Incremental impact test and simulation of prestressed concrete beam

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Abstract

This paper presents both experimental and analytical approaches on the dynamic behavior of a prestressed concrete (PC) beam under incremental impact loading. First, the weight-dropping test was performed in order to investigate the impact behavior of a PC beam under the condition that the weight is constant and the dropping-height increases. Second, the incremental impact analysis was developed in order to simulate the incremental impact test results by using an elastic-plastic dynamic analysis method based on a beam element model. Finally, the computational results are compared with the experimental ones.

1 Introduction

A large number of prestressed concrete (PC) rock-shed structures have been constructed in the mountain area in Japan (e.g. Ishikawa [1]), because of both advantages of short construction period and high strength of a PC member. Recently, the design method is now required to change from the allowable stress design to the ultimate limit state design from the viewpoint of the rational impact resistant design of PC rock-shed structures. In order to find the ultimate limit state of a PC member under a single impact loading, the dynamic behavior of PC beam has been examined by dropping the weight only one time from the height exceeding the predicted limit height (e.g. Ishikawa [2]). However, the dynamic behavior of a PC beam under incremental impact loading has not been clarified yet mainly from the viewpoint of an analytical approach. This paper presents both experimental and analytical approaches for the dynamic behavior of a PC beam under the incremental impact loading. First, the incremental impact test is
beam under the incremental impact loading. First, the incremental impact test is performed for a PC beam by increasing the dropping-height under a constant weight. Second, an analytical method is developed in order to examine the dynamic behavior of a PC beam under the incremental impact loading. Herein, an incremental impact analysis based on a beam element takes into account of the strain-rate effects of materials and the change of unloading rigidity of a PC member. The computational results are compared with the experimental ones.

2 Experimental Approach

2.1 Weight-Dropping Incremental Impact Test

The incremental impact test was carried out to examine the dynamic behaviors of a PC beam by increasing the dropping-height gradually under a constant weight. In this test a weight (W=9.8kN) is dropped at the center of a simply supported PC beam as shown in Fig. 1. The expanded polypropylene (EPP) and steel plate with depth 4.5 mm were placed on the beam instead of sand-cushion in order to prevent the local failure of beam.

2.2 Measuring Items

The measuring items in the impact test are the weight acceleration, the support reaction and the displacement at the center of beam which are measured by the accelerometer (range 500G and response frequency 10 kHz), the load cell (capacity 100kN) and the laser-type displacement sensor (range 300 +/-100 mm and response frequency 915 Hz), respectively, as shown in Fig. 1.

2.3 Specimen

The specimen was provided as PC beams with length 2.5 m (span length 2.0 m) and section 150 X 250 mm as shown in Fig. 2. It is noted that the shearing-span ratio a/d=
Table 1: Section size and material properties

<table>
<thead>
<tr>
<th>Material</th>
<th>Item</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Beam</td>
<td>Width</td>
<td>15cm</td>
</tr>
<tr>
<td></td>
<td>Depth</td>
<td>25cm</td>
</tr>
<tr>
<td></td>
<td>Length</td>
<td>250cm</td>
</tr>
<tr>
<td></td>
<td>Span</td>
<td>200cm</td>
</tr>
<tr>
<td>Concrete</td>
<td>Compressive strength</td>
<td>49.4N/mm²</td>
</tr>
<tr>
<td></td>
<td>Diameter</td>
<td>D6</td>
</tr>
<tr>
<td>Rebar</td>
<td>Diameter</td>
<td>D6</td>
</tr>
<tr>
<td></td>
<td>Yield stress</td>
<td>370N/mm²</td>
</tr>
<tr>
<td></td>
<td>Tensile strength</td>
<td>530N/mm²</td>
</tr>
<tr>
<td></td>
<td>Fracture elongation</td>
<td>26.5%</td>
</tr>
<tr>
<td>Stirrup</td>
<td>Diameter</td>
<td>D10</td>
</tr>
<tr>
<td></td>
<td>Pitch</td>
<td>5cm</td>
</tr>
<tr>
<td></td>
<td>Depth</td>
<td>21cm</td>
</tr>
<tr>
<td></td>
<td>Width</td>
<td>11cm</td>
</tr>
</tbody>
</table>

Figure 3: Impact load-time relation

5.0 is herein adopted in order to make flexural failure occur. The section size and material properties are shown in Table 1.
2.4 Test Results and Considerations

2.4.1 Impact load−time relations

Figure 3 shows the impact load−time relations at each dropping-height. The maximum impact loads at H= 25, 40, 60, 80 cm are about 55, 150, 165, 170 kN, respectively, and as such, it is found that the maximum impact load increases as the dropping height increases. But it is not so changed at the maximum loads between H= 60 cm and 80 cm. This may be the reason why the rigidity of PC beam has been reduced by the accumulated damage. On the other hand, it should be noted that the elapsed time of applied load decreases as the increase of dropping-height, i.e., the impact velocity. This may be due to the equality of momentum and impulse \( mv = \int P(t) dt \).

![Figure 3: Impact load−time relations](image)

2.4.2 Displacement−time relations

Figure 4 illustrates the displacement−time relations at each dropping-height. It is noted that the maximum displacement at H= 25, 40, 60, 80 cm are about 0.15, 1.3, 3.0, 6.0 cm, respectively, and the times at the maximum displacement are 130, 80, 70, 60 ms, respectively.

It should be also noted that the residual displacements at H= 25, 40, 60, 80 cm become 0.02, 0.2, 1.0, 5.0 cm, respectively, and they increase as the increase of dropping-height.

![Figure 4: Displacement−time relation](image)
2.4.3 Impact Load ~ displacement relations

Figure 5 represents the impact load ~ displacement relations at H= 25, 40, 60, 80 cm obtained by eliminating the time axis of Figs. 3 and 4. It is found that the starting displacements at H= 25, 40, 60, 80 cm are coincided with the residual displacements 0, 0.02, 0.2, 1.0 cm at H= 0,
2.4.4 Incremental Impact Load – Displacement Relation

Figure 6 illustrates the incremental impact load – displacement relation obtained by superimposing Figs. 5 (a), (b), (c) and (d). It is found that the envelope curve obtained by the incremental impact load – displacement curves represents the dynamic load – displacement curve obtained by the high-speed loading analysis (Ishikawa [2]). It is also noted that the unloading rigidities at H= 40 and 60 cm become smaller than the initial rigidity at H= 25 cm. This may be due to the restoring force of prestressed tendons.

3 Incremental Impact Analysis

3.1 Analytical Model

The incremental impact analysis is developed to estimate the dynamic behavior under incremental impact loading by using a simple analytical model, i.e. beam elements as shown in Fig. 7. Herein, the following factors are included in the analysis. ① the breaking of prestressed tendon or the failure of compressive concrete, ② the elements of weight and shock absorbing material (EPP) are added to the beam elements in Fig. 7, ③ the strain-rate effect in the constitutive laws of materials as shown in Fig. 8, ④ the

Figure 7: Analytical model in the impact test 25, 40, 60 cm, respectively. It is also noted that the breaking of prestressed tendon has occurred at H= 80 cm which is less than the dropping-height H= 86 cm in the single impact load condition (Ishikawa [2]). This may be due to the accumulated damage by incremental impact loads.

Figure 8: Constitutive laws of materials
First, the strains of concrete and prestressed tendon in the PC beam can be found by using the assumption of remaining plane after deformation. Second, the stress-strain curves of concrete and prestressed tendon are adopted by considering the strain-rate effects. Third, the compressive resultant $C_i$ and the tensile resultant $T_i$ can be found by using the sectional discrete element method and, as such, the relation between the bending moment $M_i$ and curvature $\Phi_i$ can be obtained and is replaced into the bending moment $M_i$ ~ end rotation $\theta_i$ relation as shown in Fig. 9 by multiplying the plastic hinge length (herein, 2/3 of effective height of section is assumed) by the curvature. In $M_i$ ~ $\theta_i$ relation, the change of unloading rigidity is assumed as shown in Figs. 9 and 10.

3.2 Stiffness Matrix of Beam Element

The impact analysis has been developed in order to find the impact load ~ displacement relation of PC beam by using the Newmark $\beta$ method. First, the bending moment ~ end rotation relation in Fig. 9 is expressed as follows:

$$ M = kq $$

(1)

where, $M$: the end bending moment vector, $q$: the end rotation vector, $k$: the internal stiffness matrix of beam element.

In case of elastic condition at both ends, $k = k_e$, which indicates the elastic internal stiffness matrix of beam. In case of elastic-plastic condition at either end, $k = k_{ep}$ is expressed as follows:

$$ k_{ep} = k_e - k_e N_a \left[ N_a^T K_e N_a + H_a \right]^{-1} N_a^T k_e $$

(2)

where $k_{ep}$: the elastic-plastic stiffness matrix of beam element, $N_a$ and $H_a$ are coefficient matrices in the active condition of the following yield function $\psi$.

$$ \psi = N^T M - H \lambda - M_p \leq 0 $$

(3)
where, $\psi$: the yield function vector, $H$: the hardening coefficient matrix, $\lambda$: the plastic coefficient vector, $M_P$: the bending plastic capacity.

When the unloading is occurred at either end, the unloading rigidity is changed corresponding to the end rotation as shown in Fig. 10. Therefore, the external stiffness matrix of beam element can be obtained as follows:

$$K_i = B_i^T B_i$$

(4)

where, $B_i$: the compatibility matrix.

3.3 Incremental Impact Analysis

3.3.1 Equation of Motion

The equation of motion can be derived from the dynamic equilibrium considering the damping term and is devied into two categories, i.e., one is the known part and the other is the unknown part.

$$\begin{bmatrix} m_{uu} & m_{uk} \\ m_{ku} & m_{kk} \end{bmatrix} \begin{bmatrix} \ddot{u}_u \\ \ddot{u}_k \end{bmatrix} + \begin{bmatrix} D_{uu} & D_{uk} \\ D_{ku} & D_{kk} \end{bmatrix} \begin{bmatrix} \dot{u}_u \\ \dot{u}_k \end{bmatrix} + \begin{bmatrix} K_{uu} & K_{uk} \\ K_{ku} & K_{kk} \end{bmatrix} \begin{bmatrix} u_u \\ u_k \end{bmatrix} = \begin{bmatrix} P_k \\ P_u \end{bmatrix}$$

(5)

where, $\ddot{u}, \dot{u}, u$: the acceleration, velocity and displacement vectors, respectively, $m, D, K$: the mass, damping and stiffness matrices, respectively, $P$: the external force vector. The subscripts $k$ and $u$ mean the known and unknown quantities, respectively.

In order to solve Eq. (5), the unknown quantities $\ddot{u}_u, \ddot{u}_u, u_u$ at time $t + \Delta t$ can be obtained by using the first line of Eq. (5) and the Newmark $\beta$ method as follows:

$$u_{u,t+\Delta t} = \ddot{u}_{u,t} \Delta t^2 + \frac{\Delta t^4}{2} \dddot{u}_{u,t} + \beta N \Delta t^2 (\dddot{u}_{u,t+\Delta t} - \dddot{u}_{u,t})$$

(6a)

$$\dot{u}_{u,t+\Delta t} = \dot{u}_{u,t} + \frac{\Delta t}{2} (\dddot{u}_{u,t+\Delta t} + \dddot{u}_{u,t})$$

(6b)

$$\dddot{u}_{u,t+\Delta t} = \left[ m_{uu} + D_{uu} \frac{\Delta t^2}{2} + K_{uu} \beta N \Delta t^2 \right]^{-1}$$

(6c)

Therefore, the unknown external force vector $P_u$ can be found from the second line of Eq. (5) as follows:

$$P_u = m_{ku} \dddot{u}_k + m_{kk} \dddot{u}_k + D_{ku} \ddot{u}_u + D_{kk} \ddot{u}_k + K_{ku} u_u + K_{kk} u_k$$

(7)

3.3.2 Incremental Impact Analysis Procedure

The analysis is performed by the following procedure.

1. By giving the sectional and material properties, the mass matrix and the damping matrix are first computed.
The bending moment~curvature (end rotation) relation is found by using the sectional discrete element method and is assumed as the three linear relations as shown in Fig. 9.

As the external force, the momentum \( mv_0 \) due to initial velocity \( v_0 = \sqrt{2gh} \) is transformed into the impulse \( P_0t_0/2 \) as shown in Fig. 11. Thus, the converted external force \( P \) during short period is given at the loaded point instead of initial velocity \( v_0 \) as follows:

\[
P = \frac{P_0}{t_0} \quad (0 \leq t \leq t_0)
\]

where \( P_0 = 2mv_0/t_0 \), \( m \): the weight mass, \( t_0 \): the elapsed time, herein, \( t_0 = 10^{-5} \) sec.

The unknown quantities \( \ddot{u}, \ddot{u}, u, \) and \( P_e \) are computed by using Eqs. (6) and (7).

The dropping-height \( H \) is increased and the computing procedure of (3), (4) is repeated by using the values at the previous dropping-height \( \bar{H} \) as the initial ones, until the breaking of prestressed tendon or the failure of compressive concrete of PC beam section.

4 Computational Results by Incremental Impact Analysis

The following input data is used in the analysis:

\[
K = 6.43 \times 10^{10} N \cdot cm, \quad M_{11} = 4.51 \times 10^5 N \cdot cm, \quad M_{22} = 6.65 \times 10^6 N \cdot cm,
\]

\[
H_1 = 1.62 \times 10^8 N \cdot cm, \quad H_2 = 0.098 N \cdot cm, \quad \theta_e = 0.07
\]

4.1 Impact Load~Time Relations

The thick lines in Fig. 3 show the impact load~time relation computed by the incremental impact analysis. It is found from Fig. 3 that the computed results simulate well the measured ones, especially, the two peak load values at \( H=80cm \) in Fig. 3 (d) coincide with the measured ones. The reason why both values show good agreements may be caused that the dynamic behavior of shock absorbing material (EPP) as shown in Fig. 8 (d) is adopted precisely in the computation.

4.2 Displacement~Time Relation

The thick lines in Fig. 4 illustrate the displacement~time relation computed by the proposed analysis. It is also noted that both computed and measured curves show almost good agreements with each other by considering the change of unloading rigidity.
corresponding to the end rotation.
It is recognized that the displacement at H= 80 by computation becomes infinity due to
the breaking of prestressed tendon, although the elapsed time is little different.

4.3 Impact Load~Displacement Relation

The thick lines in Figs. 5 and 6 indicate the impact load~displacement relations and
incremental ones obtained by analysis, respectively. It should be also noted that both
curves show almost good agreements and, as such, it is found that the proposed
incremental impact analysis can simulate well the results obtained by the incremental
impact test.

5 Conclusions

The following conclusions are drawn from this study.
(1) The dynamic behavior of a PC beam was found by performing of incremental impact
test which increases the dropping height as the constant weight.
(2) The breaking of prestressed tendon was occurred as the ultimate limit state of a PC
beam under incremental impact test.
(3) The maximum impact load increases generally as the increase of dropping height, but
it is not so changed between H= 60 and 80 cm, because of accumulated damage of a PC
beam.
(4) The displacement increases as the increase of dropping height and, finally, it
becomes infinity due to the breaking of prestressed tendon.
(5) The envelope curve of incremental impact load~displacement relation coincides
with the dynamic load~displacement curve obtained by the high-speed loading analysis.
(6) The incremental impact analysis was developed by repeating the single impact
analysis using the values of previous dropping-height as the initial ones.
(7) It was confirmed that the proposed method can simulate well the incremental impact
load~displacement relations obtained by incremental impact test.

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