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The reconstruction of the sandstone cupola of the Frauenkirche in Dresden

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Abstract

The entire original cupola of the Frauenkirche (Church of Our Lady) was built in Saxon sandstone with iron anchor rings. In the early stages of planning the reconstruction of this unique dome, the engineers analysed the original structure. It was concluded that it would be possible to reconstruct in sandstone masonry because it would produce the obvious strength. To master the problem of cracking, prestressed anchor rings of steel bandages are to be provided under the faced stone layer of the outer wall of the dome.

1 Introduction

For about 200 years the cupola crowned the silhouette of Dresden city. On approaching the city, one's first impression was of the dome of the Frauenkirche dominating the skyline. It was the symbol of the city centre. It is important that not only the physical appearance be reproduced but the cupola also, which was a brilliant achievement of its civic master builder George Bähr. In his time it was not usual to calculate such a structure and its construction was based on the experience at the time. The introduction of the science of structural engineering in the construction of buildings began at the end of the 18th century and is linked with the names of Coloumb and later Navier[1]. From the time after the original erection of the Frauenkirche, we are familiar with the dispute on the stability of dome of St. Peter's Basilica in Rome[1].

Figure 1: The skyline of the city with the dome of the Frauenkirche (view from the tower of the city hall prior to 1945, photo: SLUB, Deutsche Fotothek Dresden)

It is therefore very important for the engineers responsible to reconstruct not only the outer manifestation of the dome but also the inner structure with its different elements in its original materials of mortar, stone and iron.
The most important points in the analysis and research done by the engineers with respect to the cupola have been

- the load-bearing behaviour of the dome
- the stress level in the masonry
- the actual effect of the original anchors
- the weather protection with the outer sandstone layer
- the necessary engineering additions of our time.

The following chapters describe the findings of the analysis and planning for the re-erection in sandstone of the masonry dome with due consideration to its original form and structure.

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2 Original structure

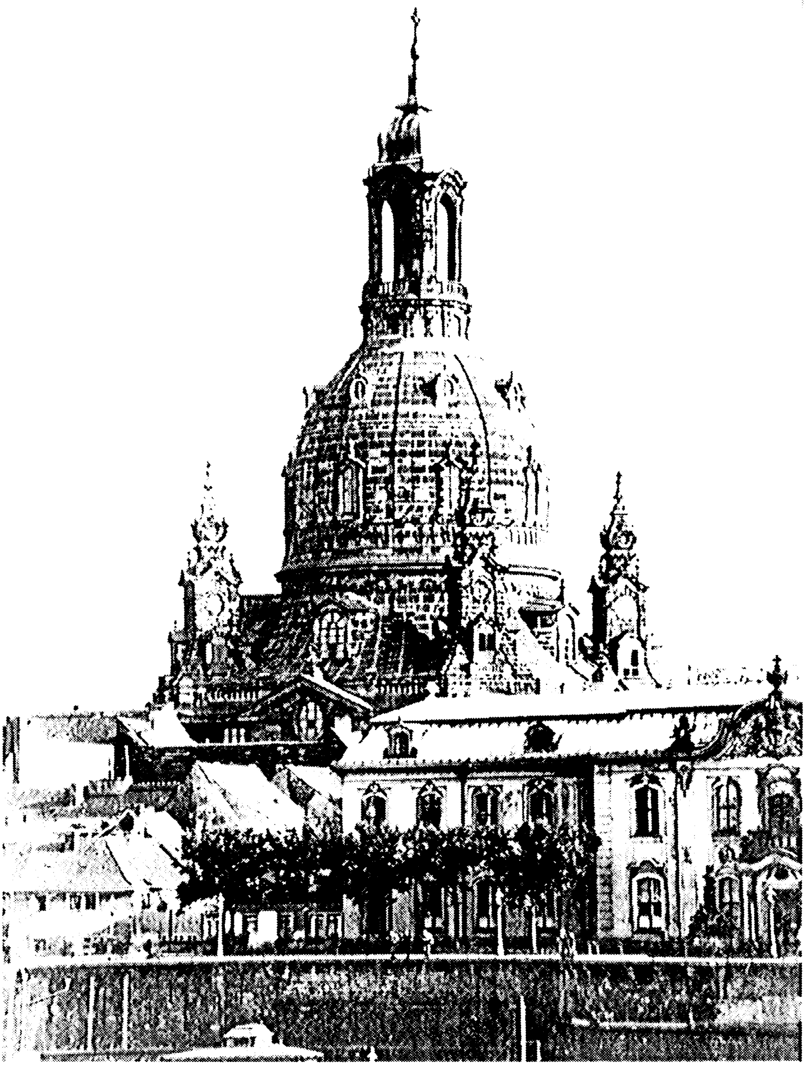

Figure 2: View on the dome of the Frauenkirche from the river Elbe prior to 1945 (photo: SLUB, Deutsche Fotothek Dresden)

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The cupola of the Frauenkirche was built over the years 1733 to 1736. In the first design the builder master intended using wood for the structure of the dome. However, some years after the beginning of the construction George Bähr thought about a solid dome roof with the aim of increasing its solidity and durability. Furthermore, the copper roof cladding needed for the wooden cupola would have been very expensive. After a long and intensive dispute on the load-bearing behaviour of the supporting structure he obtained permission for the construction of the cupola in sandstone in August of the year 1733.

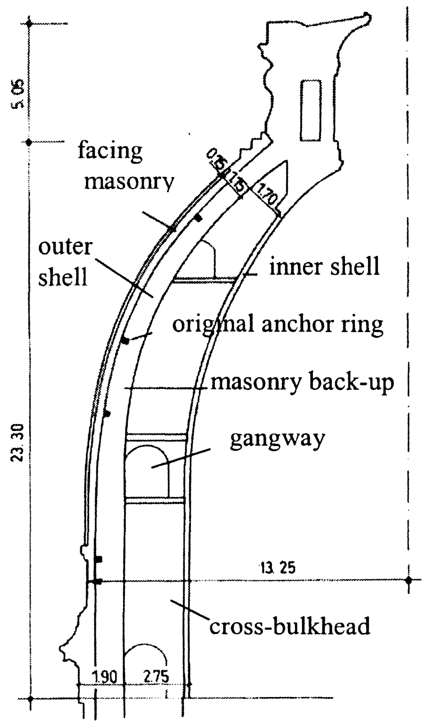

Figure 3: Longitudinal section through the original cupola

The dome of the Frauenkirche was a double-shell construction, which consisted of an outer shell of 1.3 to 1.5 m thickness and an inner shell of 0.25 m thickness. Both shells were interconnected by transverse masonry walls (see fig. 3). The outer shell had no additional roofing skin. Weather protection was provided by a sandstone top layer of approximately 0.25 m thickness, which was connected to the masonry backup. Wrought-iron ring anchors were located between the top layer and the lining masonry (see fig. 4). They consisted of straight rod elements with a cross section of approximately 4.5 x 5.0 cm² to 8.0 x 6.0 cm² and a length of approximately 3.7 to 4.6 m. The single bars have been connected by wedge locks (fig. 5). One wedge was hammered from above into two connections, which lay one on top of the other. The anchor rings have been fully enclosed in the sandstone masonry. There were four rings in the outer wall of the cupola and one in the pressure masonry ring on the top.

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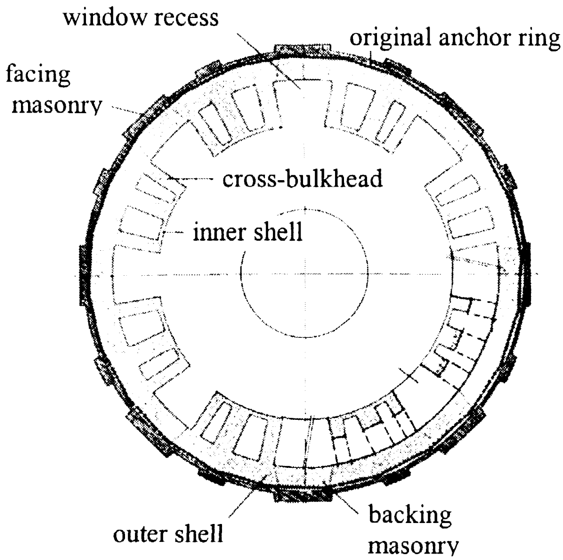

Figure 4: The cross section of the cupola at the height level of the tambour with the location of the iron, polygonal anchor ring polygonal anchor ring

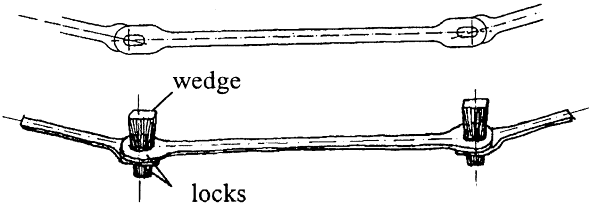

Fig. 5 Wedge-lock-connection of the single bars

The material found during the archaeological clearance of the rubble has been subjected to an analysis of its components and strength. The iron was very clean with regard to its chemical composition. The iron was quite heterogeneous and had a high content of phosphorus, causing it to be very brittle. The material test established that the strength lay between 300 and 400 N/mm². The yield strength amounted to 200 N/mm². The iron corresponded to building steel St 33[2].

The cupola had relatively large window openings permitting the light to penetrate. It was topped by a lantern, approximately 24 m in height, bearing a cross.

In the 20s and 30s of this century, it became necessary to carry out extensive

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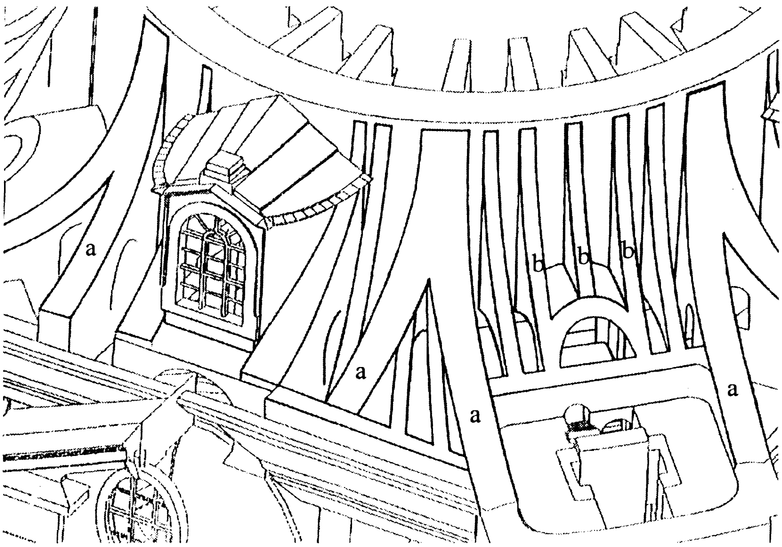
restoration work on the church. In the repairs under the guidance of Georg Rüth from 1936 to 1942 the cupola was strengthened with reinforced concrete anchor rings on the inside of the outer dome wall. Rüth decided on this measure because of slipping in the wedge-lock-connections and thermal stretching in the iron anchor rings under the finishing layer (see [3,4]).

3 The reconstruction of the stone cupola in its original structure and geometry

The intention is to reconstruct the sandstone cupola according to archaeological principles and with due consideration to modern engineering expertise. Today we are able to rebuild this cupola in its original structure without constructional errors. Neither the load-bearing performance of the cupola nor the load distribution necessitates a change in the basic geometry and structure of the cupola, as G. Zumpe suggests[5] with his bell-shaped dome.



a... 'Spieramen' (shear wall)
b... 'Schwibbogen' (arch buttress)

Figure 6: The so called 'Schwibbogen' and 'Spieramen' (source: J. Lauterbach, IPRO Dresden)

He worked out the idea of the pyramidal flow of loads on the basis of present developments in the sciences of Mechanics and Mathematics but without considering the knowledge base of the master builders at the beginning of the 18th century[1]. It is common archaeological practice to reconstruct an artefact in its original form if known. There are limits to the degree to which that aim can be realised in the reconstruction of the Frauenkirche because the reconstructed artefact is to be used as a church. We therefore had to respect the necessary safety level of a structure in our time. However, these requirements do not necessitate a change in the basic structure. The building erected in its original form shows us and the next generations the level of expertise master builders and scientists had in the first part of the 18th century.

In the past, most of the cracks and damage occurred in the zone of the negative curvature of the roof surface out of the main cupola, because of settlement and pushing-up of the supporting structure. This region was constructed like a roof nestling up to the actual dome structure. The so-called 'Schwibbögen' bore the roof plates made of Saxon sandstone which were approximately 25 cm thick. The 'Schwibbögen' was constructed like a rafter in a wooden structure. It was not the intention of George Bähr to distribute the high loads of the cupola onto the outer walls via these structural elements. In his explanation of his structural concept he says that the '...8 pillars are each connected by means of two strong 'Spieramen' to the corner towers and the main outer walls...[1, p. 44].

Unfortunately the term 'Spieramen' was coined at that time and does not reappear in the vocabulary used by the master builders and structural engineers. This explains how it could happen that even Gaetano Chiaveri interpreted the word differently. In his expert opinion dating from the year 1738, he referred to the 'Schwibbögen' as the 'Spiramini' [6,p. 102].

However, George Bähr explained in his hand-written presentation at the session of the town council on the 4th of August 1733 that the principle of the load distribution was similar to that of a pyramid but he could not quantify the kind of load distribution exactly. He was after all a master carpenter and not a scientist.

We know from the crack pattern in the longitudinal section[4] that there was a force flow to the outside but not to the extent actually necessary. Bähr also understood the phenomenon of tensile hoop forces which he wished to compensate with the iron anchor rings. But when he combined these forces with the iron elements there was no flow in the supporting structure and so they could not influence the load distribution in the masonry below it. Bähr also located the main corpus of the cupola slightly to the inside of the pillars. This was a usual method of construction at the time, we need only think of the so-termed 'pendentive cupolas' [7]. But this fact produces a high measure of bending moment in the pillar which leads to additional flexural stresses on the inner part of the pillars in the region of the capitals.

The cracks could have been avoided or limited if George Bähr had inserted a further effective anchor ring at the height level of the main cornice. We found reference to such an anchor ring in the records of the orders for the anchor

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materials during the building time of the church (see[2]), but we could not find it during the archaeological clearance[8] of the rubble in 1993/1994.

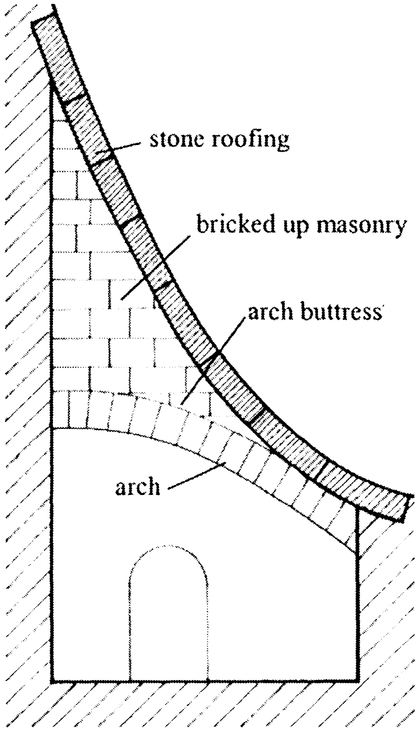

Figure 7: Longitudinal section through the negative curved stone roof to cover the area between the outer contour of the building and the drum of the dome in the original structure

George Bähr built a structure which bore the heavy stone cupola for about 200 years. That was his achievement. And so we are obligated to reconstruct the original cupola with his bearing drum, the belt arch ring and the y-shaped 'Spieramen' (shear walls).

Three main cautious engineering additions of our time guarantee that cracks in the path measure will not occur in the future:

- the differentiated quality of the masonry according to the stress flow (especially in the regions subject to high loads with thin joints and selected stone quality [9,10]) and
- the additional anchor steel ring with load flow correction to limit cracking and attain the required safety level.

In addition to that, the ground is pre-loaded so that settlement will be limited and less than at the time of construction in the 18th century[11].

The issues and problems relating to building physics in the sandstone top layer make high demands on planning and the selection of material. Here, questions of building material with regard to sandstone and the jointing material to be used as well as the design in view of durability must be investigated. This work is currently in progress and comprises theoretical and experimental studies.

4 Analyses of the original cupola and the necessity of moderate prestressing

For the FEM analysis a structural-mechanical model was used at first from the upper edge of the inner dome up to the lower edge of the lantern (see fig. 9 and [12]). This part has a height of 23.8 m and an outer diameter of 25.6 m at the bottom. 24 ribs connect the outer and the inner shell. On the base they have a spacing of 2.5 m which is reduced up to the top of the cupola. Two windows are located, one above the other, at a position corresponding to one eight of the circumference. A spiral passage is allowed to rise to the top of the cupola. Each of the ribs is interrupted three times in height by a gateway. Because of the geometrical symmetry and the high proportion of dead load only a quarter or an eighth of the whole dome needs to be analysed.
Sandstone masonry can be modelled as a linear elastic material as long as no significant tensile stresses occur in the wall. This assumption may be permitted because the aim is to reconstruct the dome in such a way that no significant tensile stresses occur.
The dome was supported in a fixed position on the defined parting plane. The symmetry of the cutting planes is prescribed by the symmetrical boundary conditions, taking into account the originally considered rotational symmetrical load of the dead weight. In a second step this model was extended up to the lower level of the inner cupola.
At first, different finite elements and different meshes were tested. In this way it was possible to find a model close to reality.
The FEM analyses were carried out in the linear range, since the intention is to prestress the cupola moderately for the reconstruction[13] and structural deformations are limited. Thus, no tensile stresses will occur in meridian and tangential directions. Initially, the cupola was considered separately under the aspect of compatibility with the supporting construction. The cupola was first calculated without prestressing in order to determine the necessity of this prestressing. The case of a prestressed cupola was subsequently implemented by introducing preelongated rods at the element edges of the outer surface. In this way it was possible to determine the effective necessary prestressing force for every anchor ring.
The comparison of the tensile hoop stresses with and without prestressing shows the necessity of prestressing when we try to produce a dome structure without cracks in the main masonry with due consideration to long life and durability. The influence of the openings can be seen in the stress flow along the middle line of the outer wall of the dome (see fig. 9).

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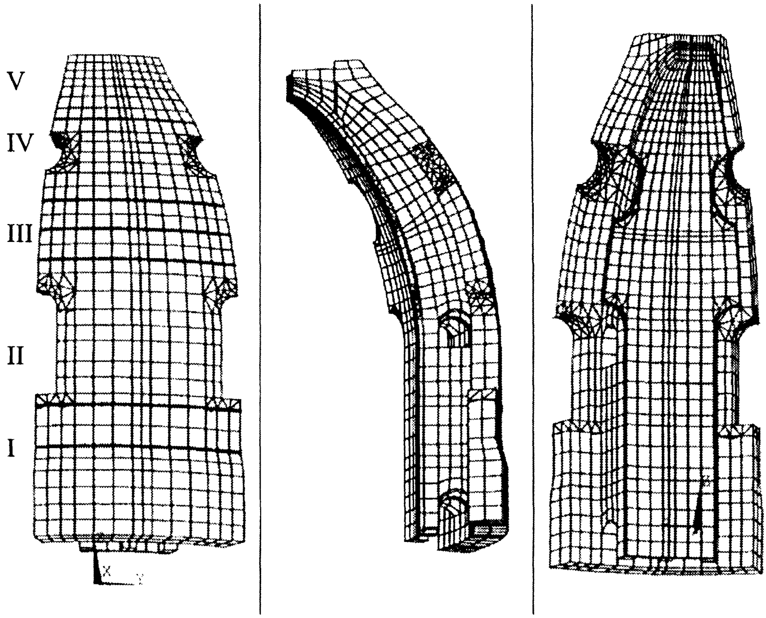

Figure 8: The FEM-model of an eighth of the dome (source: H. Bergander)

After estimating the load effects of wind, snow and temperature, a deliberate additional pressure in the circumferential direction of

$$\sigma_{\phi req.} = 0.2\ \text{N/mm}^2$$

was assumed. The existing average annular tensile stresses and the given additional pressure were used to determine the necessary extent of stress and the required effective prestressing forces (see table 1).

Region	$\Delta\sigma_\phi$	A_i	$F_{p\ eff.} = \Delta\sigma_\phi \cdot A_i$
	MN/m²	M²	MN
I	0.247	10.12	2.50
III	0.332	9.02	3.00
V	0.200	4.71	0.94

Table 1: Required effective prestressing forces $F_{p\ eff}$

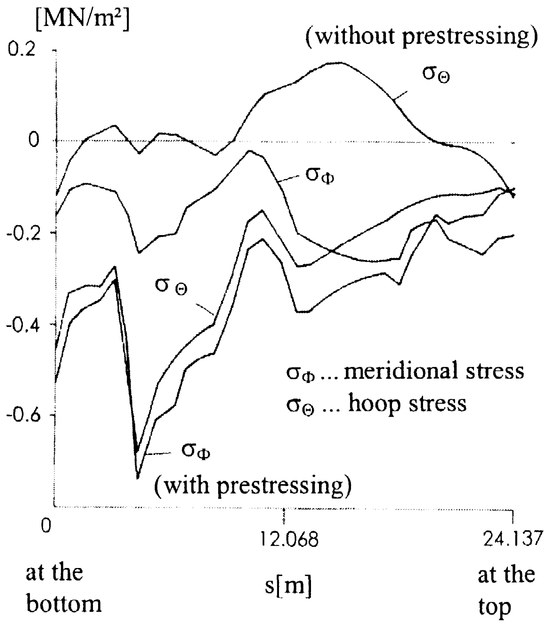

Figure 9: Stress flow along the middle line of the external wall of the dome with and without prestressing

The FEM analyses show that it is possible to reconstruct the dome in normal sandstone masonry in spite of the stress that arises (see[14]). The loads can be carried by the backing masonry alone without taking account of the sandstone surface layer so that the load-bearing capacity of the dome still exists even if the surface layer is replaced in the future.

5 Structural versions of moderate prestressing

In the tests carried out the prestressing types were divided into two main groups:

- prestressing elements between the covering sandstone layer and the load-bearing masonry of the outer wall,
- external prestressing elements on the inner side of the outer wall.

The first version comes very close to the historical example but the disadvantage here is that it is difficult to check and maintain the anchor rings. On the other hand, the external prestressing on the inner side of the dome is accessible at all times, can be checked and if necessary replaced. This version requires additional measures however to direct the forces out of the ring anchorage and into the sandstone masonry.
However, it is not important from the engineering point of view if materials are used with which a lot of experience has been gained. In the case of iron or steel bands we have the experience of the durability in the original church. It

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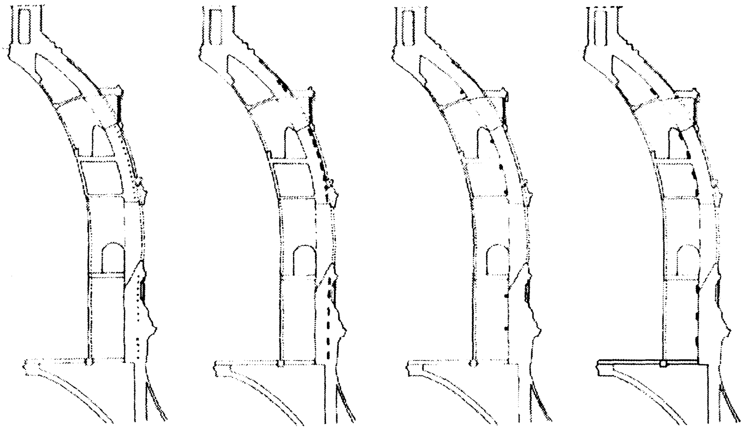
is more important in the case of strand tendons because this kind of anchorage was first used in our century (20th).



Figigure 10: Structural variations in anchoring and prestressing the dome

5.1 Internal prestressing

In the case of internal prestressing, the strand prestressing and the universal band steel prestressing were examined in different varieties of material. The description ‘internal’ marked the location according to the outer wall of the masonry. Internal means that the anchors lie in the masonry wall, external means that the tendons lie outside the main masonry shell (on the inner side of the cupola).

In the case of the strand prestressing the tendons can be distributed continuously over the prestressing areas (intervals of approx. 0.5 m) so that they give a balanced stress flow. By contrast, the universal band steel rings are at greater distances (approx. 1.10 m) and therefore give a more varied stress pattern.

For the individual types of prestressing loss the corresponding prestressing proportions were calculated, which together with the required effective prestressing give the total amount of the prestressing force. The prestressing forces are then applied to the element edges of the volume model and the stresses are calculated again. The required checks are then carried out with these calculation results (working state, ultimate state). In the results of these analyses the prestressing forces per tendon and actual cross section (strand tendons St 1570/1770, band steel tendons St 52 and StE 460/StE 690).were determined.

The anchorage and prestressing points were structurally perfected and checked for feasibility. The strand tendons can be prestressed with standardised elements. The usual tensioning jacks and the ordinary stress saddle can also be used by the universal steel tendons. The length of the universal steel tendons depends on the possible manufacturing length of a maximum 10 m. The use of butt strap joints with high-strength screws for coupling the parts was considered. The additional prestressing losses which would occur as a result were taken into account. For the stressing process itself prestressing was examined with the aid of cables and single bar tendons.

5.2 External prestressing

This type of prestressing was treated in the same way as in the methods described in the preceding paragraph.
Special rear hanging trusses are used to direct the forces into the dome masonry from the exposed anchor ring at the inside of the outer wall (see also[13]). This kind of prestressing is similar to the variety of anchorage used by Rüth (see above). In addition to the plate-anchorage of the anchor bolts in the sandstone masonry, other special monolithic and prefabricated anchor blocks were examined also. The advantage of this variety of anchor hoops is that they are always accessible and verifiable. However, producing and inserting them is more costly than internal prestressing.

6 The type of prestressing and rebuilding of the stone cupola chosen

The investigation of stress flow has shown that the compressive stresses in the meridian direction of the cupola are sufficiently extensive to allow the dome to be constructed entirely in sandstone. With respect to the tensile ring stresses, the necessary overpressure in the serviceability state can be reached with a moderate prestressing of the anchor rings. The cupola masonry can be carried out on the basis of the 'Mauerwerksrichtline' (guidelines for planning and executing natural stone masonry during the reconstruction of the Frauenkirche[14]).
The covering stone layer is connected to the masonry back up through bonding courses.
The effects of temperature and moisture on the outer surface have no significant influence on the load-bearing behaviour of the main dome structure. These effects are the focus of further investigations which are in progress. They are very important for the long life of the facing stone layer and the joint mortar.

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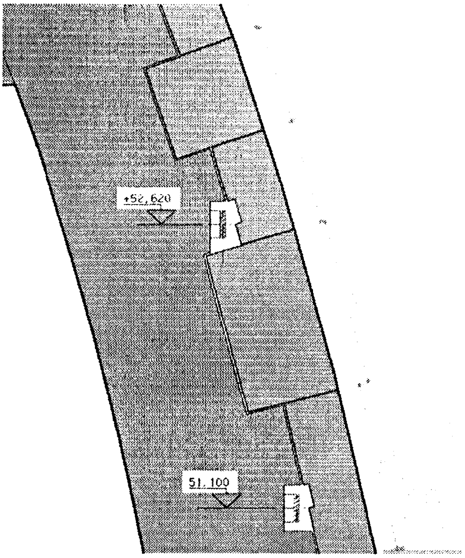

Figure 11: Longitudinal section of the planned stone cupola with the location of the new anchors (source: J. Lauterbach, Ipro Dresden)

The type of prestressing was chosen with consideration to the advantages and disadvantages of the versions examined. The peculiarities of this project played an important role in the decision taken.
The main selection criteria were:

- the long life of the re-constructed church
- the similarity of the technical solution to the original one
- the experience with this kind of solution in the needed space of time
- the robustness of the version
- the necessity of replacing the used tendons

In consideration of the specified points, the engineers responsible, the foundation trust Stiftung Frauenkirche, the architects and the curators of monuments decided to use the internal anchor version with steel bandages.
The design of the prestressed ring anchors has been decided upon in the meantime. Six flat steel ring anchors made of high-quality fine-grained structural steel are to be used (StE 690). They have dimensions of 3 x 10 to 3 x 15 cm, depending on the required prestresses. In order to keep the friction between straining ring and masonry to a minimum, the individual sections of the straining ring are supported on Teflon bands. The thin Teflon layer for reducing the friction between the prestressing tendons and the masonry are connected to the prestressing bandages. The friction losses to be considered here are minimal.
In an earlier version, the intention had been to use only shims to support the anchor bandages but this would have entailed too much effort for the execution on the building site. So the engineers responsible improved on this solution by means of the continuous support of the single anchor elements while inserting and prestressing. The permitted tolerance of the anchor bandages in this form and location in the masonry was also checked. In this way it was possible to reduce the costs.

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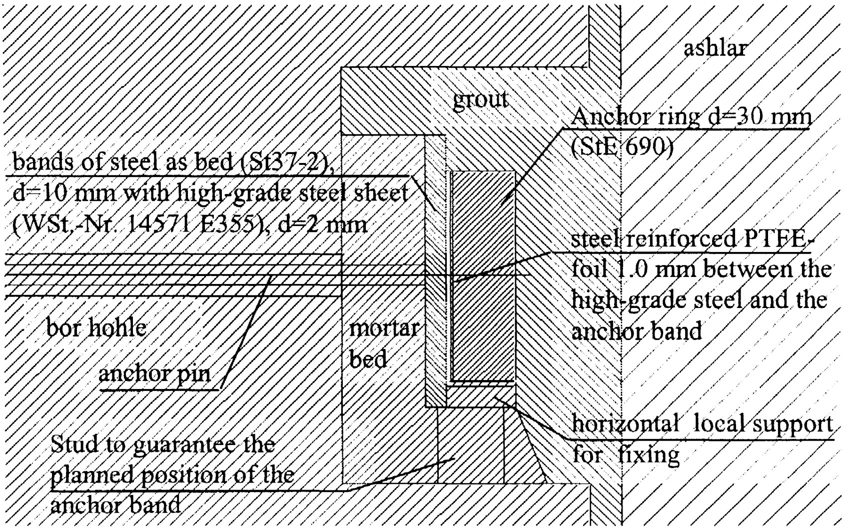

Figure 12: The cross section through the anchor bandage in the built in situation

After tensioning, the remaining straining channel will be injected with cement mortar, filling in the spaces between the Teflon shims. The solution created in this way is characterised by its high durability.

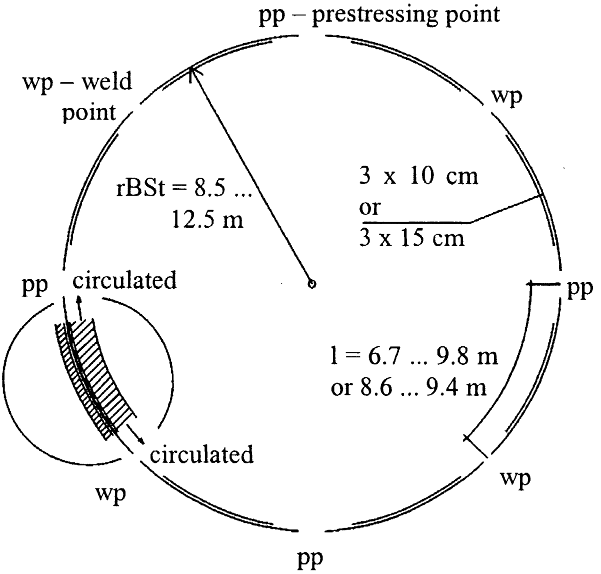

Figure 13: The division of the whole anchor ring in single elements

The anchor bands will be divided into 8 single elements in accordance with manufacturing possibilities. Two parts of the curved single elements will be welded on the building site. The four parts of the anchor ring thus arising will be built into the designated location. At the connecting points, the single bands will be connected with screw bolts after prestressing. It was decided to use butt strap joints with high-strength screws for coupling the parts of the anchor bands.

7 Conclusion

The archaeological idea of reconstructing the church will be completed with the construction of the dome in Saxon Sandstone with steel anchor rings. The only additions from our time are the moderate prestressing and the technologically improved implementation. That's the main aim of the engineers responsible, to reconstruct the church in its original structure with small, unobtrusive improvements arising out of our present-day superior knowledge of engineering. The church could have stood about another 200 years without collapsing, which was to the credit of the builder master and his assistants. The structure collapsed in an extreme situation due to the fire which occurred. For that reason, the pillars have been improved in the reconstruction by using a selected stone material of a high quality and by employing thin mortar joints.
That is why we cannot change the structure in the principle. It is therefore logical to re-erect the sandstone cupola in its original form with the added

benefit of our current engineering knowledge and subject to higher safety requirements than before.
The construction of the cupola above the tambour drum will begin in the year 2001 and will be finished in June 2004 with the topping-out ceremony of placing the cross on the top of the lantern.

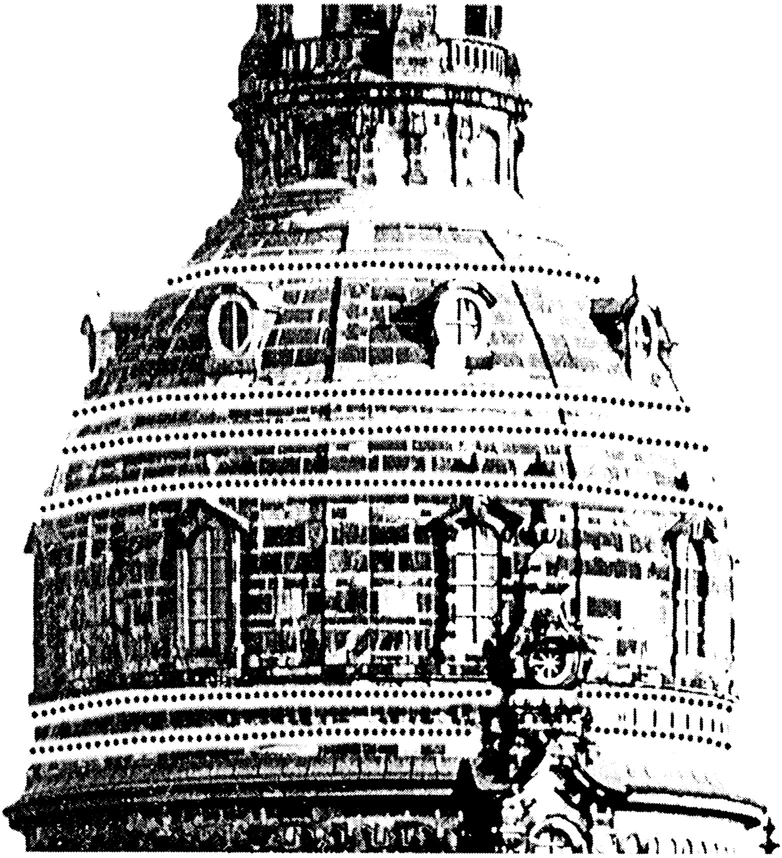

Figure 14: View of the cupola with the highlighted location of the anchor rings

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