three shock confluence grows with the elapse of time. The Mach stem starts to be diffracted after passing the equator and is gradually curved being exposed to Prandtl-Meyer expansion. The curved Mach stem formed an envelope of curved surface. Its convergence at the rear stagnation point, by exposition is equivalent to shock wave focusing. Unlike wedge cases, the merger of the curved Mach stem created higher pressures at the rear stagnation point and hence the speed of reflected Mach stem at the rear stagnation point is faster than what expected in a cylinder case. In the visualization of axisymmetric Mach reflections their slip surfaces are not clearly observable in Fig.4.

Many wavelets are observable behind the incident shock wave. It implies that double Mylar diaphragms were not ruptured very uniformly, which produced a distorted jet accompanying a skewed shock front. In a small shock tube, such a skewed shock front would become planer with its propagation. However, in a 300 mm dia. Mylar diaphragm a symmetrical petal shapes are hardly produced. Such non-uniformly ruptured diaphragms always produced a distorted shock front.

Effect of asymmetric wave motion is typically observed at the later stage when the reflection of shock waves from the shock tube side wall arrived at the sphere surface, for example, as seen in Fig. 4d. Waves on right and left hand sides look slightly deviated.

### 4.1.2 Pressure measurement
Bryson and Gross visualized for the first time Mach reflections over a sphere placed in a horizontally positioned shock tube [5] supported with many thin strings from shock tube walls. Heilig [11] measured the critical transition angles from regular to Mach reflection over a cylinder and analyzed pressures at the reflected point over wedges by using two-shock theory and found that the pressure behind the reflected shock becomes a maximum when it arrived at the critical transition angle. In order to verify this trend, he tried to measure pressure distributions by distributing pressure transducers over a cylinder surface.

Ben-Dor, Takayama, and Kawauchi [12] determined the critical transition angles of regular to Mach reflection over a convex cylinder and Mach to regular transition over a concave cylinder. They collected these values for wide range of shock Mach numbers. Takayama and Sasaki [3] later tried to identify the effect of radii of curvature on the critical transition and visualized reflected shock transitions over convex and concave cylinders. They concluded that the critical transition angles at given shock Mach number was no longer consistent but varied with radii of curvature of curved walls. The result contradicts analytical predictions based on two shock theory and also three shock theory in which the existence of wall boundary layers is totally neglected.
Figure 4: Comparison between high speed video images and interferometric ones: (a) 152 µs; (b) 312 µs; (c) 448 µs; and (d) 808 µs.

The result clearly indicates that the effect of Reynolds numbers on the reflected shock transition should be taken into account. In those days numerical schemes were not very well refined so that in order to resolve these puzzling results we have maintained our continuous effort and waited for over 20 years. We have eventually finally understood the effect of Reynolds numbers on the reflected shock transition over curved walls [13].

Itoh and Takayama [4] determined unsteady drag force induced upon shock loading on a cylinder and compared experimental results with numerical ones. A good agreement was obtained between them. The numerical unsteady drag force clearly showed the existence of a maximum value at the time instant when the incident shock wave arrived at the transition point and the unsteady drag force decreased monotonously to the value corresponding to steady flow behind the shock wave. Their trend agreed reasonably well with Heilig's experimental data.
However, it is not yet clearcut whether or not the shock wave loading on spheres will produce a similar trend to cylinders. Figure 5 shows pressure time variations of 2rd, 4th, 6th, 9th and 11th pressure transducers distributed over the sphere surface as shown in Fig. 2. Abscissa designates the elapsed time in ms and ordinate the pressure in kPa. Upon shock arrival at the frontal stagnation area the overpressure shown in ch 1 becomes a maximum. Pressures in ch 2 transducer successively indicate pressure jumps of arrivals of the incident shock at these transducers and immediate increases in pressures following the incident shock designate reflected shock waves. Since the ch 4 transducer is located very close to the transition point, its pressures behind the reflected shock wave becomes a maximum. This observation qualitatively supports the result over wedges [11].

4.1.3 Unsteady drag force
In Fig. 6 time variation of drag force is shown. Ordinate designates the drag force in N and abscissa does the elapsed time in ms. Drag forces were measured by two accelerometers, ch 1 and ch 2 designated by thin lines and compared with the numerical result in a thick line. In both experiment and simulation the time zero should have been taken as the time instant when the incident shock arrived at the frontal stagnation point. However, data obtained from accelerometers were processed as temporal average of 23 output data so that the zero position was slight shifted to a negative value.

The processed output signals out of two accelerometers in ch 1 and ch 2 appear to be identical. However, very minute deviation from each other would be created mostly due to slight asymmetry of wave motions. Agreement between these measured drag forces and numerical result is very good up to the elapsed time up to 0.8 ms. At this time instant the reflected shock wave from the shock tube sidewall arrived at the sphere and hence the comparison with the numerical simulation no longer meaningful.
At about 0.08 ms from the arrival of the shock wave, unsteady drag force is a maximum. In the case of a cylinder [4] the drag force became a maximum when the reflected shock transition took place. It is numerically found that in the case of a sphere the critical transition angles over spheres are nearly identical with those over cylinders. However, unlike the cylinder case, maximum drag force appeared when shock moved far closer to equator. This is caused due to the fact that the projected surface area over which shock propagates increases proportional to the square of the distance the shock propagation. Therefore, the drag force still increases after passing the transition point until it reaches close to the equator.

It is confirmed for the first time that at about 0.4 ms when the diffracted Mach stem converged at the rear stagnation point the drag force took negative value, which continued about 0.3 ms. This fact is well interpreted from results of visualization and also pressure measurements. When Mach stem merged the pressures directed to upstream became high enough to cancel out the positive drag force. However, it is noticed that since such resulting drag forces depend merely upon shock Mach number, it does not always take negative values but is a minimum. This is a preliminary result and the effect of shock Mach numbers on the generation of unsteady drag forces, in particular occurrence of negative ones, will be investigated in the near future. It will be worthwhile to mention that, in using the present vertical shock tube, aerodynamic force measurement can be successfully conducted.

The time variation of drag force shown in Fig. 6 disagrees with that over cylinders but the value peak drag force is larger. If the sphere is not rigidly suspended but weakly held in air, the impulse deduced from this unsteady drag force would accelerate it working it more downward. The impulse so far estimated at least at earlier stage is significantly larger than that estimated form steady drag coefficients. In modeling dusty gas shock tube flows, the Stokian drag coefficient or at best its modified formula is traditionally employed. In
modeling shock induced droplet shattering, such drag coefficients are frequently used. Although the range of the Reynolds number under study is in the order of magnitude of hundreds of thousand and the present result is most unlikely applicable directly to dusty gas shock tube flows or shock induced droplet shattering. However, it should be re-considered whether or not the trend of the unsteady drag coefficients presented in Fig. 6 may exist in the range of smaller Reynolds number cases.

4.2 Shock/arrayed sphere interaction

Figure 7 shows 30 infinite fringe sequential holographic interferograms of Ms = 1.2 shock wave propagation through arrayed spheres. These interferograms were constructed running shock tube experiments under identical initial conditions and at each run visualization was carried out with delay time of every 5 µs. The interferometric images so far were processed on PC and edited in the form of animated display. When a planar shock impinged the arrayed spheres similar wave interaction as already observed in shock/a single sphere interaction takes place. Transmitting shock waves are exposed to three-dimensional expansion waves so that every time shock interacted with distributed spheres such a process is repeated. Hence shock waves are more effectively attenuated than their propagation through arrayed cylinders.

Figure 7: Sequential holographic interferograms of shock propagation through arrayed spheres at Ms = 1.2 in air.
5 Concluding remarks

Experimental and numerical studies of shock interactions with a single sphere and arrayed spheres were carried out. Results obtained are summarized as following:

(1) Unsteady drag force was measured upon shock loading on a sphere by using accelerometers installed in the sphere model. The drag force is a maximum before shock arrival at equator. Negative drag force appeared when a diffracted Mach stem merged at the rear stagnation point. This trend agreed very well between measurement and numerical simulation.

(2) The time history of unsteady drag force over a shock loaded sphere appears to be very different from that of cylinder. From this observation and time variation of unsteady drag forces it should be noticed that drag coefficients by used for dusty gas shock tube flow analysis seem to be non-physical and also for shock interaction with liquid droplets, the effect of unsteady drag force on their deformation, at least at earlier stage, should be reconsidered.

(3) High speed video recording with combination of double exposure holographic interferometry successfully revealed contributions of individual wave interactions to results of pressure measurements and also drag force measurements.

(4) Shock interaction with arrayed spheres was primarily investigated. Three-dimensional expansion waves and complex wave interaction promoted shock wave attenuation much more effectively that that with cylinders.

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