Reconstruction of the Frauenkirche Dresden, structural proof-checking using a complete 3D FE-model

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Abstract

The application of the Finite Element Method for the analysis of complex masonry structures is illustrated using the reconstruction of the Frauenkirche in Dresden as an example. In context with the proofing procedure this analysis method is applied to establish completely independent comparision calculations. In a model of half of the church all major structural components as cupola, wall- and tower-buttresses, pillars and choir are included. This complete model allows the checking of the load path and the complicated geometry as well as several parametric studies. The effects of different structural and foundation stiffness ratios can also be investigated. This contribution shows that the chosen calculation method using a complete three dimensional system, which first of all seems to be an unusual approach in relation to a masonry building is, however, a substantial tool for such comparison calculations. These complete independent comparison calculations cause a considerable amount of work which are nevertheless appropriate regarding such a significant building.

1 Introduction

In October 1992 the responsible governmental authority for planning laws and building regulations of the capital Dresden entrusted Professor Dr.-Ing. Jörg Peter with the proofing work for the reconstruction for the Frauenkirche in Dresden. The task of a proof engineer covers the checking of the structural statical design, which comprises the design calculations and the execution drawings, worked out by the commissioned design engineers and architects - which are the engineering joint venture Prof. Jäger, Radebeul/Prof. Wenzel, Karlsruhe as well as IPRO Dresden - in order to guarantee

with respect to public interest above all the stability of the building. Besides this statutory task the proof engineer has, in this case, also advisory functions and is therefore a member of the chief planning group consisting of representatives of the client, the engineers and the architects. Therefore Prof. Peter was included in the basic considerations for the design of a conclusive structural concept in the early stages in 1992. At the same time the question arose on how to check the submitted design documents. It was decided to carry out completely independent comparison calculations using a 3D FE-model. This made it possible to check not only the chosen statical systems and stresses but also the complicated geometry of the baroque church.
An approximate statical analysis by Jäger/Wenzel (in 1993) proofed that, on the basis of today's technical knowledge and established structural design rules, the reconstruction of the Frauenkirche can be realized by using a certain extend of historical building material and by following closely the original drawings of George Bähr.
Own reflections and comparison calculations confirmed already at this stage the correctness of the approach regarding the structural concept for the reconstruction of the Frauenkirche [1].

Within the context of the approval design Jäger/Wenzel presented extensive investigations and calculations for proofing. According to the three main structural parts the analysis is divided into
- main cupola
- pillars with wall- and tower-buttresses (Spieramen)
- foundation.

In addition to calculations 'by hand' several FE-programs have been used by Jäger/Wenzel. The calculations of the main dome and the foundation were done with the program system ANSYS, partially considering a non-linear approach. In the final design calculations the Spieramen are represented in a 3D-model using a program by the software-house MÜCKE. The 'ring effect' is hereby simulated through radially positioned springs.

2 Complete FE-model

The 3D program system 'Finite Elemente' of the INFOGRAPH Corp. Aachen has been applied for the comparison calculations. Figure 1 shows the modified model, according to the latest knowledge about geometrical proportions, representing half of the church with all major structural parts from the level of ±0,00 m to + 62,40 m. Nonsymmetry in the overall system, caused by the one-sided choir, is directly taken into account.

The cutaway view (figure 2) of an earlier model shows how the main cupola is seated with the outer shell on the Tambour and with the inner shell on the inner cupola. One tower buttress, one wall buttress and one pillar form, in a Y-shaped position, a so-called Spieramen.
All comparison calculations are performed using linear-elastic material properties. This assumption is justified, as long as there are no cracks or open joints in the masonry sections. Small local zones with concentrated principal tensile stresses, e.g.

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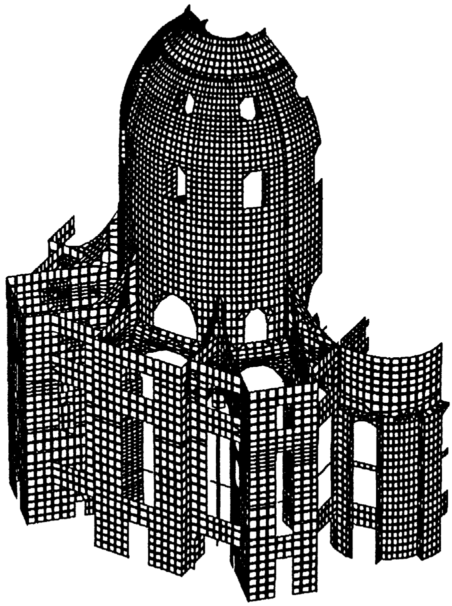

Figure 1:
Finite-Element-mesh of half the structure used in the analysis

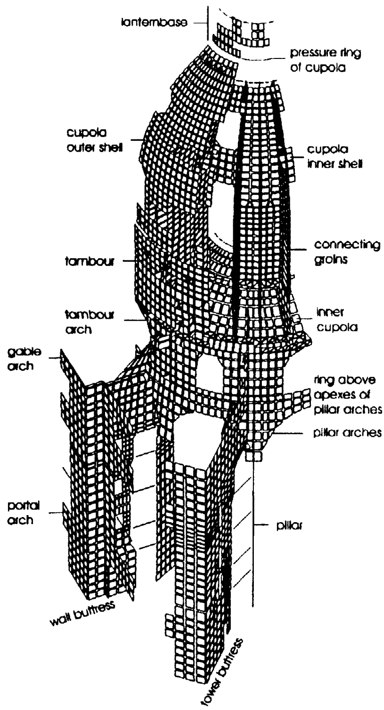

Figure 2:
Finite-Element-mesh, partial 3D-view, designation of main structural parts

within the area of openings have to be considered seperately. However, they don't affect the overall behavior. Regions with extensive tension zones, which inevitably would lead to cracks in the masonry, are made 'tension-resistant' by applying additional means e.g. placing of steel anchors.

2.1 Main cupola

In a first phase, comparison calculations for the main cupola had been performed on a partial system, which comprised an eighth of the cupola beginning at the lower edge of the inner cupola at a level of + 32,62 m up to the lantern base [1].
In the second phase the finite element analysis comprises the complete system in order to represent the load path of the main cupola onto the supporting Spieramen.
As the cupola built by Bähr, the new sandstone cupola will have ring anchors which will resist the hoop tensile forces. According to the design of Jäger/Wenzel the original wrought-iron ring anchors are substituted by 6 prestressed flat steel anchors whose positions are shown in figure 3. The figure gives also in a vertical section a comparison of the distribution of the normal forces Nx in hoop direction with and without prestres-

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sing. Figure 4 shows in a 3D-view these forces for the loading case 'self-weight' on the left side and 'self-weight + prestressing' on the right side. The resulting hoop tensile forces between the window openings of the cupola and at the upper and lower edges of those openings will be compensated by applying the chosen level of prestress, that means a moderate compressive stress will remain in circumferential direction. The right side of figure 4 shows the corresponding amounts of the prestressing forces for the different sections. They are introduced into the analysis through the application of external forces, which are acting as uniform distributed loads in radial direction (dotted areas). Effects of creep, shrinkage and the temperature strains of the flat steel bands are considered.

The resultant of the normal forces in the meridional direction stay inside the core, that means there are no tensile forces in this direction.

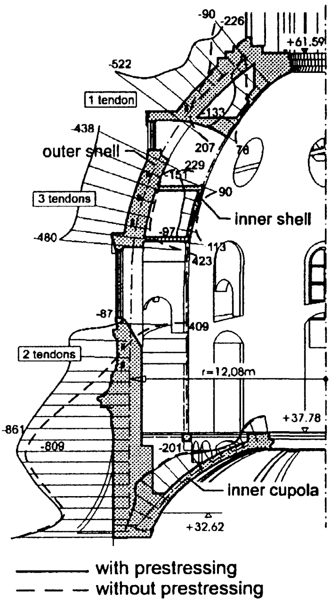

Figure 3:
Vertical section of main cupola in tower axis C, normal forces Nx in ring direction [kN/m]

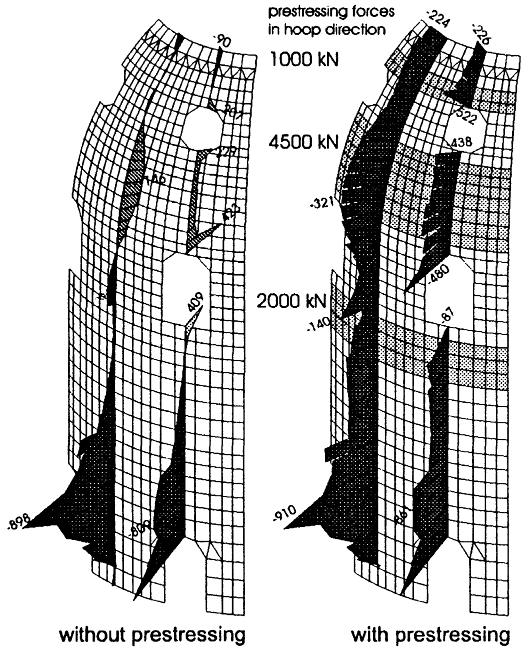

Figure 4:
Comparison of normal forces Nx in ring direction of outer shell [kN/m]

Below the Tambour the main cupola breaks up into eight Tambour-arches, which are seated onto the pillar-arches at the level of the gallery (+ 26,42 m), which itself merge into the eight inner pillars at the level of + 18,37 m (see figure 5). The statical concept by Jäger/Wenzel provides a polygonal steel tension ring besides the pillar-arches at a level of +25,50 m connected to 16 radially placed tension members, which are fixed at the far ends at the anchor blocks, in the wall- and tower buttress (figures 5, 6, 7 and

8) to correct the load path in comparison to the design by George Bähr (see also chapter 2.2).

The positive effects of this tensile ring anchor (main tension ring) on the supporting structural parts below the dome and in particular onto the masonry ring above the apexes of the pillar-arches have already been mentioned in [1–3]. Contrary to the system shown in [2] and [3] the almost polygonal shape of the pillar-arches (horizontal projection) has been introduced into the FE-model. The inner edge merges from a polygonal 8-corner-shape at a level of + 18,37 m into a circular shape at a level of + 26,42 m. The outer edge remains polygonal.

Figure 5 shows the view of three pillar-arches and the corresponding tambour in the direction of axis B, that is directly on the great arch above the southern main portal with a free span of ca. 9,90 m. In figures 5a and 5b the principal stresses are shown for the loading cases 'self-weight without prestressing of the main tension ring' and 'self-weight with prestressing of the main tension ring'. The prestressing force of the steel ring is introduced through initial strains in this ring, which is modelled in the 3D-system by polygonal positioned members. These strains ar based on the forces determined by Jäger/Wenzel, 2440 kN for the wall buttress and 3440 kN for the tower buttress, which are acting in the direction of these walls. We can recognize that the principal tensile stresses (bold trajectories) vanish almost completely. Only limited local zones with low tensile stresses remain over the pillar-arches. The actual polygonal shape of the masonry 'ring' corresponds to the lined support in vertical direction by the Spieramen-walls. That's why the bending moments, contrary to a circular shape of the ring, are considerably smaller over the vertical axis.

2.2 Spieramen

As already mentioned earlier one tower buttress, one wall buttress and one pillar form a so-called Spieramen. The vertical section in axis B (figure 6) shows as an example the elevation of a wall buttress with the FE-mesh in the background. The Spieramen-arches between the walls and the pillar are modelled using truss-elements. The size of the openings are in accordance to the execution drawings. The anchor block for the radially placed tension member is located at the upper part of the wall buttress. The external horizontal supporting force, which is activated by prestressing of the main tension ring, is introduced into the structure through this anchor block at the level of + 25,50 m.

Masonry stresses in vertical direction are shown in figure 7 for different sections. Hereby the self-weight, live loads and the influence of the settlement mould of the foundation are taken into account. Effects from temperature variation within the tension ring have to be superimposed onto these values.
The real stresses in the masonry sections of the pillars are smaller than the 'admissible stresses', which are in the range of appr. 6,0 to 8,5 MN/m2. Therefore there is an adequate safety margin e.g. in case of greater differential settlements.

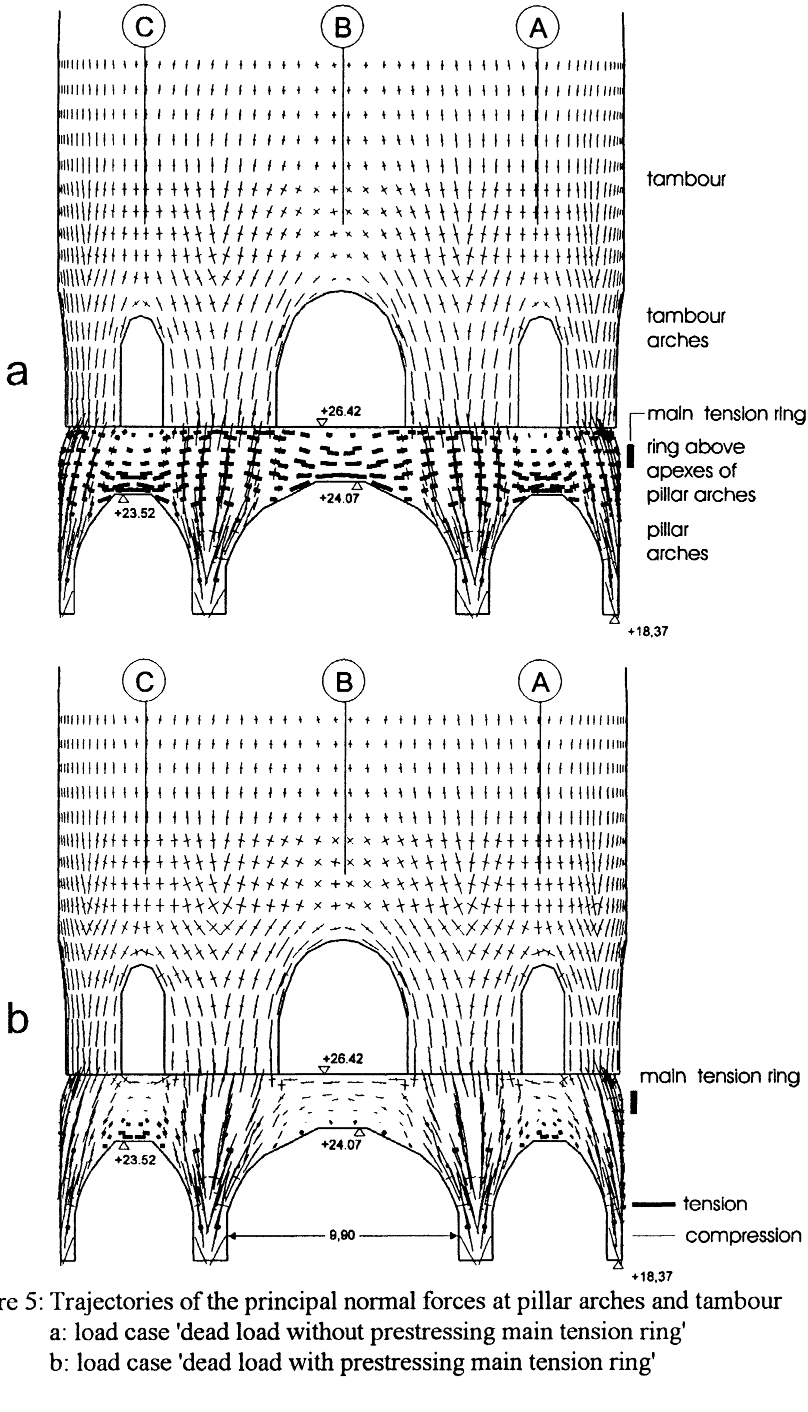

Figure 5: Trajectories of the principal normal forces at pillar arches and tambour
a: load case 'dead load without prestressing main tension ring'
b: load case 'dead load with prestressing main tension ring'

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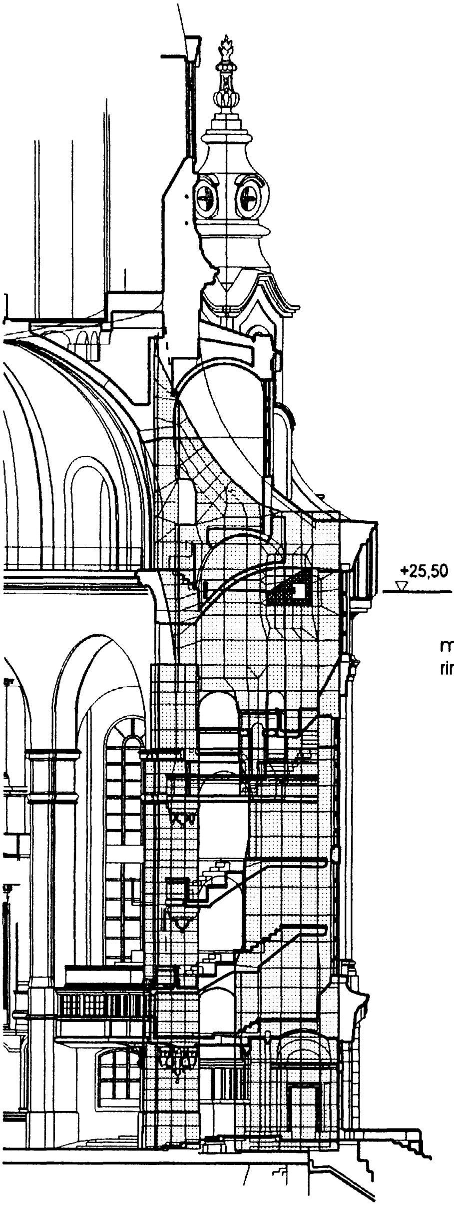

Figure 6: Vertical section at axis B with elevation of wall buttress, FE-mesh in the background

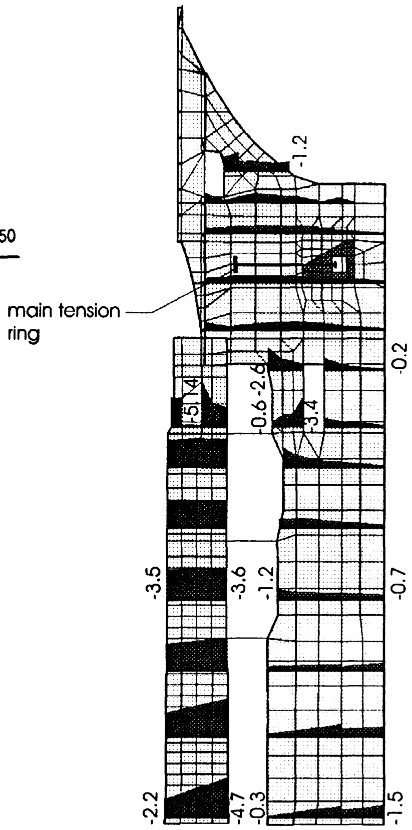

Figure 7: Vertical stresses σ_y [MN/m²] in wall buttress, load case: 'dead load with prestressing of main tension ring'

A considerable significance is attached not only to the pillars but also to the head of the Spieramen. Extensive damage due to cracks had been investigated earlier in this area. In order to indicate the cause for the occurrence of cracks, the stresses in horizontal direction are shown in figure 8 for the loading cases 'without/ with prestressing'. As a result of the horizontally prestressed main tension ring figure 8b reveals that the tension forces disappear in the upper part of the wall buttress.

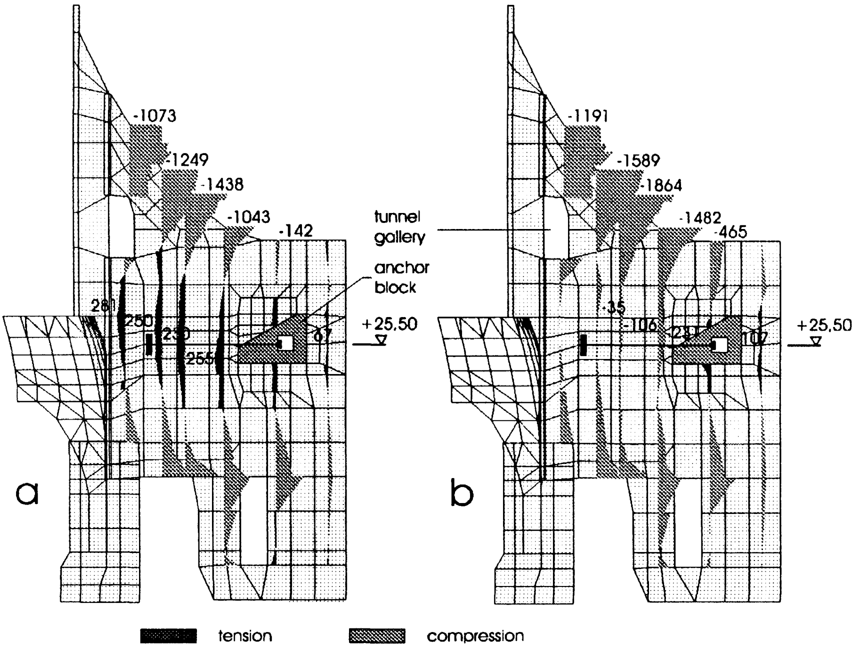

Figure 8: Head of "Spieramen" as upper part of the wall buttress, horizontal normal forces N_X [kN/m]
load cases:
a: dead load without prestressing of main tension ring
b: dead load with prestressing of main tension ring

In the remaining masonry section above the opening of the tunnel gallery, a diagonal compressive strut appears between the inclined upper surface of the anchor block and the tambour. The masonry joints are built correspondingly and are therefore inclined. Detailed comparisons between the results by Jäger/Wenzel and the comparison calculations show good agreement and confirm the design for the action of the inclined compressive strut above the tunnel gallery.

The sufficient agreement between the comparison and the design calculations is shown exemplary in figure 9 for the masonry stresses at the top level of the foundation (± 0.00 m). The columns of the tables show under II the values determined by Jäger/Wenzel and under I the comparison values. In order to determine the stresses the

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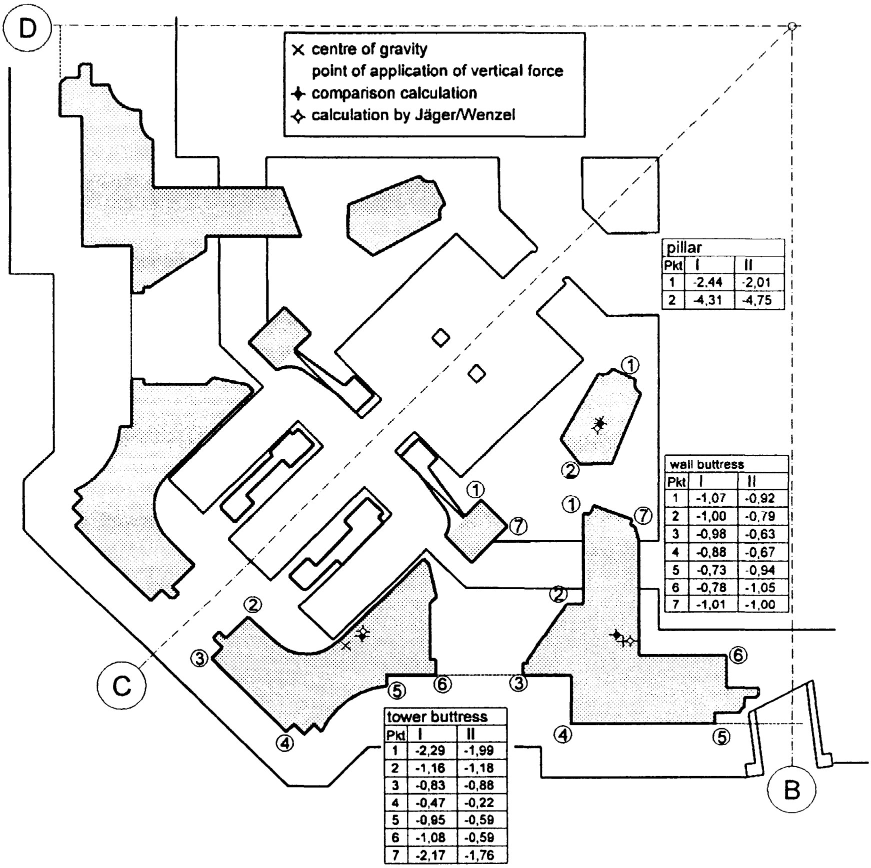
internal forces at the nodes in the FE-system are added. Subsequently the local position of the resultant forces is calculated. These values are used in the actual section to determine the stresses due to normal forces and bending moments.
The masonry stresses show an overall good correspondence even if at some points the values differ slightly higher. Independent of the chosen analysis method the calculated stresses can however be resisted by the masonry sections with a sufficient safety margin.



pillar		
Pkt	I	II
1	-2,44	-2,01
2	-4,31	-4,75

wall buttress		
Pkt	I	II
1	-1,07	-0,92
2	-1,00	-0,79
3	-0,98	-0,63
4	-0,88	-0,67
5	-0,73	-0,94
6	-0,78	-1,05
7	-1,01	-1,00

tower buttress		
Pkt	I	II
1	-2,29	-1,99
2	-1,16	-1,18
3	-0,83	-0,88
4	-0,47	-0,22
5	-0,95	-0,59
6	-1,08	-0,59
7	-2,17	-1,76

Figure 9: Comparison of points of application of vertical forces and vertical stresses in the masonry at the top level of the foundation [MN/m^2]

3 Foundation masonry

In order to check the presented calculations by Jäger/Wenzel regarding the foundation of the church, the foundation masonry is idealized as a continous slab. The FE-model hereby consists of pure plate elements (figure 10). In the region of the main cellar, the burial chambers and the choir cellars a reduced idealized slab stiffness was used. The analysis by Jäger/Wenzel modelled the foundation resting on a 10 m deep soil layer extending 10 m beyond the outer edge of the foundation. The comparison calculations were done by using the deformation modulus. The calculation model does not allow to define deformation conditions along the axis of symmetry, therefore the whole slab had to be generated as FE-model.

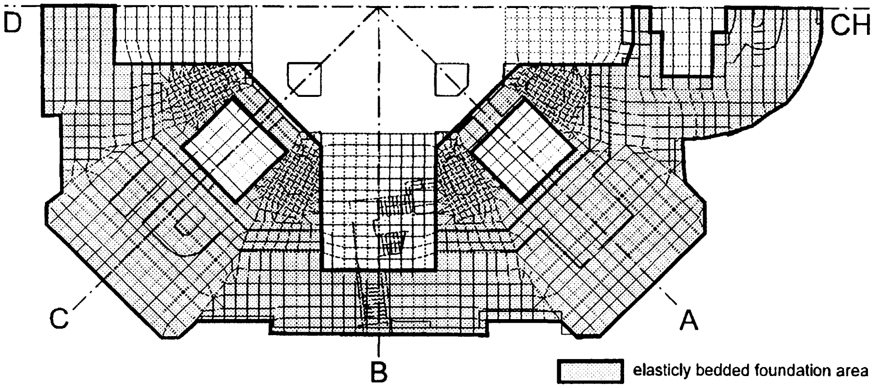


Figure 10: Finite-Element-mesh of foundation masonry, Represenation of the southern half

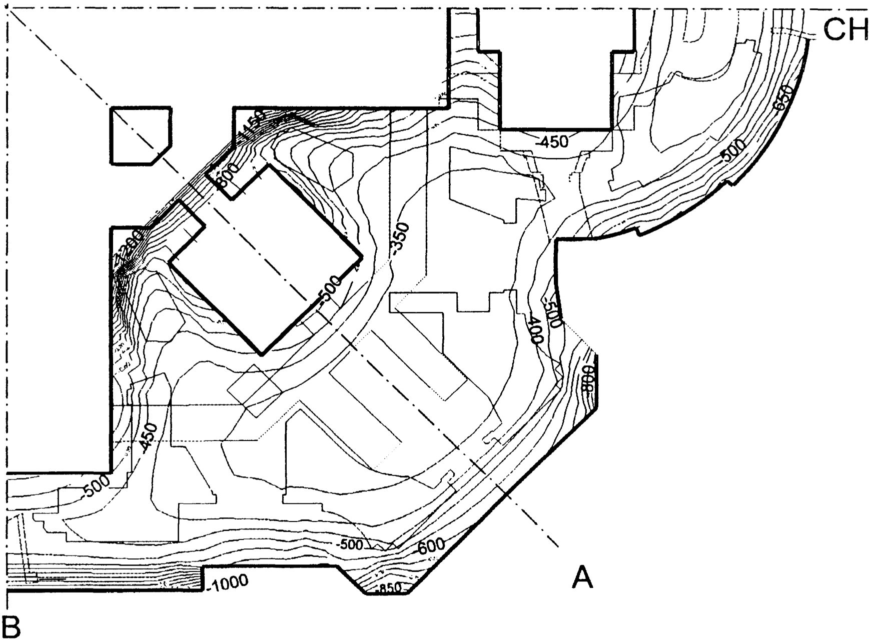


Figure 11: Soil pressures [MN/m²] at foundation level –7,15 m, load case ' dead load including foundation and prestressing of main tension ring'

Figure 11 shows the soil pressures in the foundation level using contour lines. The foundation dead load is hereby considered. The analysis shows that the admissible soil pressure of 1000 kN/m^2, given by the soil report, is reached only at corner areas, especially towards the middle of the church. The high local soil pressures at outsticking corners, which have been determined by the analysis will be reduced or redistributed such that the full value would not appear in reality.

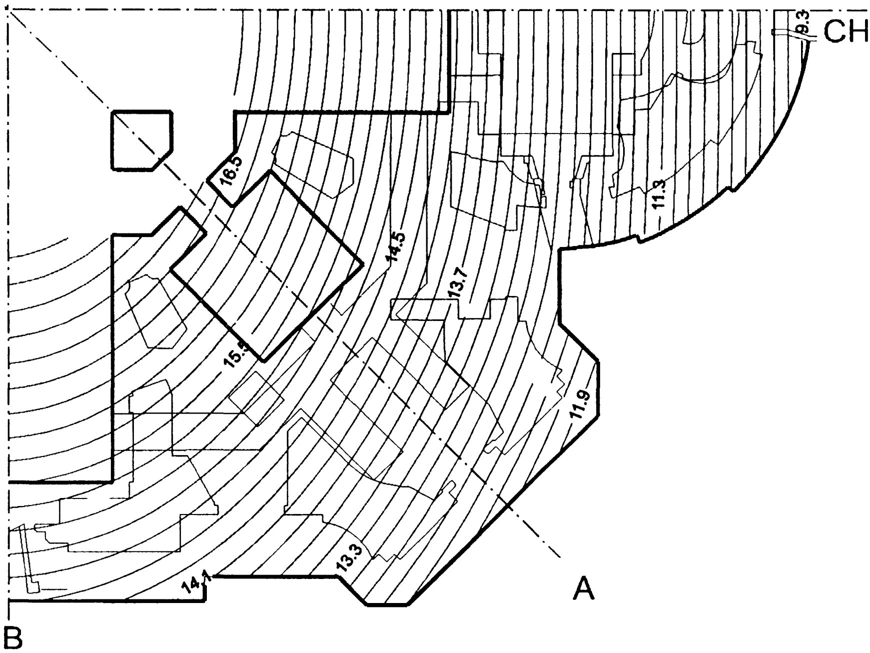


Figure 12: Settlements [mm] at foundation level –7,15 m, load case: ' dead load without foundation and prestressing of main tension ring'

Considering elastic-plastic material properites, Jäger/Wenzel determined pressures slightly under 1000 kN/m^2. Besides meeting the requirements of admissible soil pressures also the calculated settlements are checked. Figure 12 shows the contour line presentation of the relevant settlements for the reconstruction without considering the foundation dead load. The resulting "settlement mould" is clearly visible. The average settlement is, according to the comparison calculation and the analysis by Jäger/Wenzel, appr. 15 mm.

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