The typology of three-centre masonry vaults in the Venetian tradition. A method of analysis

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Abstract

The need to check the static and mechanical behaviour of arched masonry structures, widely used on the Venetian mainland throughout the nineteenth century, requires investigation of the relationship between technological aspects - the choice and use of materials, the use of temporary works, intuition and devices coded by tradition - and structural problems. The method proposed makes use of diagnostic investigations on the site and of historic research into the constructive principles, the rules of the craft, coded by technical treatises, for determining the surrounding conditions as support for the methods of calculation. The former make it possible to arrive by intuition at the empirical rules used by generations of master builders, the latter to assess their validity and limits in the light of scientific knowledge.

The paper presents the first results of the research.

1 Introduction

There is a dual interest in the study of the low-profile masonry arch with three centres: on one hand the widespread use of this model in constructive tradition in Vicenza, probably imported by craftsmen from the Lugano area and coded by Palladio1, on the other the need to check its behaviour during use and under breaking stress. Among the many examples present on the Venetian mainland, the writers investigated some arched structures located in the historic centre of
Vicenza and in the wool-manufacturing town of Schio. For greater clarity of explanation, the cases studied are listed with reference to a conventional numerical code, for the purpose of critical comparison.


Structure 2: Convent complex of S. Biagio, 1522-1580, Vicenza: pavilion vault with lunettes, rooms of the first cloister - south-west wing.

Structure 3: Fogazzaro residence and mill, 1805-1809, Schio; barrel vault, main hall - basement floor - central span.

These structures were investigated, to various extent, in schedules that combined a sequence of direct analyses, metrical and stratigraphical measurement, checking the static behaviour and stress according to the monitored physical and geometrical parameters.

2 Technological characteristics and mechanical behaviour of the masonry vault with three centres

The structural typology investigated belongs to the category of thin masonry vaults made of bricks joined with mortar with a lime binder. From the mechanical point of view they present different cracking mechanisms depending on the method of construction. The characteristic behaviour of these structures is to adapt the formal geometry, in the event of kinematic phenomena involving the piers, or as a result of stress which generates pressure curves in the individual voussoirs outside the middle third - the limit state of cracking - or at the edges of the section - limit state of tilting - or, finally, a greater inclination, with respect to the normal inclination of the voussoir, of the angle of friction of the masonry - limit state of slipping.

They are therefore stable and safe constructions, only in working conditions similar to historical ones. They show noticeable limits on dynamic action, rapidly reaching collapse in the case of horizontal thrust induced by seismic phenomena, due to the disconnection and disjointing of the constituent parts. It was due to the earthquakes in the early years of this century that their use was abandoned, with the consequent gradual loss of the technical and technological knowledge acquired in the traditional building site, now completely forgotten.

2.1 Cataloguing the vault elements

The formal type may be defined as a set of parts, each one of which has the task of performing precise static and mechanical functions by means of its dimensions, geometric characteristics and material composition. In the most general case, 4 elements may be distinguished: piers, voussoirs, backings and fillings.

In the type investigated, the technological and scientific importance of the pier element is generally negligible. Venetian tradition uses the polycentric type with a low profile, generally covering underground environments in constructions with a masonry framework organised in a tripartite pattern. This is the case of
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Structures 1 and 3, both floors of bodies with a single span in the former and of three spans in the latter. The absorption of horizontal thrust, one of the warring thoughts of pre-modern builders, is in this case carried out by the ground for the external perimeter spans, and is cancelled in the internal spans, due to the symmetry of the load and structure, typical in tripartite buildings. The oversizing of the piers, which in this case are real substructures of the entire building, is implicit in the larger sections with which the masonry foundation structure is built (amounting to 82 cm in structure 3), and in the use of materials with a greater weight per unit volume, such as rough-hewn calcareous stones or actual courses of stone laid with an orderly bond. Finally, the springing lines of the foundation structures are generally terrains with acceptable geotechnical parameters, consolidated and not subject to yielding in normal load conditions.

In all three cases studied the vault has a low profile with three centres, with a rise-span ratio of 1/3 (1), 1/5 (2) 1/4.5 (3) and a mean span of 606 (1), 593 (2) and 647 cm (3), longitudinal development 912 (1), 594 (2) and 1250 cm (3). The radii of curvature are in the ratio $r_1/r_2/r_3$ of $1-2.9-3.3$ with $r_1 \approx 130$ cm (1), 1-2-10 with $r_1 \approx 120$ cm (2-3). The thin vault is built with a uniform thickness equal to a head of bricks, 14-15 cm, laid on edge with the larger dimension, 25-28 cm, parallel to the longitudinal axis, in offset courses. Structure 3 shows an increase in thickness (14+14 cm) corresponding to the change in curvature. The lime mortar joints are in all cases on the intrados, with a thickness of between 1.5 and 2 cm, and they increase considerably on the extrados.

With the aid of endoscopic techniques (for which see Chapter 3 below), the dimensional characteristics and the material composition of the backings were identified, with the characteristic curvilinear form of the intrados and with discontinuity of the extrados due to the uncertain bond and to the use of heterogeneous debris, compacted with lime mortar. Though they do not form homogeneous spurs of the vertical masonry skeleton, they may be considered integral with the pier on account of the recurrent scarfing, assisting it in the absorption and in the vertical organisation of the resultant of the pressure curve. We may therefore assume a physical springer height greater than the height of the pier measured at the intrados, respectively 100 cm (2) and 91 cm (3).

The fillings are all made of non cemented materials of various kinds and compositions, with a prevalence of aggregate with a small diameter, 3-4 mm, calcareous and brick, and waste materials with a low specific gravity.
Figure 1. Geometrical comparison of structures 1-3 and reconstruction of the profile of structure 3 according to Temanza's design (1733)

Figure 2. Technological section of structure 3 and vault's parts indication
2.2 Intuitions and contrivances of the historic building site in search of stability

The search for equilibrium in the pre-modern building site was entrusted to the intuition of the behaviour of the thrusting structure. The only sizing criteria that were known and handed down, based on geometrical procedures, were functional more for constructive requirements the for the transmission of forces to the ground through a structure with a resistant form. Though they had no scientific experience, generations of craftsmen continued to build structures that made up for their lack of theory in statics with continuous experimentation, to the extent that they drew up a code of rules that were valid within the narrow interval of the variable factors involved: rise-span ratio, springer height, thickness of the resistant radial sector, transverse section of the piers, choice and use of materials. The construction of a vault required the use of temporary works, of which the constructive technique complied with precise control objectives in the delicate phase of striking. Centring was not limited to a passive aid in the construction phase, but became a tool for checking the stability of the fault at the time of slackening the temporary supports, so that it was possible to take corrective action while work was still in progress.

Figure 3. Example of centring: tightening and slackening mechanisms

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The explanation of how centring works is closely linked with the mutual relationship that exists between the elements that compose it: a vertical framework, transverse primary frame, made of planks shaped according to the desired geometrical form, and a secondary longitudinal frame composed of nailed boards or lathing. The primary frame is anchored to the ground with flat laid boards which, by means of counterposed wedges, regulate the slackening or tightening of the centring springer piers. At the time of striking, which implies an operation lasting several days, any unbalance was corrected without the collapse of the vault. Site experience had consolidated the use of contrivances: the decrease of the span by constructing shaped piers, the increase of the resistant thickness of the arches that composed the vault, the increase of loads on the extrados of the sections between the bearings and the reins with the construction of backings, or lightening the loads exerted on the crown voussoirs, making use of fillings of loose material. All these elements are found in the examples considered. In addition to the solutions listed we must mention the use of wooden struts inserted between the reins and the piers, with the aim of generating a state of coercion on the sections that were forced to rotate to form hinges on the intrados. All these contrivances vouch for the intuitions of the mechanical behaviour of the vaulted structures and arches. Barrel vaults with a low profile present typical cracking mechanisms, characterised by lesions on the crown of the intrados and on the reins of the extrados.

3 Methodological tools

Work on existing structures is often justified by structural tests which limit the analysis of the behaviour in the present state or the expected project behaviour to numerical data. The calculation models were formulated on general basic assumptions and require approximate hypotheses, the validity of which must be checked and must correspond to the physical behaviour of the structure. The determination of the surrounding conditions and the knowledge of the characteristic values examined becomes less of a foregone conclusion for pre-modern structures, whose complexity often defies simplified systems of analysis. This methodological contribution aims to review a resistant type, the polycentric vault, in the light of technological tools, providing an articulated, preventive cognitive criterion as a support for numerical checking. To conclude, the first results of the application of the method are given. The structures were investigated with a metrical survey, planned by subdividing the profile of the intrados into voussoirs, each of which was composed of 3 or 4 bricks (3), proceeding from the pier to the crown; for plastered structures (1 and 2) we applied the criterion of subdividing the span into regular intervals of about 50 cm each. For each voussoir or arch the heights were measured with respect to a common horizontal reference, which is necessary in conditions of imperfect flatness, typical in historic buildings. The broken lines obtained were interpolated so as to calculate centres of curvature and radii which approximate the real profile with greater precision. The geometrical models were compared
with the coded patterns drawn from the technical treatises, obtaining a reading of the evolution of the polycentric type between the sixteenth and the nineteenth century, characterised by the use of gradually smaller curvatures. The stratigraphic survey of the parts made use of the endoscopic technique (3) on holes with a small diameter, made in the intrados, at regular intervals, on the resistant radial crown and, where conditions allowed it (2), with some tests on the extrados, carried out on samples, and checking of the backings and fillings. In this way it was possible to reconstruct the typical technological sections and to identify the precise position of the section in which the resistant thickness changes. To assess the presence of kinematic phenomena and deformation, the cracking picture was analysed. In case 3, monitoring revealed localised deformation near the reins, developed along the whole directrix without lesions on the intrados; this later allowed an interpretation of the results obtained with the analytical check. The visual analysis of the joints with lime mortar did not reveal a loss of cohesion of the material, excluding the hypothesis of "spontaneous" failures which occur when the resistance of the mortar tends to be nil. The cognitive data obtained were organised in a system and allowed the determination of the physical model for analysis: the span and the thicknesses of the real thin vault, the component materials, the pattern of constraints which best approximates the working behaviour of the structure.

Figure 4. Typical technological section of structure 3 and endoscopic investigation
After having completed the geometrical and stratigraphic analysis and the analysis of the cracking picture, a number of experimental tests were carried out on the state of stress exerted on the vaults and on their tensional response. The flat jacks diagnostic technique was used, checking the value of the mean axial tension \( \sigma_{\text{mean}} \) with a rectangular single jack, only for the station corresponding to the springer measured (structure 3), that is for the section subject to most stress. The experimental figure obtained was used as a reference value and for comparison with the numerical values of the analytical modelling. In the cases studied, the behaviour of the vaulted structures was compared to the three-hinge pattern both for the pavilion type (2) and for the barrel type (1 and 3). The heaviest loads for the stability of the structure were assessed, distinguishing the contributions by the dead load of the components, by any accidental loads in the present state and in the hypothesis of the future working life.

The calculation models used were the following: Mery's graphic method\(^6\) for checking stability and Heyman's plastic method\(^7\) for checking safety. The technological pattern obtained enables us to define the surrounding conditions and to check the basic hypotheses of the two methods. Of the three cases studied, an in-depth analysis was made only for structure 3. Mery's need to define the pattern of restraints a priori is satisfied in the interpretation of the position of the hinges found. In both methods the determination of the loads involved and the definition of the geometry of the profile need special investigation, respectively: stratigraphic survey, to check the weights per unit volume of the component parts, and topographic survey. The analysis of the cracking picture simplifies the search for the plastic hinges by trial and error in Heyman's hypotheses, and it is a useful tool for checking Mery's hypothesis. The application of the graphic method does not satisfy the condition of stability for the geometrical model used, which interpolates the broken line found. But if the pressure curve is superimposed on the real profile, while the condition for failing within the kern is still not satisfied, the lie inside the edges of the intrados and the extrados is checked. An explanation is therefore found for the localised deformation between the springer sections and the reins along the whole longitudinal development, which probably occurred on striking, as a consequence of the structure's need to achieve a balanced configuration by modifying its own geometry. The values obtained in the two methods, comparing the condition of the dead load alone with an overload of 50 N/m\(^2\), are summed up in table 1. with relation to the thrust values at the crown and on the bearing. The interpretation of the numerical data allows us to check that the structure is subject to much lower values of \( \sigma_{\text{compression}} \), 1/9, with respect to \( \sigma_{\text{breakage}} \), 3.1-4.6 N/mm\(^2\). The mean tensional values calculated, \( \approx 0.48 \) N/mm\(^2\), come close to the experimental measurements obtained with the flat jacks, \( \approx 0.45 \) N/mm\(^2\). But the sections between reins and the subject to tensile stress. This indication, together with the results of the plastic method with reference to the minimum dimension of the arch in the situation immediately prior to collapse, \( d=11.2 \) cm (dead load) and \( d=12.3 \) (dead load + overload), strongly suggests interacting with caution if the structure is to be reused. This type of structure does not present limit states of slipping, as in each section the angle
formed by the resultant of the pressure curve with the perpendicular never exceeds $6^{\circ}.25$ quite lower $\sigma_{\text{friction}}$ of the masonry.

### Table 1. Comparison of the thrust values (T) at the crown and on the bearing with the two methods

<table>
<thead>
<tr>
<th></th>
<th>$T_{\text{crow}}$ [N]</th>
<th>$T_{\text{bearing}}$ [N]</th>
<th>$T_{\text{crow}}$ [N]</th>
<th>$T_{\text{bearing}}$ [N]</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>DEAD LOAD</td>
<td>with OVERLOAD</td>
<td></td>
<td></td>
</tr>
<tr>
<td>MERY</td>
<td>607.2</td>
<td>677.8</td>
<td>938.1</td>
<td>1037.0</td>
</tr>
<tr>
<td>HEYMAN</td>
<td>538.6</td>
<td>610.0</td>
<td>819.9</td>
<td>924.0</td>
</tr>
<tr>
<td>$\Delta %$</td>
<td>12.7</td>
<td>11.1</td>
<td>14.4</td>
<td>12.2</td>
</tr>
</tbody>
</table>

### 4. Conclusions

The results obtained in these few cases studied suggest that we should extend the method described to other examples in the Veneto area, to refine the parameters used in the analytical models and to plan pilot operations of recuperation while respecting the individual character of the structure.

The tests with flat jacks were carried out under the direction of the engineer Paolo Soardo of the Material Tests Laboratory of the Province of Verona.
References


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