Structural response to a train of blast pulses

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Abstract

Air blast acting on structures in the real world rarely takes the idealised form of instantaneous pressure rise followed by smooth exponential decay. Reducing a composite load history to a simple equivalent pulse in calculations to determine damage risks overlooking additional damage that may result as a coincidence of repetitive blast pulses with the dynamic behaviour of the structure. This paper describes the response of a model structure to a train of explosively generated pressure pulses. Air-blast frequency content is analysed by means of Fourier transformation and related to structural response as a possible complement to existing methods. Correlation is sought between blast energy spectral density and structural response. Results indicate that whilst the model structure accumulated energy when successive blast pulses were in phase with the dominant mode of vibration, the interval between blast pulses need not be regular and there are limits on phase difference. Consequently blast energy spectral density may give misleading indication as to how a structure responds to a train of pulses. The conclusions are relevant to how structures might respond to internal explosions, blast in high-rise urban areas, multiple-charge demolitions and hazard assessment for distributed explosive storage sites.

1 Introduction

Blast waves often take the form of a train of pulses or an extended pulse containing several distinct shocks. Examples are blast waves containing pressure peaks from internal or external reflections, blast waves from multiple explosions such as linked demolition charges or military bombardment, and blast waves which might possibly occur when assessing risk hazards of
distributed explosive storage sites. It might be supposed that the most damaging circumstances for a structure subjected to repetitive blast loading occur if successive pulses add to movement imparted by earlier pulses (i.e. they are in phase), whilst the least damaging circumstances occur if successive pulses oppose movement imparted by earlier pulses (i.e. they are out of phase). In the former case, a relatively large amount of energy from the blast wave could be accumulated by the structure, implying vulnerability to repetitive loads at low overpressure with long distances from explosions required to provide safety. In the latter case, relatively little of the energy of the blast wave could be accumulated in the structure, implying vulnerability only to repetitive loads at high overpressure and safety at shorter distances. It could be expected that the most damaging circumstances might arise when the frequency of the pulses is close to the frequency of structural response since resonance is a cause of excessive and sometimes damaging vibrations in other situations. The context of this paper is an investigation into whether the frequency of blast loading might cause damage independently of peak over-pressure or total impulse. As an analogy, the work is investigating the degree to which structural members such as panels will behave like reeds in a vibrometer. The aim of this paper is to describe how correlation between a structure's natural frequency spectrum and a blast wave's energy density spectrum can be used to predict structural response.

Pressure-impulse diagrams are commonly used to predict damage to a structure from a combination of peak pressure and impulse, by determining the damaging deflections of the structure modelled as a single-degree-of-freedom linear-elastic mass in response to a simplified pulse as described by Biggs [1]. The validity of results from a PI diagram depends on assumptions, some of which may not be met when the blast load takes the form of many pulses. For example, in the quasi-static region of a PI diagram, a structure is normally considered to be at rest before any load is applied, and when loaded, the structure is able to respond quickly such that deflection is proportional to load. Given that assumption, it might be hypothesised that the maximum deflection in response to a train of pulses is proportional to the maximum peak in the train. However, the structure is not at rest when the second and subsequent pulses arrive and deflection is most unlikely to remain proportional to load independent of load history. Calculations using the quasi-static region of a PI diagram may therefore be invalid. In the impulsive region of a PI diagram where duration of loading is considered to be much shorter than the time in which the structure reaches maximum deflection, relevant assumptions for the use of PI diagrams hold true provided that the complete blast history is used to derive the loading impulse. Parts of the blast history where pressure falls below atmospheric must be included since a structure pushed as much as it is pulled will return to rest as negative parts of the pulse act against deflection caused by positive parts. The validity of calculations using the impulsive region of a PI diagram is therefore likely to be unchanged by the correct use of a simplified blast pulse. The dynamic region of a PI diagram where the duration of loading is within 0.4 to 40 times the structure's first mode natural period is where any
resonance effects might be expected to appear. The increased duration of a blast wave with many pulses will tend to extend the duration of loading and hence extend the dynamic region into the impulsive region. Overall, PI diagrams are likely to be useful only for impulsive conditions, and the longer the train of pulses, the fewer structures will qualify as responding impulsively.

Where a structure can be modelled with a suitable degree of accuracy, numerical methods can predetermine structural response to a given blast history, be it a single pulse or many pulses, provided that where many pulses exist, they are taken into account. Whitney et al [2] compare errors in selected numerical methods arising from simplifying a triple-pulse to a single-pulse and assess such errors as significant. However, even if pulses are taken into account, it is not always possible to model a structure accurately and there may be merit in studying the effect of pulses from a different perspective.

Blast waves are normally complex, in the sense that they are not periodic. To assess any significance of frequency in blast loading, a resonant model structure was subjected to an artificial ('worst-case') periodic blast wave and the results analysed by techniques normally used to determine the response of structures to vibration from cyclical loads such as rotating or reciprocating machinery. In this paper the validity of one technique is assessed to discover any significance of frequency in a train of blast pulses.

2 Experimental

Blast waves were generated in the form of pulse trains by firing nine electric detonators placed 500mm apart along the centre line of a blast-tube 12m in length and 298mm internal diameter. Blast waves travelled to the mouth of the blast-tube where they acted against a model structure. Separation between the shock pulses was adjusted by setting the time interval between detonator firing signals and for results described later the pulses arrived at approximately 5.6ms intervals equivalent to 178 pulses per second.

The model structure, oriented 'face-on' to blast at the mouth of the blast-tube, was a steel strap 25mm wide and 0.8mm thick. The strap was held in tension across a 'letterbox' slot 250mm in length cut in a thick steel plate and allowed to vibrate freely in the manner of a beam with clamped ends. Steady-state natural vibration of the strap was set to desired frequencies using a loudspeaker driven by a signal generator to find resonance and by adjusting masses attached to the front centre of the strap.

A photo-diode was placed in the blast-tube to detect times at which detonators burst. A small accelerometer was set on the centre reverse face of the strap. A strain gauge was set on the strap part-way between clamp and mid-point. Blast pressure gauges were set in the mounting plate surrounding the strap. The mounting plate formed one panel of a robust sealed chamber so that blast pressure could not get behind the plate and pressure differential remained relative to atmospheric.
In one firing the model structure was set to a steady-state natural frequency of 177Hz and its response curve recorded from accelerometer signal. It was then subjected to a regular train of blast pulses where the pulse frequency was set close to 177Hz with the intention of setting a resonant response. This is referred to as 'Shot R'. In another firing the strap was tuned to 158Hz by the addition of masses and was subjected to an arbitrary disordered train of blast pulses where the pulse interval and detonator size were irregular with the intention of avoiding resonant response. This is referred to as 'Shot I'. Sensor outputs were recorded by a digital storage oscilloscope for Shot R and Shot I. The steady-state natural frequency was checked and found to be unchanged after each shot.

3 Results

3.1 Sound-induced steady-state resonance

The resonant response of the strap set at 177Hz is shown at Figure 1. The response curve for the strap set at 158Hz is similar and omitted for brevity.

![Response curve for model structure described](image1)

Figure 1: Response curve for model structure described

3.2 Pressure results and strain results from Shot R and Shot I

Pressure history and strain history from Shot R are shown at Figure 2. The pressure data displayed in the figure were processed by discrete Fourier transformation [3] of the form shown in Equation 1, a process commonly used to derive spectra for electrical systems.
\[
X(k) = \sum_{n=0}^{N-1} x(nT)e^{-jk\omega nT}, \quad k = 0, 1, \ldots, N - 1.
\]

where:

- \(x(n)\) = \(n^{th}\) input value for total of \(N\) samples
- \(X(k)\) = \(k^{th}\) output value for total of \(N\) coefficients
- \(k\) = harmonic number being integral multiples of \(2\pi/NT\)
- \(T\) = sampling time interval
- \(\omega\) = angular frequency of first harmonic \(2\pi/NT\)

The output \(X(k)\) is a list of coefficients in the form of complex numbers from which can be found the magnitude, \(|X(k)|\), and phase angle, \(\phi\), for a set of sinusoids of frequencies at integral multiples of \(2\pi/NT\). If the complete set of sinusoids is superimposed paying regard to magnitude and phase, the original time-varying record \(x(n)\) can be closely recreated. For quantities to be accurate, the system being analysed must be linear so that the principle of superposition can be applied. In this case the units of \(x(n)\) were Pa, the units of the output magnitude \(X(k)\) are PaHz\(^{-1}\). If the magnitude \(|X(k)|\) is normalised by allowing the pressure to act over unit area, and the resulting force is normalised by allowing it to act over unit distance, the units become JHz\(^{-1}\) and the pressure spectral density can be regarded as energy spectral density. High values of \(|X(k)|\) denote high energy and when plotted on a graph against their respective frequencies, peaks represent frequency bands containing high energy with the most potential to cause structural response. The result of transformation from time domain to frequency domain for Shot R is shown in Figure 4.

Whilst the strain gauge data at Figure 2 was predominantly due to elongation of the strap, a small part of the signal would also have been due to bending. Depending on the (unknown) shape taken by the strap, the bending contribution to the strain gauge signal could have caused significant errors. This contribution has been neglected and to denote lower precision, strain is shown only as being relative or comparative rather than absolute. Strain data was transformed into the frequency domain and high values of \(|X(k)|\), i.e. peaks in the energy spectral density, denote high strain energy. The energy spectral density for Shot R is shown at Figure 6.

In similar manner, the pressure history and strain history from Shot I are shown at Figure 3, the blast pressure spectral density is shown at Figure 5 and the relative strain spectral density is shown at Figure 7.
Figure 2: Pressure history and strain history for Shot R
Figure 4: Blast pressure spectral density for blast-wave Shot R

Figure 5: Blast pressure spectral density for blast-wave Shot I
Figure 6: Relative strain energy spectral density for blast-wave Shot R

Figure 7: Relative strain energy spectral density for blast-wave Shot I
4 Discussion

The question under investigation is whether there is correlation between blast pressure spectral density and strap natural frequency. The simplest correlation would occur if the model described behaved as a narrow-band oscillator vibrating at resonance and taking energy from a broad-band load spectrum only in the neighbourhood of its resonant frequency.

It can be seen from Figures 4 and 5 that the model resonator was subjected to excitation from broad spectrums, though energy from both shots was concentrated in bands. The energy spectral density of Shot R at Figure 4 shows peaks at the blast pulse frequency of 177Hz and harmonics required to describe the pulse shape, along with unattributable or random energy. The peaks appear in broad agreement with the Fourier series that can be derived analytically for a saw-toothed waveform of similar shape to the blast wave. The energy spectral density for Shot I shows less periodicity as would be expected given the less ordered nature of the blast pulses, but since the pulse train had nonetheless some structure, i.e., it was far from being white noise, energy is not evenly distributed across the spectrum.

It can be seen from the strain traces in Figures 2 and 3 that the model responded differently to Shot I than it did to Shot R. The response to Shot R shown in Figure 2 was periodic but there is a marked change in waveform at each pulse loading. The strain spectral density at Figure 6 shows three peaks at approximately 210Hz, 420Hz and 750Hz consistent with the strap vibrating in its 1\textsuperscript{st}, 2\textsuperscript{nd} and 3\textsuperscript{rd} modes as described in Figure 8. The frequency of first-mode vibration appears to have been shifted from 177Hz to about 210Hz as would be expected in the presence of other active modes.

The response to Shot I shown in Figure 3 was periodic with no marked change in waveform at each pulse loading, but instead the amplitude increases. The strain spectral density at Figure 7 shows a single peak at about 750Hz consistent with vibration in 3\textsuperscript{rd} mode only. The model has clearly not behaved as a narrow-band oscillator drawing energy from the excitation spectrum only in the neighbourhood of its resonant frequency. There is little correlation between the widely distributed blast energy spectrum of Shot I at Figure 5 and the strain energy spectrum showing a dominant peak at 750Hz in Figure 7.

Close inspection of the traces at Figure 2 show that although the blast pulses from Shot R were more regularly spaced than in Shot I, there were small but
significant random variations in the interval between pulses caused by lack of precision in the apparatus used. It could be inferred from the strain record that the strap was set into cyclical motion by the first pulse with a period measured to be about 1.3 ms, corresponding to the third mode, and that successive pulses arrived at random points thereafter in the strap’s response cycle, each altering its pattern of vibration. The changing waveforms can be explained by Cases 1 and 4 in Figure 9 overleaf.

Close inspection of the traces at Figure 3 shows that although the blast pulses from Shot 1 were less regularly spaced than for Shot R and could not be controlled any more closely, in this particular instance, they just happened to act against the strap at the same point each time in its third mode cycle. On the section of trace shown, pulses can be seen to have impinged at 0 rad, 8\pi rad, 16\pi rad, and 22\pi rad, with each increasing the accumulating strain. In effect, the structure has acted rather like a child’s swing in that intermittent short pushes just as the swing starts to move forward have caused a larger amount of energy to be accumulated than a sinusoidal force applied over many cycles but often out of phase with the swing’s natural frequency, thereby tending to remove as well as add energy.

This example shows difficulties in regarding the strap as a narrow-band oscillator and in seeking to calculate its dynamic response through correlation of steady-state response spectra with blast wave energy density spectra. It would be difficult to predetermine, for the purposes of calculation, which natural modes will exist and whether their frequencies have to be offset by the presence of other modes (that is to say how stiffness coefficients should be applied) if these factors are strongly dependent on stochastic time-varying blast loads. Moreover, the strap does not behave as a narrow-band oscillator drawing energy from the neighbourhood of its resonant frequency. Its behaviour could be described as super-harmonic i.e. it is being driven by a sub-multiple frequency, except the sub-multiple changes erratically. Sub-multiples of 3 and 4 appear in Figure 3, and other low integers should achieve the same effect. Simple Fourier analysis did not reveal the magnitude of this partly randomised energy.

The discussion so far has been limited to idealised circumstances. Structural response to blast in the real world contains many more uncertainties such as angle of blast wave incidence, coalescence of blast pulses, damping factors and so on. So even if a correlation technique of the type described could be made to work, random factors would make results inaccurate. However, many uncertainties also apply to other methods to calculate structural response to blast. Evidence that a train of pulses can cause additional damage may help towards ‘worst case’ assessments or more realistic interpretation of PI diagrams.
CASE 1
Blast pressure opposes movement of 1st mode loop M, damping vibration at ~210Hz
Blast pressure assists movement of 3rd mode loops Q and Q', amplifying vibration at ~750Hz

CASE 2
Blast pressure opposes movement of 1st mode loop M, damping vibration at ~210Hz
Blast pressure opposes movement of 3rd mode loops Q and Q', damping vibration at ~750Hz

CASE 3
Blast pressure assists movement of 1st mode loop M, amplifying vibration at ~210Hz
Blast pressure assists movement of 3rd mode loops Q and Q', amplifying vibration at ~750Hz

CASE 4
Blast pressure assists movement of 1st mode loop M, amplifying vibration at ~210Hz
Blast pressure opposes movement of 3rd mode loops Q and Q', damping vibration at ~750Hz

Figure 9: Modulation of vibration by successive blast pulses at instant of loading
5 Conclusions

It is concluded that:

- The amount of energy transferred by successive blast pulses to each mode of the model structure was dependent on phase difference between loading pulse and mode of vibration.

- Which particular modes of vibration were set up in the model was dependent on how the blast load was applied.

- The resonant structure described was increasingly strained when successive blast pulses were in phase with the natural frequency of its dominant mode of vibration, but that successive pulses need not occur at regularly spaced intervals for increasing strain to occur.

- The response of the resonant model structure described cannot be determined simply from correlation between the structure’s response curve and the energy spectral density of the blast wave.

- The resonant model structure described did not behave as a simple narrow-band oscillator drawing energy from the blast wave in the neighbourhood of its resonant frequency.

References

