Simplified analysis of square plates under explosion loading

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Abstract

The methods most frequently used for preliminary evaluation of explosion damage are the empirically based pressure-impulse charts of constant damage levels, single-degree-of-freedom dynamic analysis of equivalent spring-mass systems, and the energy and rigid-plastic theoretical solutions. Experimental data for the explosion loading of square plates having nominal dimensions of 125 mm, 1 m and 3 m will be discussed and compared with some simplified theoretical predictions. Some comments will be offered on scaling effects and on the accuracy of simple methods of analysis for dynamic loadings ranging from those characterised as quasi-static to impulsive loadings.

Notation

E  Young’s modulus of elasticity
H  Plate thickness
L  Half-span length of square plate
\( \lambda \)  Dimensionless kinetic energy
\( M_0 \)  Fully plastic moment capacity of plate section per unit length
p  Pulse pressure load
\( p_1 \)  Peak pulse pressure load
\( \rho \)  Density
\( \sigma_0 \)  Plastic flow stress
T  Natural period of vibration
\( t_d \)  Load pulse duration
1 Objective

Structural response to explosion loading is a specialised area of structural dynamics incorporating a variety of analytical tools of varying complexity from multi-degree-of-freedom finite element methods to equivalent one-degree-of-freedom spring-mass systems. Approximate techniques allow rapid assessment and screening of a large number of components and scenarios than would otherwise be possible with computationally intensive engineering models. These methods are usually sufficiently accurate for assigning priorities and making decisions on the need for more detailed analysis. The purpose of this article is to compare some theoretical solutions with experimental data on pulse pressure loaded and impulsively loaded square plates and to show that simplified methods in most cases give adequate engineering estimates of structural response to explosion loading. Thus, by considering the nature of the problem initially, much effort can be avoided in generating unnecessary detailed analysis.

2 Experimental data

2.1 Data on 3 m square plates

A structural response test of six specially designed blast panels, typical of those used offshore, was conducted as part of the Joint Industry Project (JIP) on Blast and Fire Engineering for Topside Structures [1] sponsored by the Health and Safety Executive and several oil and gas companies. All the panels were made from steel plate and, with the exception of one panel, stiffened on one side with structural sections welded to the plate. The panels were supported in different ways ranging from knife-edge supports to fully fixed. A partially confined gas explosion generated the transient pressure loading on the panels. For the purposes of this article, the unstiffened panel will be compared with some simplified theoretical predictions. The panel dimensions of this panel are 3.06 m by 3.16 m by 6 mm thick. The edges of the plate were welded to a 200 mm by 12.5 mm thick steel hollow section frame. The average static material properties derived from coupon tests were 306 MPa for the yield stress and 455 MPa for the ultimate tensile stress. Load pulse data obtained from transducers adjacent to the panel was averaged to give a peak overpressure of 146 kPa, a rise time to peak pressure of 46.5 ms and a load duration of 83.5 ms. Linear displacement transducers measured a peak displacement of 161 mm and a residual displacement of 60 mm at the centre of the panel. Plastic hinges were observed in the four corners of the panel at 45° to the edges of approximately 400 mm in length. No tearing of the welds between the plate and the frame was detected.
2.2 Data on 1 m square plates

Small-scale tests [2] were conducted at the University of Liverpool on clamped square plates measuring 1 m by 1 m by 2 mm thick, approximately one-third scale of the 3 m JIP test plate. The plate was bolted and clamped to a rigid support plate and loaded by means of a transient differential pressure device. All the plates were restrained rotationally with some plates having in-plane restraint and others with no in-plane restraint. Some plates were stiffened with ribs welded to the plate. The loading and response data given in Table 1 comes from the unstiffened plate tests. The material properties derived from static and dynamic uniaxial tensile tests after averaging the data are 186 MPa for the yield stress, 299 MPa for the ultimate tensile stress, 44.7 and 2860 for the Cowper-Symonds (equation see Reference [3]) constant D at yield and UTS, respectively, and 4.7 and 3.1 for the Cowper-Symonds constant q at yield and UTS, respectively.

Table 1: Test data on 1 m fully clamped square mild steel plate loaded dynamically [2].

<table>
<thead>
<tr>
<th>(p_t) (kPa)</th>
<th>Impulse (Ns)</th>
<th>(t_r) (ms)</th>
<th>(t_d) (ms)</th>
<th>Max. centre disp. (mm)</th>
<th>Final centre disp. (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>103.0</td>
<td>3100</td>
<td>32.0</td>
<td>61.2</td>
<td>44</td>
<td>18</td>
</tr>
</tbody>
</table>

2.3 Data on 125 mm square plates

Small-scale dynamic tests were reported in Reference [4] on 2.69 mm thick fully clamped square plates measuring 127 mm by 127 mm. The external blast loading was generated using sheet explosive and was characterised as an initial impulsive velocity distributed uniformly on a plate. The plates were made from hot rolled mild steel having a yield stress of 251 MPa, an ultimate tensile stress of 311 MPa and a 25.4% rupture elongation.

3 Theoretical solutions

3.1 Rigid-plastic methods

A quasi-static, rigid-plastic analysis of a square plate [3], which is assumed to be fully restrained axially, is presented here for use in estimating the residual displacements of the test plates. The static collapse pressure for a fully clamped, rigid, perfectly plastic plate is given by

\[
p_c = \frac{10.71M_0}{L^2}.
\]  

(1)

For \(W/H > 1\), the pressure ratio for a fully restrained, clamped square plate is given by
\[
\frac{p}{p_c} = \eta = \frac{2W}{H} \left[ 1 + \frac{1}{2} \left( \frac{H^2}{3W^2} - 1 \right) \right], \quad (2)
\]

which transforms to

\[
\frac{W}{H} = \frac{\eta \pm \sqrt{\eta^2 - \frac{4}{3}}}{2}. \quad (3)
\]

It is noted that the solution for hinged supports and fully clamped supports gives the same maximum transverse displacement for large values of W/H which are well into the membrane range of behaviour, which is the case in these tests. Equation (3) simplifies to

\[
\frac{W}{H} \approx \eta \quad (4)
\]

for W/H >> 1.

A theoretical rigid, perfectly plastic analysis was developed in Reference [5] for the response of rectangular plates subjected to dynamic loads producing large inelastic deformations. This analysis for the particular case of a uniform impulsive velocity of magnitude, \( V_0 \), predicts the maximum permanent transverse displacement of a fully clamped square plate

\[
\frac{W}{H} = \sqrt{1 + \frac{\lambda}{6} - 1}, \quad (5)
\]

where

\[
\lambda = \frac{\rho V_0^2 L^2}{M_0} \quad (6a)
\]

is the dimensionless initial kinetic energy. For a plate having a solid cross-section, \( M_0 = \sigma_0 H^2/4 \), so that equation (6a) can be written as

\[
\lambda = \left( \frac{\rho}{\sigma_0} \right) \left( \frac{2LV_0}{H} \right)^2. \quad (6b)
\]

It is shown in §8.5.4 of Reference [3] how the observations of Perrone and Bhadra [6] can be used to modify the above analysis in order to predict the residual, or permanent, transverse displacements of a plate made from a strain rate sensitive material which is characterised by the Cowper-Symonds equation and the associated material constants D and q. For the particular case of a square plate

\[
\frac{W}{H} = \sqrt{1 + \frac{\lambda}{6n} - 1}, \quad (7a)
\]

where

\[
n = 1 + \left[ \left( \frac{\sigma_0}{\rho} \right)^{\frac{1}{2}} \left( \frac{H}{2L} \right)^2 \frac{\lambda}{3\sqrt{3DL}} \right]^{\frac{1}{q}} \quad (7b)
\]
and \( n \sigma_0 = \sigma_0' \) is an estimate of the dynamic flow stress.

### 3.2 Energy methods

Energy solutions are extremely useful for predicting peak displacements and stresses. The quasi-static energy solution for a square plate, which is considered to be a membrane, pinned on all edges, with in-plane restraint, is derived here for use in estimating the maximum and residual displacements of the 3.11 m and 1 m square test plates. Since the deformations in the test plates are so large, it was felt that the bending strain energy contribution could be neglected.

For a square plate, which is considered to be a membrane, with pinned supports, a possible deformed shape is

\[
w = W \cos \frac{\pi x}{2L} \cos \frac{\pi y}{2L},
\]

where \( L \) is the half-span of the plate, \( w \) is the transverse displacement at any point, \( W \) is the transverse displacement at the centre of the plate, and \( x \) and \( y \) are the plate co-ordinates with their origin at the centre of the plate.

For either bending or extension, the strain energy per unit volume is given by

\[
\frac{SE}{Vol} = \int_{vol} \left[ \sigma_{xx} \varepsilon_{xx} + 2\sigma_{xy} \varepsilon_{xy} + \sigma_{yy} \varepsilon_{yy} \right].
\]

The subscripts \( xx \) and \( yy \) represent the normal stresses and strains and the subscript \( xy \) represents the shearing stresses and strains. For an elastic plate, the general solution becomes

\[
\frac{SE}{Vol} = \frac{E}{2} \varepsilon_{xx}^2 + \frac{E}{2(1+v)} \varepsilon_{xy}^2 + \frac{E}{2} \varepsilon_{yy}^2.
\]

In a rigid, perfectly plastic plate that obeys the Von Mises maximum shear strain energy per unit volume yield criterion, when the normal biaxial stresses equal \( \sigma_0 \), the shearing stress is zero. Therefore, for a square plate, the rigid-plastic solution gives

\[
\frac{SE}{Vol} = 2\sigma_0 \varepsilon_{xx},
\]

according to the Von Mises yield criterion, and when assuming that \( \varepsilon_{xx} = \varepsilon_{yy} \) for a square plate loaded uniformly over the entire area. The strain \( \varepsilon_{xx} \) associated with membrane behaviour is

\[
\varepsilon_{xx} = \frac{1}{2} \left( \frac{\partial w}{\partial x} \right)^2.
\]

The external work done by the load is given by

\[
W_k = \iint p w \, dx \, dy,
\]

where \( p \) is the uniformly distributed pressure load and \( w \) is the transverse displacement defined by equation (8).
Equating the extensional strain energy to external work in an elastic analysis gives the quasi-static solution

\[
W = \sqrt[3]{\frac{512pL^4}{9 + \frac{1}{16(1+\nu)}}} EH\pi^6. \tag{14}
\]

Equating the extensional strain energy to external work in a plastic analysis gives the quasi-static solution

\[
W = \frac{64pL^2}{\sigma_0 H\pi^4}, \tag{15a}
\]

without strain rate effects included, and

\[
W = \frac{64pL^2}{\sigma_0' H\pi^4}, \tag{15b}
\]

with strain rate effects included.

4 Comparisons

4.1 Rigid-plastic analysis

For \(2L = 3.11\) m (average of 3.06 and 3.16 m) and \(\sigma_0 = 306\) MPa, then equation (2) predicts a value of 11.97 for \(\eta\) when using \(p_1 = 146\) kPa. Using equation (3), gives the ratio \(W/H = 11.94\) and for \(H = 6\) mm, \(W = 71.6\) mm. This calculation assumes that the plate is fully restrained axially and that the pulse loading causes a quasi-static response. It also ignores the important influence of material strain rate sensitivity for steel, which can be significant even at relatively low strain rates. Using Perrone and Bhadra [6] to estimate material strain rate effects, a value of 0.01 sec\(^{-1}\) was computed. Now using the usual values for the constants \(D = 40\) sec\(^{-1}\) and \(q = 5\) in the Cowper-Symonds equation [3] gives a new value for \(\sigma_0\) in \(M_0\) so that equation (2) now predicts \(W = 60.2\) mm for the permanent maximum transverse displacement of this 3.11 m, 6 mm thick steel plate subjected to a quasi-static pressure of 146 kPa. This compares with a measured value of 60 mm.

For \(2L = 1\) m and \(\sigma_0 = 186\) MPa, then equ (2) predicts \(\eta = 12.875\). Using equ (3), gives the ratio \(W/H = 12.85\) and for \(H = 2\) mm, \(W = 25.7\) mm. Including material strain rate effects as in the previous example, and using \(D = 44.7\) sec\(^{-1}\) and \(q = 4.7\) derived from the dynamic materials test data for the Cowper-Symonds equation, gives a new value for \(W = 21.5\) mm for the permanent maximum transverse displacement of this 1 m, 2 mm thick steel plate subjected to a quasi-static pressure of 103 kPa. This compares with a measured value of 18 mm.

The above calculations could be repeated for the triangular pulse loading taken to be a dynamic loading rather than a quasi-static one. A relatively simple
Theoretical procedure for estimating damage to plated structures due to a triangular pressure pulse is given in reference [7].

![Diagram](image)

**Figure 1**: Dimensionless maximum permanent transverse displacements of fully clamped square mild steel plates loaded impulsively. ♦: experimental results [4], ——: theoretical predictions using equation (7) with $D = 1300 \text{ sec}^{-1}$ and $q = 5$.

The theoretical predictions of equation (7) are now compared with the experimental data obtained for the 127 mm square plates reported in Reference [4]. The maximum strains in the $2L = 1 \text{ m}$ and $2L = 3.11 \text{ m}$ square plates are about 0.0025 according to equation (7.140) in Reference [3], and, therefore, the Cowper-Symonds coefficients should correspond to those for small strains, as used in the above calculations (i.e., $D = 40 \text{ sec}^{-1}$ with $q = 5$ and $D = 44.7 \text{ sec}^{-1}$ with $q = 4.7$). However, the maximum strains for the square plates in Reference [4] are one order of magnitude larger and range from about 0.02 to 0.1 for plates with the $W/H$ values in Reference [4].

It is well known [8] that the influence of material strain rate sensitivity is greatest for strains near to yield and, for a given strain rate, the effect decreases as the strain increases. It is observed in Reference [9], for example, that $D = 1300 \text{ sec}^{-1}$ and $q = 5$ at a strain of 0.05 for the mild steel studied by Marsh and Campbell [10]. Thus, in the absence of any dynamic material properties on the plate material used in Reference [4], the values $D = 1300 \text{ sec}^{-1}$ and $q = 5$ are used in equation (7) for the 127 mm square plates. It is evident from the comparisons in Figure 1 that equation (7) gives acceptable agreement with the maximum permanent transverse displacements of the impulsively loaded square mild steel plates reported in Reference [4].
4.2 Energy analysis

Here we use the quasi-static elastic and plastic energy solutions to estimate the maximum and maximum permanent transverse displacements of the 3.11 m and 1 m square test plates. First, a simple treatment is proposed to deal with elastic to plastic transition in the analysis. For a fully axially restrained plate, the axial stretching of the plate at mid-span is given by

\[ \Delta = \frac{1}{2} \int_0^L \left( \frac{\partial w}{\partial x} \right)^2 \, dx. \]  

(16)

Substituting for \( w \) (equation (8)) in equation (16) and integrating gives

\[ \Delta = \frac{W^2 \pi^2}{16L}. \]  

(17)

Now consider an axially loaded member made from elastic, perfectly plastic material and having length \( L \), cross-sectional area \( A \), Young’s modulus \( E \) and plastic flow stress \( \sigma_0 \). The elastic limit of stretching in the plate is given by

\[ \Delta_p = \frac{\sigma_0 L}{E}. \]  

(18)

Equating (17) and (18) gives the elastic limit transverse displacement of the plate at the centre

\[ W_p = \frac{4L}{\pi} \sqrt{\frac{\sigma_0}{E}}, \]  

(19a)

and with strain rate effects included

\[ W_p = \frac{4L}{\pi} \sqrt{\frac{\sigma_0'}{E}}. \]  

(19b)

For \( 2L = 3.11 \) m, \( \sigma_0' = 364 \) MPa, \( E = 205 \) GPa, \( \nu = 0.3 \) and using equations (14) and (19b), gives an elastic limit transverse displacement of 83 mm at a quasi-static pressure of 74.4 kPa. It is then assumed that the difference in pressure between the peak pressure loading and the elastic limit pressure loading (i.e. 146 - 74.4 kPa) will cause plastic deformation. The transverse plastic displacement is calculated using equation (15b) and found to be 52 mm. Therefore, the maximum transverse displacement is given by the sum of the elastic and plastic components, namely 135 mm and the final permanent transverse displacement, assuming wholly elastic recovery after unloading, is 52 mm. This compares with 161 mm and 60 mm respectively in the experiment.

For \( 2L = 1 \) m, \( \sigma_0' = 221 \) MPa, \( E = 205 \) GPa, \( \nu = 0.3 \) and using equations (14) and (19b) gives an elastic limit transverse displacement of 21 mm at a quasi-static pressure of 37.6 kPa. By a similar procedure to the previous example, the maximum and maximum permanent transverse displacements are calculated to be 45 and 24 mm, respectively. This compares with 44 mm and 18 mm, respectively in the experiment.
Clearly, the above calculations again assume no in-plane edge displacements. Also, these calculations are for a quasi-static loading. However, for a simple procedure, the correlation is remarkably good.

5 Discussion

5.1 Accuracy of simple methods

The response of a single-degree-of-freedom (SDOF) elastic spring-mass system to a pulse load having a triangular shaped pressure pulse with a duration $t_d$, as shown in Figure 2b, has been studied by Biggs [11]. The fundamental elastic period of vibration of the mass is $T$. It can be shown that dynamic effects exercise a very important influence on the response of the mass when $t_d/T < 1$. On the other hand, when $t_d/T$ is very large, the response of the mass is essentially static. The deflections calculated for the same peak load in Figure 2b applied statically agree with the corresponding maximum dynamic deflections within approximately 17% when $t_d/T > 1.75$.

![Figure 2: Idealised load pulses.](image)

Inertia effects are always significant when the loading is applied almost instantaneously as in the case of shock loading, illustrated in Figure 2d. Biggs also studied the effect of a rectangular pressure pulse having a finite load duration $t_d$, as in Figure 2a. It can be shown that the maximum deflection of the mass is double the quasi-static calculation (assuming the peak load is applied statically) when $t_d/T > 0.5$, approximately.

The maximum displacements associated with a load pulse having a finite rise time $t_r$ and a constant load thereafter, as in Figure 2c, are within approximately 20% of the quasi-static value when $t_r/T > 0.9$, approximately.

This approach can therefore be used to determine the limiting ratio $t/T$ associated with quasi-static response for a particular problem. The load duration,
$t_d$, can be calculated using the methods described in reference [12], and the natural period, $T$, is available or can be easily calculated for a wide range of structural elements. In more complicated structures, consisting of several types of structural elements, the natural period of individual elements can differ considerably from the global response of the whole structure. It is worth considering the response of individual elements as well as the total structure in assessing whether a particular problem is quasi-static or not. It should also be noted that the response times associated with plastic deformation are longer than the response times associated with elastic deformation of a structure due to the reduced stiffness associated with plastic deformation. On the other hand, finite deflections or change in geometry significantly affect stiffness and can lead to shorter response times for certain geometries.

The SDOF methods were used to estimate the fundamental elastic period of vibration, $T$, of the 3.11 m and 1 m square plates. The ratio $t_d/T$ was found to range from 1.02 to 0.55 for the elastic and elastic-plastic response of the 1 m square plate, respectively. For the 3.11 m square plate, the ratio $t_d/T$ ranged from 0.43 to 0.24 for the elastic and elastic-plastic response, respectively. The actual response times of these plates are likely to be much shorter than the calculated values due to the influence of finite deflections and, therefore, the ratios of $t_d/T$ will be larger than the calculated values above. This would explain the reasonably good correlation between the experimental data for the 3.11 m and 1 m square plates and the theoretical predictions using a quasi-static analysis.

In the linear elastic range, the natural period of the structure can be used in conjunction with the load duration to assess the nature of the response to a given loading. Baker et al [13] provide a classification of the response of a structure to dynamic loadings. The term $\sqrt{k/M}t_d$ is used to identify the loading regime and classify the response into one of three loading regimes, namely impulsive, dynamic and quasi-static, where $k$ is the elastic stiffness, $M$ is the lumped mass of the structure and $t_d$ is the load duration.

The importance of including material elasticity has been studied by Symonds and Frye [14], who examined the dynamic behaviour of impulsively loaded, simple one-degree-of-freedom spring-mass models with elastic, perfectly plastic or rigid, perfectly plastic springs. They found that it is reasonable to neglect elastic effects when the total external energy is much larger than the total amount of energy that can be absorbed in a wholly elastic manner. This energy ratio has to be larger than 10 approximately before the rigid, perfectly plastic model overestimates the permanent displacement by less than 10%. It has also been observed that for a pressure pulse of duration $t_d$ as in Figure 2a, the difference between the two models is a function of $t_d/T$, where $T$ is the fundamental period of elastic vibration. As the ratio $t_d/T$ increases, the accuracy of the rigid, perfectly plastic analysis deteriorates when compared to the predictions of an elastic, perfectly plastic analysis. When $t_d/T = 1/2\pi$, the energy ratio must be larger than about 20 in order to achieve an error of less than 10%. It appears, therefore, that elastic effects may be disregarded when the energy ratio is larger than about 10, provided the duration of the pressure pulse is sufficiently short compared to the natural period of the structure ($t_d/T < \sim 0.06$). Material elasticity
is important for loads with durations that are not short compared to the natural period of elastic vibration \((t_d/T > \sim 0.06)\) and when the energy ratio is so small that the dynamic loads do not cause extensive plastic deformation.

### 5.2 Scaling effects

The dimensions \(L\) of the largest steel plates studied here are nearly 25 times larger than the smallest plate specimens, though only the tests on the 1 m and 3.11 m square plates were arranged to test the laws of geometrically similar scaling. Nevertheless, the theoretical methods give acceptable predictions for the maximum permanent transverse displacements of all specimens regardless of size. This agreement tends to suggest that there is no significant departure from the laws of geometrically similar scaling.

It is well known that the phenomenon of material strain rate sensitivity introduces a known size effect, as shown in §11.3.2 of Reference [3], or

\[
\frac{\sigma_0'}{\Sigma_0'} = \frac{1 + (\dot{\varepsilon}/\beta D)^{1/q}}{1 + (\dot{\varepsilon}/D)^{1/q}},
\]

where \(\sigma_0'\) and \(\Sigma_0'\) are the dynamic flow stresses for a model having a geometric scale factor, \(\beta \leq 1\), and a full-scale prototype, respectively. \(\dot{\varepsilon}\) is an average strain rate and \(D\) and \(q\) are the Cowper-Symonds coefficients introduced earlier. Now, if we take \(\beta = 1/3.11 = 0.3215\) for the 1 m and 3.11 m plates, then using \(D = 40\) sec\(^{-1}\) and \(q = 5\), equation (20) predicts that the dynamic flow stress for the smaller plate would be about 4% larger than the flow stress for the larger plate. The strict requirements for geometrically similar scaling, which demands equality of the dynamic flow stresses at all scales [3], would be violated and lead to a 4% reduction, approximately, in the value of \(W\) according to equation (3). This is a small difference mainly because of the small value of strain rate which is estimated as \(\dot{\varepsilon} \approx 0.01\) sec\(^{-1}\) in §4.1.

The strain rates range from about 8 to 180 sec\(^{-1}\) for the square plates which are loaded impulsively in Reference [4] so that the phenomenon of material strain rate sensitivity would be much more important for this case. Thus, the effect would be noticed in experiments on steel plates having different sizes, although the influence on the maximum permanent transverse displacement would be somewhat less because the dynamic flow stress appears as a square root in equation (7a) for large values of \(\lambda\).

Clearly, it is important to estimate the influence of material strain rate sensitivity when conducting dynamic tests on small-scale models, but this can be done fairly reliably. Thus, with this proviso, the laws of geometrically similar scaling would be satisfied provided no rupture occurs, as noted in Reference [15].
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References


