

# Failure analysis of laminated architectural glass panels subjected to blast loading

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## Abstract

A 3-D nonlinear finite element model is used to predict the probability of failure of laminated architectural glass subjected to blast loading. A uniformly distributed pressure, simulating blast loading generated by a shock wave from the discharge of a bomb, is applied to one of the exposed surfaces of the laminate. In the analysis, the glass ply is modeled as an elastic material while the polymer interlayer is treated as a viscoelastic material. Two-parameter Weibull distribution is adopted to describe the cumulative probability of failure of the unloaded glass ply. A parametric study is presented showing the effect geometric and material properties of the laminate and the blast load parameters. These analytical results can be used in evaluating the blast performance of new laminated architectural glazing systems and in the design of retrofit or upgrade schemes for the existing glazing systems.

## 1 Introduction

A vast majority of non-fatal injuries from the recent bomb blasts have been attributed to airborne sharp glass fragments. The hazard of flying or falling glass during a blast or severe windstorm can be minimized by the use of laminated glass for glazing systems. Laminated architectural glazing generally consists two glass plies adhered by a polyvinyl butyral (PVB) interlayer. When subjected to severe blast pressure, even if laminated glass plate break, fragments of broken glass plies stay adhered to the interlayer, thereby reducing the potential bodily injury and property damage from flying glass.

In this paper, a 3-D nonlinear dynamic finite element model is used to predict the cumulative damage probability of inner glass ply of a laminated architectural

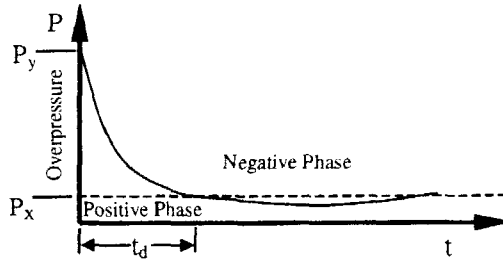


Figure1: A typical pressure-time curve for an explosive blast.

glazing subjected to a blast load resulting from a bomb explosion on ground. Based on the cumulative damage theory, a two-parameter Weibull distribution is adopted to determine the failure probability of inner glass ply of such a laminated architectural glazing.

## 2 Blast load characterization

A typical pressure-time curve for an explosive blast wave is shown in figure 1. Even though it consists of positive and negative pressure phases, most of the structural damage is due to positive phase, Kinney [1] and Baker [2].

An empirical quasi-exponential form has been used to describe the explosive blast wave of the positive phase, Kinney [1].

$$p(t) = p^0 (1 - t/t_d) e^{-\alpha t/t_d}, \quad (1)$$

where,  $p(t)$  is the instantaneous overpressure at time  $t$ ,  $p^0 = p_y - p_x$  the peak overpressure observed when  $t$  is zero,  $p_x$  is atmosphere pressure,  $p_y$  is the peak pressure when  $t$  is zero,  $\alpha$  is the decay factor and  $t_d$  is the (positive) over pressure duration. The other key blast load is the blast impulse defined as, Kinney [1] and Barker [3]

$$I = \int_0^{t_d} \eta p(t) dt, \quad (2)$$

where  $\eta$  is a yet to be determined scale factor which accounts for the effect of the reflected overpressure on the exterior building subjected to ground explosions. Substituting eqn (1) into eqn (2) yields,

$$I = \eta p^0 t_d \left[ \frac{1}{\alpha} - \frac{1}{\alpha^2} (1 - e^{-\alpha}) \right]. \quad (3)$$

For an explosion with a spherical charge of one ton (907.8 kg or 2000 lb) equivalent TNT in standard atmosphere (0.1 MPa or 14.7 psi) at 21° C (70° F), Kinney [1] has tabulated  $p^0$  (psi),  $t_d$  (ms) and  $\alpha$  for various scaled distances. By smooth curve-fitting the tabulated results of Kinney [1], the following equations for the three parameters are obtained

$$\frac{p^0}{p_x} = -1.75259 \times 10^{-3} + 22.112x^{-1} + 2121.79x^{-2} + 132456x^{-3}, \quad (4)$$

$$t_s = -40.9856 - 3.1605 \times 10^{-2} x + 24.4355 x^{1/5}, \quad (5)$$

$$\alpha = 2.73453 - 2.19511 \times 10^{-3} x - 305.832 x^{-1} + 16022.4 x^{-2} - 181202 x^{-3}, \quad (6)$$

where  $x$  is the scaled distance and  $t_s$  is the scaled positive pressure duration.

By means of the scaling law, eqns (4)-(6) can be applied to other explosion cases. For a ground explosion and not far from the explosion center, the scaled distance is expressed as, Kinney [1],

$$x = \left( \frac{w_0}{w} \right)^{\frac{1}{3}} d, \quad (7)$$

where  $w$  is the TNT equivalent of explosive energy release in the explosion to be described,  $w_0$  is that of a reference explosion (2000 lb), and  $d$  is a actual distance from explosion center. The actual positive pressure duration is obtained as

$$t_d = \left( \frac{w}{w_0} \right)^{\frac{1}{3}} t_s. \quad (8)$$

The decay parameter  $\alpha$  is not itself scaled, so that it can be obtained directly from eqn (6) at properly scaled distance from eqn (7). When blast wave encounters the building surface, it will be reflected, thereby amplifying the overpressure. Furthermore, if a blast source is placed on or near a reflecting surface, such as the ground, then the surface burst appears to have 1 to 2 times the source energy as the blast in free-air. Therefore, a scale factor needed to correct the difference between the calculation in free-air blast using formulae (1)-(8) and the experimental measure of ground blast.

Smith et al. [4] reported tests of blasting on ground to assess the capability of methods to reduce the hazards of flying glass shards after failure of the window system. Tests were conducted at different sites over a period from 1996 to 1998. There are 22 test records all of which are for 227 kg (500 lbs) TNT equivalent explosive. Peak-pressure, impulse and distance from explosion were recorded. By using eqns (3)-(6), the calculated values of peak overpressure, duration, decay factor and impulse corresponding to a given distance from the explosion center are calculated. The scale factors for the peak overpressure between the test data and the calculated data are obtained with  $\eta = 3.36$ .

Figure 2 shows the comparison of the overpressure versus distance between the test data and the calculated data with a scale factor of  $\eta = 3.36$  for an explosion of 227 kg (500 lbs) TNT equivalent. Using eqns (3)-(6), the predicted blast impulse versus distance is obtained and compared with test data shown in figure 3. Therefore, the blast wave generated by ground explosion imposes a dynamic load on objects given by

$$p_d(t) = \eta p(t), \quad (9)$$

where  $p_d$  is the applied shock pressure on the laminated glass. Once the TNT equivalent of an explosion and the distance from the explosion center are known, the overpressure, duration time, the decay factor and the impulse can be calculated by using eqns (3)-(6). In these equations, the SI unit system should be used (1 lb = 0.454 kg, 1 ft = 0.305 m and 1 psi = 6900 Pa).

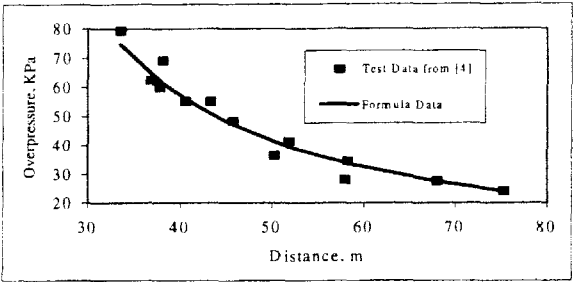


Figure 2: Comparison of overpressure versus distance between test and calculation for scale factor  $\eta = 3.36$ .

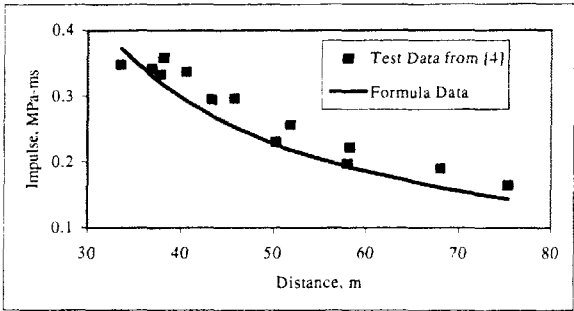


Figure 3: Comparison of blast impulse versus distance between test and calculation for scale factor  $\eta = 3.36$ .

### 3 Finite element modeling

A schematic diagram of a rectangular laminated architectural glass plate subjected to a uniform blast loading is shown in figure 4. An outer glass ply with thickness  $h_o$  and an inner glass ply with thickness  $h_i$  are adhered to a PVB interlayer with thickness  $h_p$ . The plate has total dimensions with  $2a \times 2b \times h$ . Uniform pressure simulating blast loading is applied on the exposed surface of the outer glass ply.

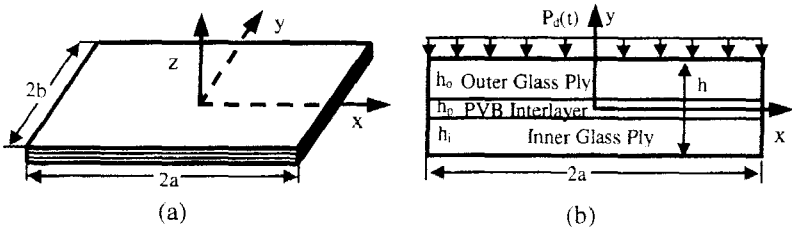


Figure 4: Schematic diagram of a rectangular laminated glass plate subjected to uniform blast loading.

The problem is solved numerically using the commercial dynamic nonlinear 3-D finite element code LS-DYNA3D developed by Livermore Software Technology Corporation [5]. The basic governing equations are given by Hallquist [6]. As most dynamic formulations, the stress components can be calculated as

$$\sigma_{ij} = S_{ij} - p\delta_{ij}, \quad (10)$$

$$p = -\frac{1}{3}\sigma_{kk}, \quad (11)$$

where  $\sigma_{ij}$  are the stress components,  $S_{ij}$  are the deviatoric stress components,  $p$  is the pressure and  $\delta_{ij}$  is the Kronecker delta.

The glass ply is modeled as a linear elastic material. The deviatoric and volumetric behaviors are given by

$$S_{ij} = \left[ \frac{E\nu\varepsilon_{kk}}{(1+\nu)(1-2\nu)} + p \right] \delta_{ij} + \frac{E\varepsilon_{ij}}{(1+\nu)}, \quad (12)$$

$$p = -\frac{E\varepsilon_{kk}}{3(1-2\nu)}, \quad (13)$$

where  $\varepsilon_{ij}$  are the strain components,  $E$  is Young's modulus and  $\nu$  is Poisson's ratio. The PVB interlayer is modeled as a linear viscoelastic material for which the deviatoric component is given by

$$S_{ij}(t) = 2 \int_0^t G(t-\tau) \dot{\varepsilon}_{ij} d\tau, \quad (14)$$

where  $t$  denotes time,  $\dot{\varepsilon}_{ij}$  is the deviatoric strain rate and  $G(t)$  is the stress relaxation modulus which is assumed to be of the form

$$G(t) = G_{\infty} + (G_0 - G_{\infty})e^{-\beta t}, \quad (15)$$

where  $G_{\infty}$  is the long time shear modulus,  $G_0$  is the short time shear modulus and  $\beta$  is the decay factor. The volumetric response is elastic, so the pressure  $p$  is computed by

$$p = -K\varepsilon_{kk}, \quad (16)$$

where  $K$  is the bulk modulus.

## 4 Cumulative probability of inner ply breakage

In general, the surfaces of glass plates are covered with minute flaws of varying geometry and orientations. These flaws are induced during the manufacturing process and through subsequent exposure to service conditions. Under tensile stress, these surface flaws tend to open and extend.

A simplified cumulative damage model, that relates to surface strength parameters  $n$ ,  $m$  and  $k$ , was developed by Beason and Morgan [7] from Brown's expression [8], to predict the probability of failure of monolithic glass plates under uniform lateral pressures. The model then was adopted by Norville and Minor [9]. These parameters represent strength characteristics of the glass surface and are independent of load duration, plate surface area, and plate geometry. A two-parameter Weibull distribution is used to characterize the cumulative probability of inner glass ply breakage for a laminated glass subjected to blast loading which is

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expressed as

$$p_f = 1 - \exp[-kB_0], \quad (17)$$

where  $k$  is surface flaw parameter and  $B_0$  is risk factor given by

$$B_0 = \int_0^a \int_0^b [c(x, y) \tilde{\sigma}_{\max}(x, y)]^m dx dy, \quad (18)$$

in which  $m$  is also surface flaw parameter and  $c$  is biaxial stress correction factor is defined as

$$c = \left[ \frac{2}{\pi} \int_0^{\alpha} (\cos^2 \theta + \lambda \sin^2 \theta)^m d\theta \right]^{\frac{1}{m}}, \quad (19)$$

where  $\lambda$  is the ratio of the minimum to maximum principal stresses,  $\theta$  is the angle of the surface flaw orientation to the maximum principal stress, and  $\alpha = \pi/2$  in this impact problem because both the maximum and minimum principle stresses on the bottom surface of the inner glass ply are tensile.

The equivalent sixty-second constant stress is

$$\tilde{\sigma}_{\max}(x, y) = \left[ \frac{1}{60} \int_0^{t_f} \sigma_p(x, y, t)^n dt \right]^{\frac{1}{n}}, \quad (20)$$

where  $\sigma_p(x, y, t)$  is the maximum principle stresses at location  $x, y$  and time  $t$ ,  $t_f$  is the load duration, and  $n$  is also surface flaw parameter. All  $n, m$  and  $k$  are material constant they are obtained from test. After getting  $n, m$  and  $k$ , with given geometry of laminated glass and blast loading history, the probability of inner glass ply damage can be predicted via eqns (17) to (20) and the stresses from the finite element model.

## 5 Results and discussion

Three material parameters  $n, m$  and  $k$  appear in the probability of failure analysis. Generally,  $n$  is fixed at a value of 16, [7-9], while  $m$  and  $k$  should be determined from blast loading tests combined with numerical blast loading simulations on laminated glass. No experimental data on  $m$  and  $k$  from blast loading tests is currently available. Norville and Minor [9] published a set of  $m$  and  $k$  for monolithic (new and aged) glass plate under constant uniform lateral pressure (or equivalent sixty-second failure load). The above data for  $m$  and  $k$  is used in the dynamic analysis of laminated glass problems in this paper. It is based on the fact that constant equivalent stress of 60-second duration has the same effect on the surface flaw as the pulse stress would during the duration to failure time  $t_f$  as in Eq. (20), [7-9]. At same time,  $n, m$  and  $k$  obtained from monolithic glass plate are used for laminated glass plate is based on the observation that outer and inner plies in laminated glass plate have the same material and flaw distribution as the corresponding monolithic glass sheet. The parameters taken from Norville and Minor [9] for new and aged glass plates are listed in table 1.

Table 1. Parameter  $n$ ,  $m$  and  $k$  of monolithic glass sheets from [9].

Series	Glass condition	$n$	$m$	$k \text{ (m}^{2m-2}\text{-N}^{-m}\text{)}$
AN	Aged	16	5.5	$7.73 \times 10^{-42}$
DAL	Aged	16	6.3	$6.08 \times 10^{-48}$
GPL-SQ	Aged	16	6.0	$7.3 \times 10^{-45}$
GPL-RECT	Aged	16	6.0	$4.54 \times 10^{-44}$
CON	New	16	9.9	$1.75 \times 10^{-76}$
SL	New	16	8.0	$2.96 \times 10^{-61}$
SQ	New	16	8.8	$1.42 \times 10^{-68}$

In the finite element calculating, the follow constants are used: for the glass plies,  $E = 72 \text{ GPa}$ ,  $\nu = 0.25$ , mass density  $\rho = 2,500 \text{ kg/m}^3$ ; for the PVB interlayer,  $G_o = 0.33 \text{ GPa}$ ,  $G_\infty = 0.69 \text{ MPa}$ ,  $K = 20.0 \text{ GPa}$ ,  $\beta = 12.6 \text{ s}^{-1}$ , and  $\rho = 1,100 \text{ kg/m}^3$ . The following baseline data were used in the analysis unless specified otherwise:  $h_o = h_i = 4.76 \text{ mm}$ ,  $h_p = 1.52 \text{ mm}$ ,  $a/h = 70$ ,  $b/h = 50$  where  $h$  is the overall thickness of laminate glass,  $w = 227 \text{ kg}$  and  $d = 68.0 \text{ m}$ . Simply supported boundary conditions are applied to the laminated glass plate. The problem is symmetric about  $x$ - and  $y$ -axis, so only one-quarter of the plate need to be analyzed. The laminated glass unit is discretized using 8-node solid elements.

Figure 5 shows the cumulative probability of inner ply failure versus impulse for laminated architectural glass with baseline data above. The plate composed of outer and inner ply corresponding to new and aged glass sheets and its parameters listed in table 1. It is seen that the new laminated glass plate can undertake higher blast impulse than aged one corresponding to same cumulative probability of inner ply failure, as expected.

Figure 6 shows the effect of PVB thickness on the cumulative probability of inner ply failure for the two typical glass plates, AN-aged and SL-new. The PVB thickness is normalized by baseline PVB thickness 1.52 mm above. It is seen that changing PVB thickness from 1 to 0.5 does not change the failure probability too much as other PVB thickness segments did. It implies the thickness of PVB chosen

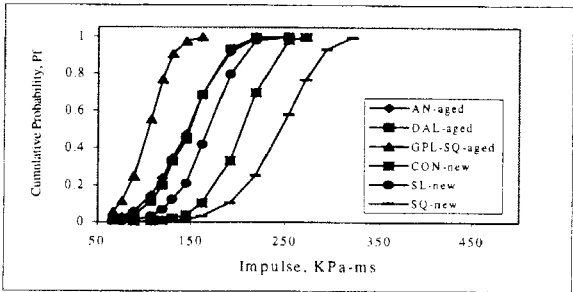


Figure 5: Cumulative probability of inner ply failure in laminated glass.

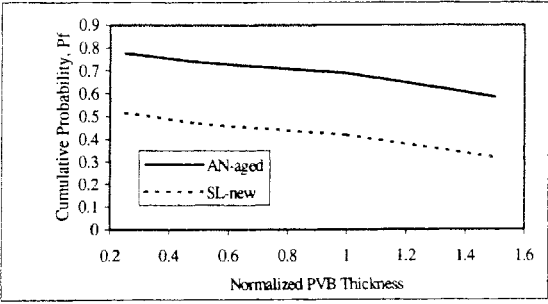


Fig. 6. Effect of PVB thickness on the cumulative probability of inner ply failure.

between 0.76 mm and 1.52 mm is better from economic consideration. Also, the new laminated glass plate has lesser failure probability than aged one with same PVB thickness.

Figure 7 shows the effects of thickness of outer and inner glass ply on the probability of inner ply failure for the two typical glass plates, AN-aged and SL-new. Increasing thickness of both outer and inner ply can decrease the probability. Especially, increasing inner ply thickness can decrease more probability than increasing outer ply thickness.

Accordingly, figure 8 shows the effect of thickness ratio of inner ply to outer ply on the probability of inner glass ply failure for the two typical glass plates. In this calculation, the total thickness of outer and inner ply is constant. It indicates that increasing the thickness ratio of inner ply to outer ply is good way to decrease the probability of inner glass ply failure without increasing total glass thickness.

Figure 9 shows the effect of area of plate exposed to uniform blast overpressure on the cumulative probability of inner glass ply failure for the two typical glass plates. The area of plate is normalized by the square total laminated glass plate

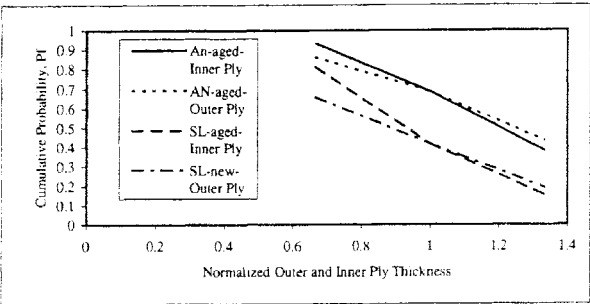


Figure 7: Effect of thickness of outer and inner glass ply on the cumulative probability of inner ply failure.



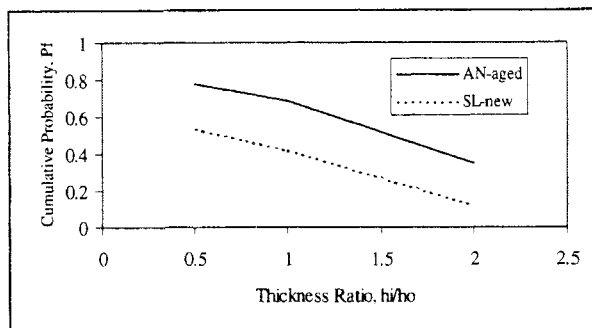


Figure 8: Effect of thickness ratio of inner ply to outer ply on the cumulative probability of inner ply failure.

thickness,  $h^2$ . It is obvious that increasing the area of plate exposed to blast overpressure will increase the cumulative probability of inner glass ply failure.

## 6 Conclusions

Surface flaw parameters,  $n$ ,  $m$  and  $k$ , can be adopted from monolithic glass plate under constant uniform lateral pressure for the blast load problem. That based on the concept of equivalent sixty-second failure load and the consideration of each outer and inner ply in laminated glass plate have same material and minute flaw distribution with the monolithic glass sheet. For a given failure probability, results indicate that the new laminated glass plate can carry higher blast impulse than aged. The thickness of PVB chosen between 0.76 mm and 1.52 mm is better from economic consideration. Decreasing outer ply thickness and increasing inner ply thickness keeping total thickness constant is an efficient design against blast loading.

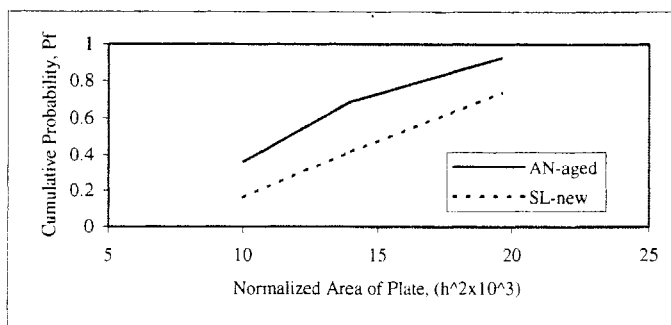


Figure 9: Effect of area of laminated plate exposed to blast overpressure on the cumulative probability of inner ply failure.

## Acknowledgments

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## References

- [1] Kinney, G.F., *Explosive Shocks in Air*, The Macmillan Company: New York, 1962.
- [2] Baker, W.E., *Explosions in Air*, University of Texas Press: Austin and London, 1973.
- [3] Barker, D.D. Blast Load Prediction Methodologies, *Protective Glazing Association: 1<sup>st</sup> Annual Educational Seminar*, Washington DC, April 21- 23, 1999.
- [4] Smith, J.L., Hall, B., & Renfro, N., Glazing Hazard Mitigation Technology, *Protective Glazing Association: 1<sup>st</sup> Annual Educational Seminar*, Washington DC, April 21-23, 1999.
- [5] Livermore Software Technology Corporation, *LS-DYNA USER'S MANUAL (Nonlinear Dynamic Analysis of Structures in Three Dimensions)*, 1997.
- [6] Hallquist, J.O, *Theoretical Manual for DYNA3D*, Lawrence Livermore National Laboratory, UCRL-52112, 1983.
- [7] Beason, W.L. & Morgan, J.R., Glass failure prediction model. *Journal of Structural Engineering*, ASCE, 110(2), pp.197-212, 1984.
- [8] Brown, W.G., A practicable formulation for the strength of glass and its special application to large plates. *Publication No. NRC 14372*, National Research Council of Canada, Ottawa, Ontario, Canada, 1974.
- [9] Norville, H.S. & Minor, J.E., Strength of weathered window glass. *American Ceramic Society, Bull.*, 64(11), pp. 1467-1470, 1985.